

**COMPUTER SIMULATION OF VIBRATION
SURVEILLANCE OF DISCRETELY IDEALISED
MECHANICAL SYSTEMS.
PART I: OPTIMAL CONTROL
AT THE ENERGY PERFORMANCE INDEX
FOR OPEN NON-STATIONARY SYSTEMS**

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Abstract: The paper discusses means of mechanical vibration surveillance using elements of control theory, dynamic optimisation, modern methods of computer simulation and measurement techniques. A mixed method of rigid and flexible finite elements is applied to discrete modelling of non-stationary controlled systems. A method of vibration surveillance in the time domain is elaborated, which utilises optimal control at the energy performance index with respect to the trajectory of given motion. An example is given of surveillance of transient vibration of a railway freight wagon weighing device.

Keywords: dynamics, control, simulation, railway weighing systems

1. Introduction

Due to the importance of dynamic phenomena for the behaviour of mechanical structures, looking for suitable means to control dynamic phenomena is the subject of many research projects concerning the development of methods of simulation and control of dynamic systems, robots and manipulators, servo-drives, lifting devices, railway truck pantographs, vehicle suspension and vehicle motion, as well as metal cutting tools and the accompanying machining processes.

By creating a discrete model of a real mechanical system we can describe dynamic phenomena with the aid of a finite number of ordinary differential equations. Thus, a discrete model becomes a calculation model. One of discrete modelling approaches is a mixed method of rigid and flexible finite elements [1–3]. Its range of applications has been extended by taking into account time changes in its configuration, caused by relative motions of several subsystems [4]. Additionally, inclusion of the coupling element in the model has brought about new possibilities in the scope of modelling of mechanical closed-loop systems (*e.g.* cutting process interaction) and

a convenient connection of substructures of a system whose configuration changes with time [2–5].

A set of intentional activities, aimed at securing the desired performance of a vibration process, is called *surveillance*. The surveillance of vibration depends upon [3]:

- monitoring of physical quantities which affect the quality of surveillance (vibration level, amplitude of displacements, forces of interaction) [6],
- generation of instantaneous values of control commands, in accordance with a proper rule being applied (the so-called strategy of surveillance) [7].

In order to solve the problem of vibration surveillance effectively, the following stages are suggested to be applied [3].

- **Computer simulation.** It is a process of *on-line* solution of a calculation model of the system under surveillance, which is an effective tool supporting the surveillance process. In case the real structure does not exist, or when identification of dynamic properties of all the items is impossible, computer simulation is a tool providing information on the expected performance of a real process, as well as valuable indications concerning the synthesis of the system of surveillance.
- **Prediction** of possible results of the strategy of surveillance. It takes place when a computer simulation of the strategy of surveillance has been done and, subsequently, real performance of the strategy is expected.
- **Implementation.** It depends upon real performance of the predicted strategy of surveillance of the vibrating structure. It is a final step of vibration surveillance.

2. Optimal control at the energy performance index with respect to the trajectory of given motion [3]

Let us consider a problem of vibration control of non-stationary systems, which have been idealised discretely using the finite element method. In order to find an effective solution of the problem, a method is proposed of *on-line* optimal control at the energy (algebraic) performance index with respect to the trajectory of given motion. It has been developed on the basis of earlier research concerning an application of the methodology of optimal control for transient vibrations of stationary conservative systems [8]. It allows:

- minimal-energy formulation of the control problem with respect to the trajectory of given motion, in the time and frequency domains,
- elaboration of an effective computer simulation algorithm,
- transient vibration surveillance of non-stationary systems,
- vibration surveillance of closed-loop time-delay non-stationary systems.

Here is a dynamic equation of a controlled non-stationary system, described in generalised co-ordinates:

$$\mathbf{M}^* \ddot{\mathbf{q}}^* + \mathbf{L}^* \dot{\mathbf{q}}^* + \mathbf{K}^* \mathbf{q}^* = \mathbf{f}^* + \mathbf{B}_u^* \mathbf{u}, \quad (1)$$

where \mathbf{M}^* , \mathbf{L}^* , \mathbf{K}^* are matrices of inertia, damping, stiffness and control, respectively; \mathbf{q}^* , \mathbf{f}^* , \mathbf{u} are vectors of generalised co-ordinates, forces and control commands, respectively.

Dynamic Equation (1) can be rewritten for state co-ordinates as follows:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{D}\mathbf{z} + \mathbf{B}\mathbf{u} \\ \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{w} \end{cases} \quad (2)$$

where $\mathbf{x} = [\dot{\mathbf{q}}^* \quad \mathbf{q}^*]$ is the state vector of the system; $\mathbf{A} = \begin{bmatrix} -\mathbf{M}^{*-1}\mathbf{L}^* & -\mathbf{M}^{*-1}\mathbf{K}^* \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$ is the state matrix of the system; $\mathbf{D} = \begin{bmatrix} \mathbf{M}^{*-1} \\ \mathbf{0} \end{bmatrix}$ – the disturbance matrix; $\mathbf{B} = \begin{bmatrix} \mathbf{M}^{*-1}\mathbf{B}_u^* \\ \mathbf{0} \end{bmatrix}$ – the input matrix; \mathbf{C} – the output matrix; $\mathbf{z} \equiv \mathbf{f}^*$ – the disturbance vector; \mathbf{y} – the output vector, whose components are responses of the system being registered; \mathbf{w} – the vector of measuring noise.

Solution of the first of Equation (2) with the aid of the state transition method shall produce:

$$\mathbf{x}(t) = \Phi(t, t_0)\mathbf{x}(t_0) + \int_{t_0}^t \Phi(t, \tau)[\mathbf{B}(\tau)\mathbf{u}(\tau) + \mathbf{D}(\tau)\mathbf{z}(\tau)]d\tau, \quad (3)$$

where $\Phi(t, t_0)$ is the solution of the homogeneous differential equation:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}, \quad \mathbf{x}(t_0) = \mathbf{I}. \quad (4)$$

Now, let us define an energy performance index, which takes into account time-varying kinetic and potential energy, with respect to the trajectory of the desired motion (determined by the generalised displacement vector, $\bar{\mathbf{q}}$, and the generalised velocity vector, $\dot{\bar{\mathbf{q}}}$), as well as the instantaneous energy of the control command:

$$J(t) = \frac{1}{2}(\dot{\mathbf{q}}^* - \dot{\bar{\mathbf{q}}})^T \mathbf{Q}_1 \mathbf{M}^* (\dot{\mathbf{q}}^* - \dot{\bar{\mathbf{q}}}) + \frac{1}{2}(\mathbf{q}^* - \bar{\mathbf{q}})^T \mathbf{Q}_2 \mathbf{K}^* (\mathbf{q}^* - \bar{\mathbf{q}}) + \frac{1}{2}\mathbf{u}^T \mathbf{R} \mathbf{u}, \quad (5)$$

where \mathbf{Q}_1 , \mathbf{Q}_2 are matrices of dimensionless weighing coefficients; \mathbf{R} is the matrix of control command effect.

The vector of generalised displacement, $\bar{\mathbf{q}}$, is obtained as a solution of the following equation:

$$\mathbf{K}^* \bar{\mathbf{q}} = \mathbf{f}_0, \quad (6)$$

where \mathbf{f}_0 is the vector of non-potential generalised forces, which are loads in given motion. In particular case: $\mathbf{f}_0 \equiv \mathbf{f}^*$.

The generalised velocity vector, $\dot{\bar{\mathbf{q}}}$, is a time derivative of vector $\bar{\mathbf{q}}$.

Matrix \mathbf{Q}_1 determines the contribution of kinetic energy of vibration (of high importance in unsteady vibration surveillance), while matrix \mathbf{Q}_2 denotes the contribution of potential energy (essential in vibration surveillance of closed-loop time-delay systems).

A variation of the performance index (5) shall get the following form:

$$\begin{aligned} \delta J = & \frac{1}{2} (\dot{\mathbf{q}}^* - \dot{\bar{\mathbf{q}}})^T (\mathbf{Q}_1 \mathbf{M}^* + \mathbf{M}^{*T} \mathbf{Q}_1^T) \delta (\dot{\mathbf{q}}^* - \dot{\bar{\mathbf{q}}}) + \\ & + \frac{1}{2} (\mathbf{q}^* - \bar{\mathbf{q}})^T (\mathbf{Q}_2 \mathbf{K}^* + \mathbf{K}^{*T} \mathbf{Q}_2^T) \delta (\mathbf{q}^* - \bar{\mathbf{q}}) + \frac{1}{2} \mathbf{u}^T (\mathbf{R} + \mathbf{R}^T) \delta \mathbf{u}. \end{aligned} \quad (7)$$

If we introduce replacements, *i. e.*:

$$\dot{\mathbf{q}}^* = \underbrace{[\mathbf{I} \quad \vdots \quad \mathbf{0}]}_{\mathbf{T}_1} \mathbf{x} = \mathbf{T}_1 \mathbf{x}, \quad \mathbf{q}^* = \underbrace{[\mathbf{0} \quad \vdots \quad \mathbf{I}]}_{\mathbf{T}_2} \mathbf{x} = \mathbf{T}_2 \mathbf{x}, \quad (8)$$

we obtain:

$$\delta \dot{\mathbf{q}}^* = \mathbf{T}_1 \delta \mathbf{x}, \quad \delta \mathbf{q}^* = \mathbf{T}_2 \delta \mathbf{x}. \quad (9)$$

Now, we calculate a variation of expression (3), thus obtaining:

$$\delta \mathbf{x} = \int_{t_0}^t \Phi(t, \tau) \mathbf{B}(\tau) \delta \mathbf{u}(\tau) d\tau = \int_{t_0}^t \Phi(t, \tau) \mathbf{B}(\tau) d\tau \delta \mathbf{u}. \quad (10)$$

The purpose of optimal control is vibration surveillance, so we assume that the control does not affect the desired motion of the system. Thus, we assume:

$$\delta (\dot{\mathbf{q}}^* - \dot{\bar{\mathbf{q}}}) \cong \delta \dot{\mathbf{q}}^*, \quad \delta (\mathbf{q}^* - \bar{\mathbf{q}}) \cong \delta \mathbf{q}^*, \quad (11)$$

and a zero variation of the performance index:

$$\begin{aligned} \delta J = & \left[\frac{1}{2} (\dot{\mathbf{q}}^* - \dot{\bar{\mathbf{q}}})^T (\mathbf{Q}_1 \mathbf{M}^* + \mathbf{M}^{*T} \mathbf{Q}_1^T) \mathbf{T}_1 \int_{t_0}^t \Phi(t, \tau) \mathbf{B}(\tau) d\tau + \right. \\ & \left. + \frac{1}{2} (\mathbf{q}^* - \bar{\mathbf{q}})^T (\mathbf{Q}_2 \mathbf{K}^* + \mathbf{K}^{*T} \mathbf{Q}_2^T) \mathbf{T}_2 \int_{t_0}^t \Phi(t, \tau) \mathbf{B}(\tau) d\tau + \frac{1}{2} \mathbf{u}^T (\mathbf{R} + \mathbf{R}^T) \right] \delta \mathbf{u} = 0 \end{aligned} \quad (12)$$

As a product of transformation of Equation (12), the optimal control command has been derived as follows:

$$\begin{aligned} \mathbf{u} = & -(\mathbf{R} + \mathbf{R}^T)^{-1} \int_{t_0}^t \mathbf{B}^T(\tau) \Phi^T(t, \tau) d\tau \cdot \\ & \cdot \left\{ \mathbf{T}_1^T (\mathbf{M}^{*T} \mathbf{Q}_1^T + \mathbf{Q}_1 \mathbf{M}^*) \left[\dot{\mathbf{q}}^* - (\dot{\mathbf{K}}^{*-1} \mathbf{f}_0 + \mathbf{K}^{*-1} \dot{\mathbf{f}}_0) \right] + \right. \\ & \left. + \mathbf{T}_2^T (\mathbf{K}^{*T} \mathbf{Q}_2^T + \mathbf{Q}_2 \mathbf{K}^{*T}) (\mathbf{q}^* - \mathbf{K}^{*-1} \mathbf{f}_0) \right\} \end{aligned} \quad (13)$$

which minimises the performance index (5). It is a function of generalised displacements, \mathbf{q}^* , and generalised velocities, $\dot{\mathbf{q}}^*$, whereas instantaneous values of the \mathbf{u} command are determined *on-line*, during the control process.

A real process of control is characterised by the presence of certain constraints imposed on the control command. If constant-value constraints are introduced during *on-line* determination of the command values, the following inequalities will restrict these values:

$$u_i(t) = \begin{cases} u_i(t), & |u_i(t)| \leq U_i \\ U_i \text{sign}(u_i(t)), & |u_i(t)| > U_i \end{cases}, \quad i = 1, \dots, i_u, \quad (14)$$

where i_u is the number of components of control command vector \mathbf{u} ; U_i is a constraint on the magnitude of component u_i of control command vector \mathbf{u} .

The possibility to consider constraints during *on-line* process performance is a significant advantage of optimal control at the energy performance index. Earlier solutions of constrained optimal control, based upon the maximum principle of Pontriagin, did not have such a feature at all [3, 7].

3. Transient vibration surveillance of railway weighing platforms during freight measurement [3]

3.1. General characteristics

Obtaining reliable information about the weight of running rolling stock (the so-called dynamic weighing) is essential in weighing bulk cargo trains (*i.e.* trains carrying coal, ore, grain, fuel or fluid substances), without necessity of stopping the train. Modern weighing techniques during running are becoming more and more popular because of prompt availability of measurement results, as well as for commercial reasons concerning the monitoring of overloading axles and wagons. Contemporary weighing devices usually have three or four weighing platforms, and the weight of whole wagons is measured simultaneously while running along the platform (Figure 1).

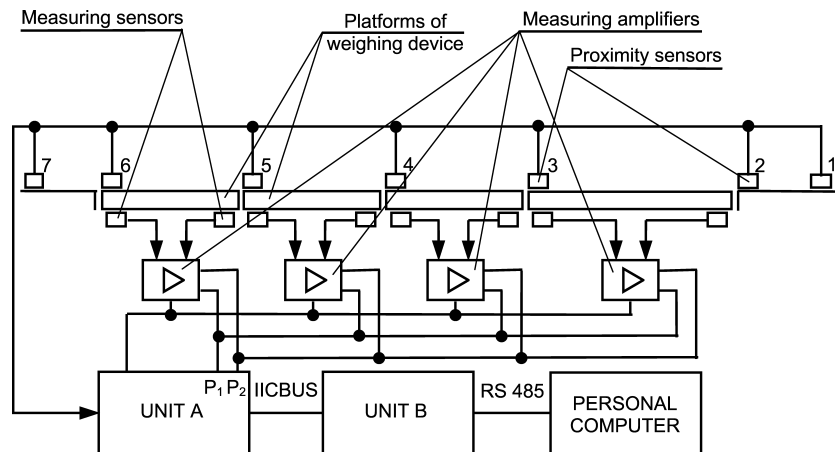


Figure 1. A scheme of 4-platform railway weighing device [9]

An important problem in the use of the railway weighing devices concerns unsteady vibrations of operating platforms. The time of vibration suspension is usually so long, that the weight measurement of a running wagon with a desired accuracy (*i.e.* about 0.1%) may be interrupted. In order to decrease the time of vibration suspension, a method of vibration surveillance by optimal control at the energy performance index has been proposed. Moreover, the contribution of the cost of the surveillance system in the total cost of the weighing device is negligible, so that its application is reasonable also from an economic point of view.

Forces of interaction of the actuators placed under the platforms have been treated as control commands. Instantaneous values of these forces are generated with a desired sampling time.

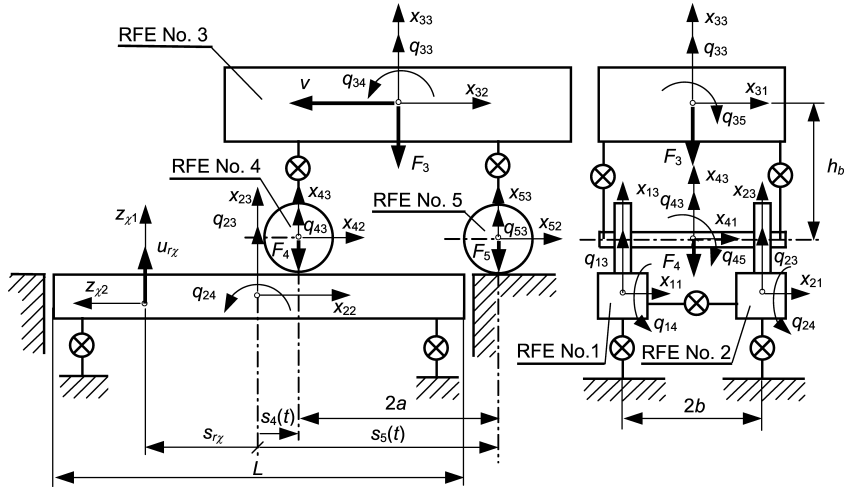


Figure 2. A discrete model of the system of a weighing platform and a 2-axle freight wagon; SDE stiffness principal axes are parallel to RFE inertia principal axes

3.2. The computation model

A discrete model of a platform with a 2-axle wagon running along has been created, based upon the following assumptions (Figure 2):

1. concerning the platform's structure:

- Each girder with an attached rail is idealised by a rigid finite element (RFE). Thus, we obtain RFE no. 1 (generalised displacements q_{13} , q_{14}) and RFE no. 2 (generalised displacements q_{23} , q_{24}). There are $i_{r\chi}$ actuators placed under RFE no. r , $r = 1, 2$, whose forces of interaction are denoted as $u_{r\chi}$, and their positions with respect to a centre of inertia of RFE no. r are determined by co-ordinate $s_{r\chi}$, $\chi = 1, \dots, i_{r\chi}$.
- Measuring sensors supporting the girders are idealised by spring-damping elements (SDE) nos. 1–4. Only vertical flexibility of these elements is considered.
- Meagre mass of the connector has been neglected, while its flexibility is idealised by SDE no. 5.

2. concerning the wagon's structure:

- The wagon's body is idealised by RFE no. 3 (generalised displacements q_{33} , q_{34} , q_{35}), while the two wheelsets, idealised by RFE no. 4 (generalised displacements q_{43} , q_{45}) and RFE no. 5 (generalised displacements q_{53} , q_{55}), are separated. External forces acting on those RFEs are gravity forces F_3 , F_4 and F_5 .
- Body suspension elements are idealised by SDEs nos. 6–9.

3. stick flexibility of the system: wagon's wheel-girder's rail has been neglected.

Thus the vibrating system of a weighing platform and a 2-axle wagon is a system whose configuration changes with time [4].

3.3. An exemplary calculation

A computer simulation of the weighing process of a running 2-axle wagon HSFV1 has been performed. The following input data were used:

- length of the girder $L = 6.5\text{m}$;
- speed of the passing wagon $v = 3.06\text{m/s}$ (about 11 km/h);
- weighing platform discrete model parameters [3];
- wagon discrete model parameters [3, 10];
- control command sampling time $\Delta t = 2\text{ms}$;
- expected dimensionless coefficient of vibration damping at measuring sensors (not provided by the manufacturer) $\zeta = 0.015, 0.006$;
- number of actuators for the girder $i_{r\chi} = 2, r = 1, 2$;
- co-ordinates of actuators $s_{r1} = -2.5\text{m}, s_{r2} = 2.5\text{m}, r = 1, 2$;
- maximum force of the actuator $\Delta F = 300\text{N}$.

For an instantaneous configuration of the system, eigenfrequencies and eigenmodes were determined. In the case of the wagon running along the platform we observe (Table 1):

- eigenmodes of the wagon with eigenfrequencies below 3Hz (nos. 1, 2 and 3; due to significant energy dissipation in the suspension elements of the wagon, such vibrations do not require a surveillance);
- eigenmodes of the weighing platform, whose eigenfrequencies are above 60Hz (nos. 4, 5, 6 and 7; high vibration level may interfere with the results of weight measurement, which justifies the necessity of their surveillance).

The results of the computer simulation (see Figure 3) have shown that after damping out the unsteady vibration of the platform we can weigh the wagon with a desired accuracy. An analysis of time plots of vertical vibrations of the girder (RFE no. 1) at the points of setting the sensors (Figures 3a and 3c) allows us to separate two times of measurement: t_{p1} and t_{p2} . Time t_{p1} relates to the running of the first axle of the wagon along weighing platform, but time t_{p2} – to the running of the second axle. If we consider the wagon's mass to be constant, the weighing will be performed by summing instantaneous indications of the platform's sensors, obtained at measuring times t_{p1} and t_{p2} . It should be noted that, due to the desired dimensions of the platform and of the wagon, the axles are not simultaneously on the platform.

Time $t_{p\alpha}$ is measured from an instant for which the following condition has been assumed:

$$|q_{ci} - \bar{q}_{ci}| \leq \varepsilon_i, \quad i = 1, 2, \quad (15)$$

where q_{ci} is the instantaneous vertical displacement of the girder at the point of setting the sensor no. i ; \bar{q}_{ci} – instantaneous vertical displacement of the girder at the point of setting the sensor no. i , in the desired motion; ε_i – allowable difference of instantaneous displacements of the girder at the point of setting the sensor no. i .

In order to extend the times of measurement t_{p1} and t_{p2} , a methodology of optimal control at the energy performance index for a discrete non-stationary system is applied, whereas:

$$\begin{aligned} \mathbf{Q}_1 &= Q_1 \cdot \text{diag}[1, 1, 1, 1, 0, 0, 0], & \mathbf{Q}_2 &= Q_2 \cdot \text{diag}[1, 1, 1, 1, 0, 0, 0], \\ \mathbf{R} &= \text{diag}[R_i], & i &= 1, \dots, 4. \end{aligned} \quad (16)$$

The values $Q_1 = 1 \cdot 10^6$, $Q_2 = 0$, $R_i = 1 \text{ s}^2/\text{kg}$ have been determined in a trial-and-error approach [3, 7] for the purpose of the simulation. They produce instantaneous changes of the acting forces, which refer to the abilities of piezoelectric actuators. Neglecting the contribution of potential energy ($Q_2 = 0$) means that real determination of the optimal control command requires only recording the vibration velocities of the girder, more readily obtainable than displacements. The application of optimal control has resulted in the expected extension of times of weight measurement t_{p1} and t_{p2} (Figures 3b and 3d).

The control by optimal command determined at the constraint of $\Delta F = 300 \text{ N}$ and generated with the sampling time of $\Delta t = 2 \text{ ms}$ significantly extended measurement times t_{p1} and t_{p2} (Figures 3b and 3d), which is particularly noticeable for the smaller damping in the system ($\zeta = 0.006$).

4. Conclusions

An alternative method of vibration surveillance by optimal control at the energy performance index has been elaborated, with respect to the trajectory of the desired motion. Its main advantage is that instantaneous values of optimal control command

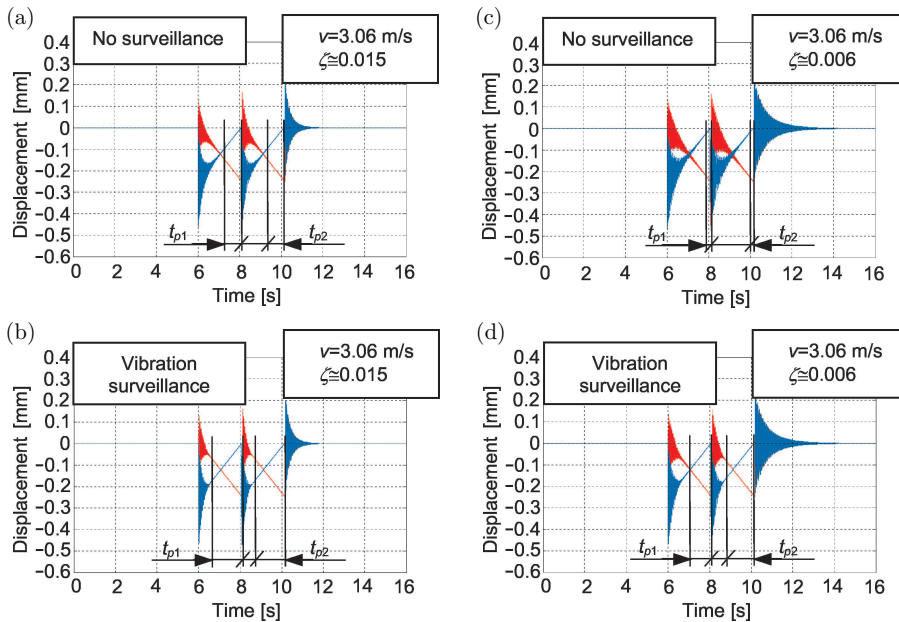


Figure 3. Results of a computer simulation of the weighing process during passage of a 2-axle HSFV1 wagon; red and blue lines represent vertical vibrations of the girder (*i.e.* RFE no. 1) at the point of setting the sensor at a side of the wagon rolling off and on the platform, respectively

Table 1. Eigenfrequencies and eigenmodes of the system of a weighing platform and a 2-axle HSFV1 wagon

No. of eigenmode	1	2	3	4	5	6	7
Eigenfrequencies [Hz]	1.45	2.63	2.93	66.68	124.52	128.65	130.59
q_{13}	0	0.003	-0.009	1	1	-0.075	1
q_{14}	0	0	0	0.008	0.454	1	-0.933
q_{23}	0	0.003	-0.009	1	-1	-0.075	-1
q_{24}	0	0	0	0.008	-0.454	1	0.933
q_{33}	0	1	0.019	-0.001	0	0	0
q_{34}	0	-0.002	1	0	0	0	0
q_{35}	1	0	0	0	0	0	0

are determined *on-line*, during the surveillance process. Consideration of constraints while performing the process is also possible.

The usefulness and practical meaning of the method of optimal control at the energy performance index for *on-line* vibration surveillance of non-stationary systems, has been demonstrated. The above conclusion has been confirmed by the results of a computer simulation of unsteady vibration of a system consisting of a weighing platform and a railway wagon. The efficiency of surveillance has been demonstrated and suitable weighing times have been significantly extended. The level of control command and the desired values of sampling time indicate that real performance of surveillance is possible. The proposed solution, based upon piezoelectric disks in a stack, is a serious concept of actuator performance. The above demonstrates that the method of surveillance is actually applicable.

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