

NATURAL FREQUENCIES AND MODES SHAPES OF TWO RIGID BLADED DISCS ON THE SHAFT

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Abstract: The dynamic behavior of a rotor consisting of two bladed discs on a solid shaft is considered. The effect of shaft flexibility on the dynamic characteristics of the bladed discs and the coupling effects between the shaft and bladed disc modes are investigated. Results presented for various cases with differing blade flexibility show clearly the coupling effects in a bladed disc-shaft system. Interference diagrams are developed, from which the dynamic behavior of a system can be predicted for differing flexibility relationships between the solid shafts and the bladed discs.

In this study, the global rotating mode shapes of flexible bladed disc-shaft assemblies have been calculated. Rotational effects such as centrifugal stiffening have been accounted for, and all the possible couplings between the flexible parts have been allowed. Gyroscopic effects have been included in the shaft with two discs. Calculated natural frequencies obtained from the blade, shaft, bladed disc and shaft with two discs have been checked to determine resonance conditions and coupling effects. The calculations have shown the influence of the shaft on the natural frequencies of the bladed discs up to one nodal diameter frequencies. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameter modes of the single bladed discs. The bending modes of the shaft are coupled with one nodal diameter modes of the bladed discs. It is shown that including the shaft in the bladed discs model is important from a designer's point of view and can change the spectrum of frequencies considerably.

Keywords: natural frequencies, bladed discs-shaft

Nomenclature

f – non-dimensional frequency,
 k – number of nodal diameters,
 $K(e^{jk\omega}, \Omega)$ – stiffness matrices with respect to rotational speed Ω ,
 $M(e^{jk\omega})$ – mass matrix,
 N – number of rotor blades,
 Ω – rotational speed.

1. Introduction

In turbomachinery, the study of the vibration characteristics of individual components such as blades, discs and shafts has been established as an important part of design. However, the vibration characteristics of an individual components can change considerably when these components are assembled to form a system, due to the coupling effects among these constituent components. The vibration characteristics of a blade in a gas or a steam turbine can change due to flexibility of the discs and that of the shaft on which the discs are mounted. Furthermore, there may be interactions between blades on various stages of the turbine due to flexibility of the discs and the shaft.

Two independent approaches are commonly used to analyze the dynamic behavior of turbomachinery rotating assemblies. The rotordynamics approach is concerned with disc-shaft systems. The shaft is mostly modeled by using beam finite elements and disc flexibility is not considered (Berger [1], Rao [2]), and also the flexibility is concerned (Chivens [3]). The bladed disc approach deals with flexible discs (Rządkowski [4]). There are a few papers on the free vibration of a bladed disc on a shaft (Dubiegeon *et al.* [5], Huang *et al.* [6], Jacquest-Richardet *et al.* [7], Kanki *et al.* [8], Khader *et al.* [9], Okabe *et al.* [10]). Authors of these papers assert that the inertial effects generated on the shaft by the vibration of the bladed disc are important for the modes associated with zero and one nodal diameter modes.

In this paper the natural frequencies of a rotating single blade, a bladed disc, discs on a shaft, a shaft and two bladed discs on a shaft are checked to discover resonance conditions and coupling effects. Calculations have shown there to be an influence of the shaft on the natural frequencies of the bladed discs up to one nodal diameter frequencies. The torsional frequency of the shaft with two discs is coupled with the zero nodal diameter modes of single bladed discs. It follows from the calculations that including the shaft in the bladed discs model is important from a designer's point of view and can change the spectrum of frequencies considerably. When flexible bladed discs are mounted on a flexible shaft, the resultant system has vibration characteristics dependent on the coupling between the vibration modes of the individual components. In studies of these vibration characteristics the system cannot be treated as two independent systems, one being flexible bladed discs on a rigid shaft and the other being discs with rigid blades on a flexible shaft.

2. Description of the model

Free analysis of the dynamic of rotating structures is very important for the design and development of turbomachines. First of all, a designer must remove all rotors critical speeds from an engine's operational range. Usually, dynamic analysis of rotors is performed with special rotordynamics software, as finite element programs traditionally do not consider gyroscopic effects of rotating structures.

MSC NASTRAN 2003 has a special capacity for analysis of rotating structures including gyroscopic effects – DMAP alter RIDGYROA.Vxxx. Making use of this capacity, researchers can use one finite element model for complex dynamic analysis of turbomachines, including in model all rotors, bearings, casings and mounting parts.

Rotor critical speed is calculated at any value of the precession coefficient or an arbitrary precession speed.

The simplest rotor has a shaft and a disk with polar and diametric inertial moment I_P and I_D [2]. The rotor is rotating with rotational speed ω . The shaft has a bending deformation from the unbalanced loads' action and the point of disk mounting makes a turn on the angle α . The deformed axis of the shaft is performing precession motion with angular velocity Ω . Assuming, that the rotor has uniform circular precession and values of deformations, precession speed, *etc.* remains constant. In this case, the gyroscopic moment from disk to shaft is as follows:

$$M_{GIRO} = \Omega^2 \alpha I_D \left(1 - \frac{\omega I_P}{\Omega I_D} \right). \quad (1)$$

Depending on the correlation between the directions of angular velocities ω and Ω , the precession may be forward (same directions) or backward (opposite directions). The ratio between the speeds is the precession coefficient, $s = \omega/\Omega$. When absolute values ω and Ω are equal, the precession is referred to as synchronous (forward or backward), and if $s \neq 1$ the precession motion is non-synchronous.

In MES model the rotor at a given node has four degrees of freedom: two displacement, u and w , and two slop about the X and Z axes, θ and ψ , respectively. Then, if the nodal displacement vector δ of the central disc is:

$$\delta = [u, w, \theta, \psi], \quad (2)$$

Lagrange's equation [11] is as follows:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\delta}} \right) - \frac{\partial T}{\partial \delta} = \begin{bmatrix} m_d & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 \\ 0 & 0 & I_D & 0 \\ 0 & 0 & 0 & I_D \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{w} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \Omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_P \\ 0 & 0 & I_P & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = 0, \quad (3)$$

where the first matrix is the classical mass matrix and the second one is the gyroscopic (Coriolis) matrix.

3. The numerical model

The structure is composed of 24 blades mounted rigidly on a simply supported, clamped shaft with two discs (see Figure 1). The main dimensions are as follows: the disc's outer diameter is 0.3808m, the inner diameter is 0.12m, the height of the blade is 0.484m and the length of the shaft is 3.52m. The isoparametric brick elements with 8 nodes, and 3 degrees of freedom per node are used. Numerical model has 261760 DOF.

Natural frequencies obtained from the cantilever blade, the shaft, the bladed disc and the shaft with two discs are checked to discover resonance conditions and coupling effects.

In this paper geometrical cross-section blade parameters have been taken from the 4th Standard Configuration [12] (see Figure 2). The length of the blade was assumed to be $L = 0.484$ m in order to get the first natural frequency close to the first natural frequency of the shaft with two discs.

Table 1. Natural frequencies of the blade ($L = 0.484\text{m}$)

	0rpm	3898rpm
mode	frequency [Hz]	frequency [Hz]
1 B1	46.457	80.979
2 B2	210.40	224.57
3 B3	287.66	347.73
4 T	556.89	561.59

Table 2. Natural frequencies of the disc

	0rpm	3898rpm
mode	frequency [Hz]	frequency [Hz]
1 B1	2713.5	2713.4
2 B2	3908.3	3908.5
3 B3	3908.3	3908.5
4 T	4172.2	4172.3

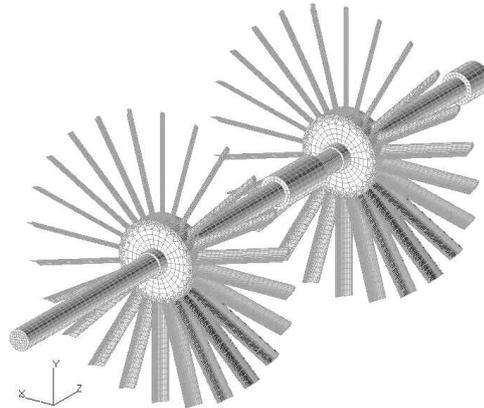


Figure 1. Two bladed discs on the shaft

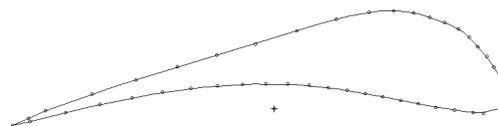


Figure 2. Cross-section of the blade

The first four natural frequencies of the non-rotating and rotating ($n = 3898\text{rpm}$) cantilever blade are shown in Table 1.

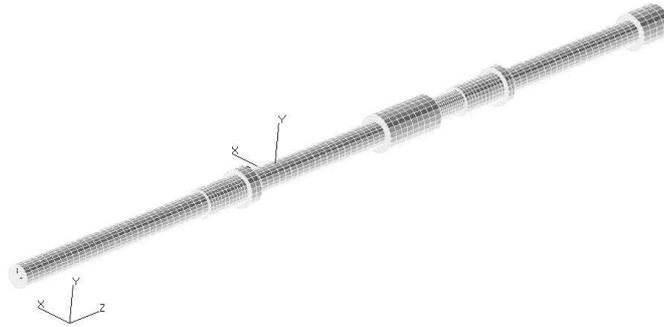
The natural frequencies of the of the non-rotating and rotating ($n = 3898\text{rpm}$) disc of thickness $h = 0.206\text{m}$, the inner diameters $d_i = 0.12\text{m}$ and the outer diameters $d_o = 0.3808\text{m}$ are presented in Table 2.

The natural frequencies of the shaft and the shaft with two discs are presented in Table 3. The boundary conditions in the cylindrical coordinates (r, ϕ, z) from the left are $z = 0, \phi = 0$, and in the position of bearings assumed $r = 0$.

The natural frequencies of the shaft with discs consist of a single torsion, a double bending and a single axial. Due to gyroscopic effects the bending frequencies are capable of splitting into backward and forward branches (see Table 4).

Table 3. Natural frequencies of the shaft and the shaft with two discs ($n=0$ rpm)

	shaft	shaft with discs
mode	frequency [Hz]	frequency [Hz]
1	105.51	45.59 (T)
2	105.51	62.07
3	165.29 (T)	62.08
4	178.11	95.96
5	178.11	95.97
6	287.58	122.82 (T)
7	287.58	224.34 (A)
8	328.19 (A)	248.89
9	486.95	248.89
10	486.95	343.31
11	554.62 (T)	343.32
12	564.47	403.34
13	564.47	403.34
14	786.66 (T)	591.36 (T)
15	1004.49	645.28 (A)
16	1004.49	897.54
17	1048.25 (A)	897.54
18	1138.52	931.71 (T)
19	1138.55	1028.40
20	1448.25 (T)	1028.50
21	1536.35	1166.00
22	1536.35	1166.0
23	1594.65 (A)	1253.10
24	1653.56 (T)	1253.10
25	1699.67	1325.40 (A)
26	1699.67	1582.10 (A)

**Figure 3.** 3D model of the shaft

Thus, the natural frequencies of the non-rotating and rotating bladed disc with 24 blades of $L = 0.484$ m have been calculated (see Figures 4 and 5).

The numerical results computed for all the possible nodal diameters are presented in Figures 4 and 5. The modes of the bladed disc are classified by analogy with axisymmetric modes, which are mainly characterized by nodal lines lying along diameters of the structure and having constant angular spacing. They are either zero ($k = 0$), one ($k = 1$), two ($k = 2$), or more ($k > 2$) nodal diameter bending or torsion modes. Series 1 is associated with the first natural frequency of the single cantilever

Table 4. Natural frequencies of the rotating shaft with two discs ($n = 3898\text{rpm}$)

mode	frequency [Hz]
1	62.016
2	62.016
3	62.223
4	62.223
5	95.918
6	95.918
7	96.258
8	96.258
9	246.2
10	246.2
11	323.56
12	323.56

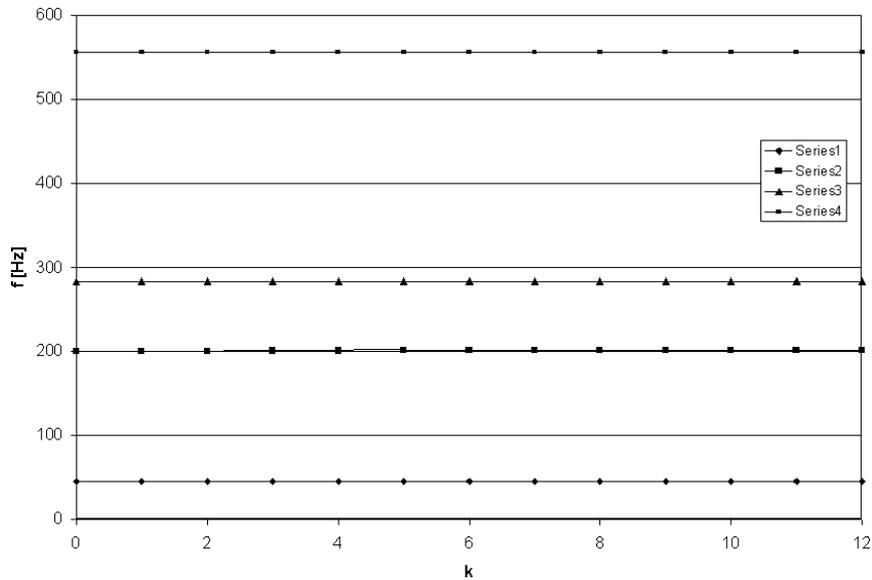


Figure 4. Interference diagram of the non-rotating bladed disc with 24 blades ($L = 0.484\text{m}$)

blade. Series 2 is associated with the second natural frequency of the single cantilever blade, and so on, where k is the number of nodal diameters.

Subsequently, natural frequencies of two bladed discs on the shaft were calculated, as shown in Figure 6.

In this figure, the upper axis shows uncoupled natural frequencies of the cantilever blade (see Table 1). The next axis shows natural frequencies of the bladed disc. Coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. Next, the natural frequencies of the shaft with two discs and the shaft are shown. The lowest axis shows uncoupled natural frequencies of the disc. The numbers in brackets show the number of nodal diameters of the bladed disc.

The spectrum of natural frequencies of two bladed discs on the shaft consist of natural frequencies connected with the natural frequencies of the first bladed disc (24 frequencies), the second bladed disc (24 frequencies) and the bending and axial

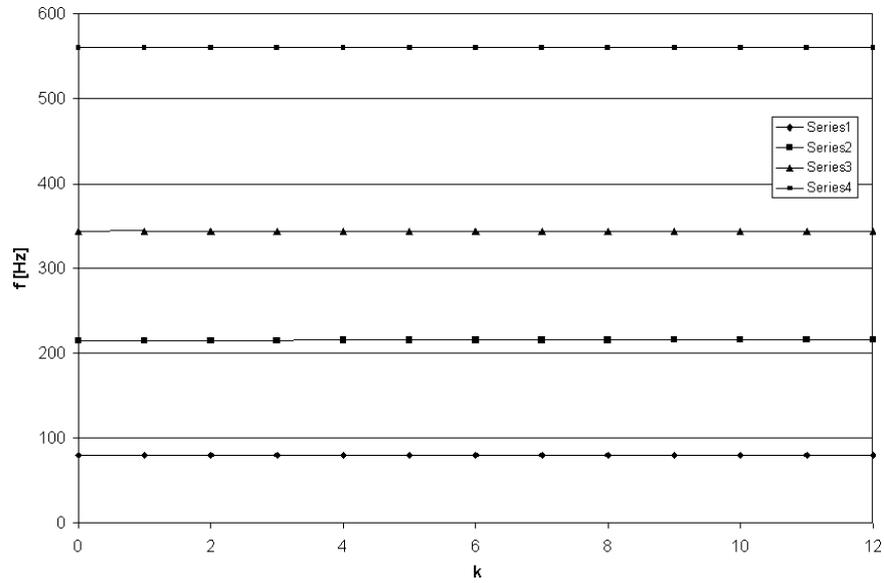


Figure 5. Interference diagram of the rotating bladed disc with 24 blades ($n = 3898\text{rpm}$, $L = 0.484\text{m}$)

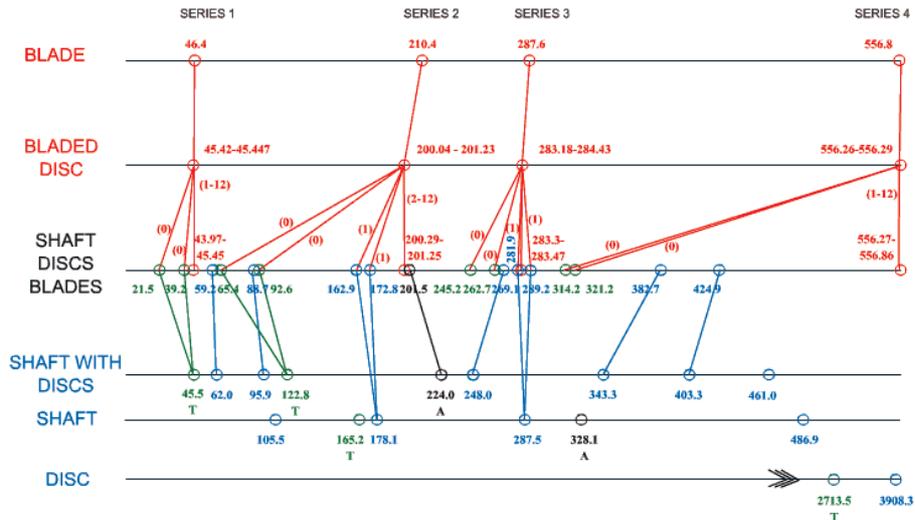


Figure 6. Natural frequencies of the non-rotating two bladed disc on the shaft ($L = 0.484\text{m}$)

frequencies of the shaft with two discs. The torsional natural frequencies of the shaft with two discs are coupled with the zero nodal diameter frequencies of the bladed discs.

The natural frequencies of the shaft-discs-blades system coupled with the uncoupled natural frequencies of the bladed disc are shown in red in Figure 6.

The uncoupled bladed disc's natural frequency of the zero nodal diameter mode is 45.42Hz (red), while the uncoupled torsional natural frequency of the shaft with two discs is 45.5Hz (green). They are so close to each other that the coupled natural frequency of the bladed discs on the shaft with zero nodal diameter split into values of

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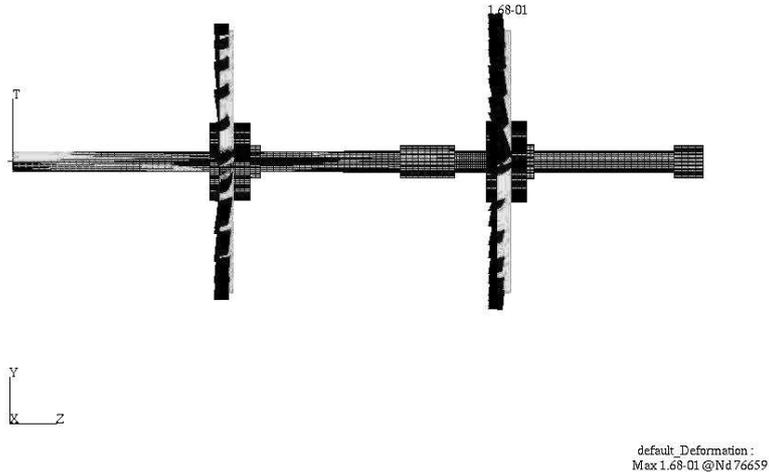


Figure 7. Coupled shaft-discs-blades mode ($k=0$); series 1, $f = 21.526\text{Hz}$

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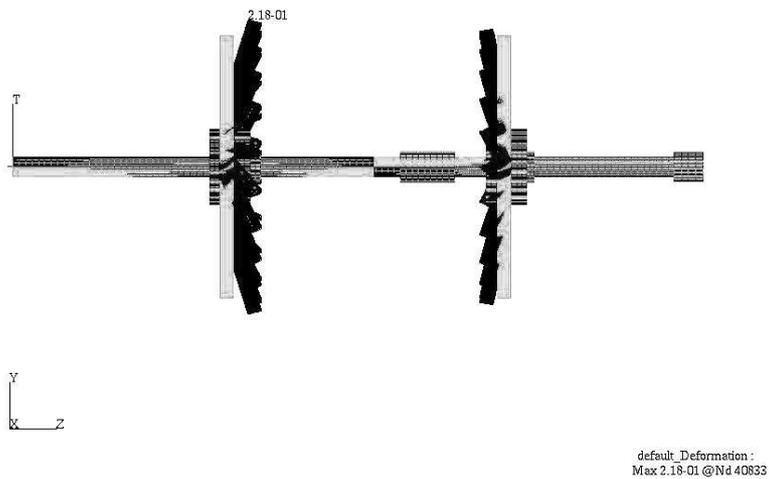


Figure 8. Coupled shaft-discs-blades mode ($k=0$); series 1, $f = 39.284\text{Hz}$

21.526Hz for the first bladed disc (see Figure 7) and 39.284Hz for the second bladed disc (see Figure 8). As shown in this example, the frequency split due to coupling effects cannot be ignored in certain situations.

The uncoupled bladed disc's natural frequency of the one nodal diameter mode is 45.42Hz (red), while the coupled natural frequencies of the shaft with two bladed discs are 43.973Hz for the first bladed disc ($k=1$, see Figure 9) and

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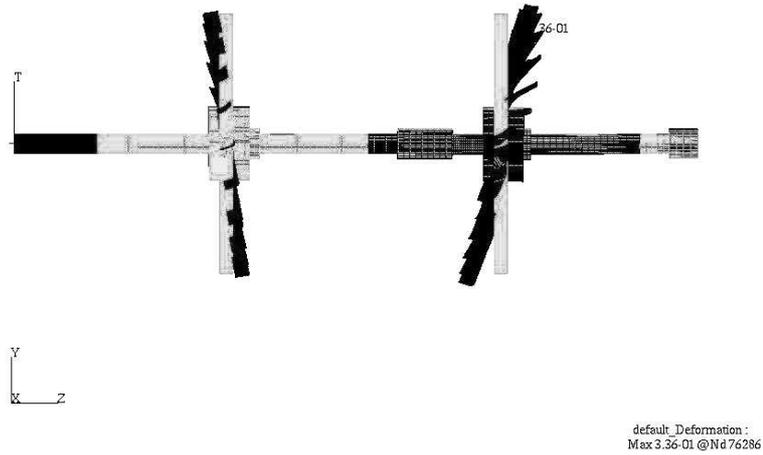


Figure 9. Coupled shaft-discs-blades mode ($k = 1$); series 1, $f = 43.973$ Hz

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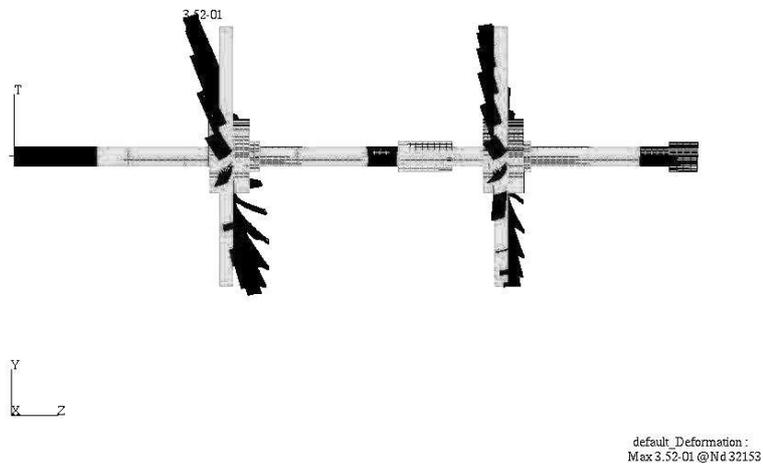


Figure 10. Coupled shaft-discs-blades mode ($k = 1$); series 1, $f = 44.976$ Hz

44.976 Hz for the second bladed disc ($k = 1$, see Figure 10). It can be seen that two bladed discs vibrate with one predominant. In these modes, shaft-bending vibration is visible.

The uncoupled bladed disc's natural frequency of the two nodal diameter mode is 45.419 Hz (red), while coupled natural frequency of the shaft with two bladed discs are 45.424 Hz (red) for the first and 45.425 Hz for the second bladed disc.

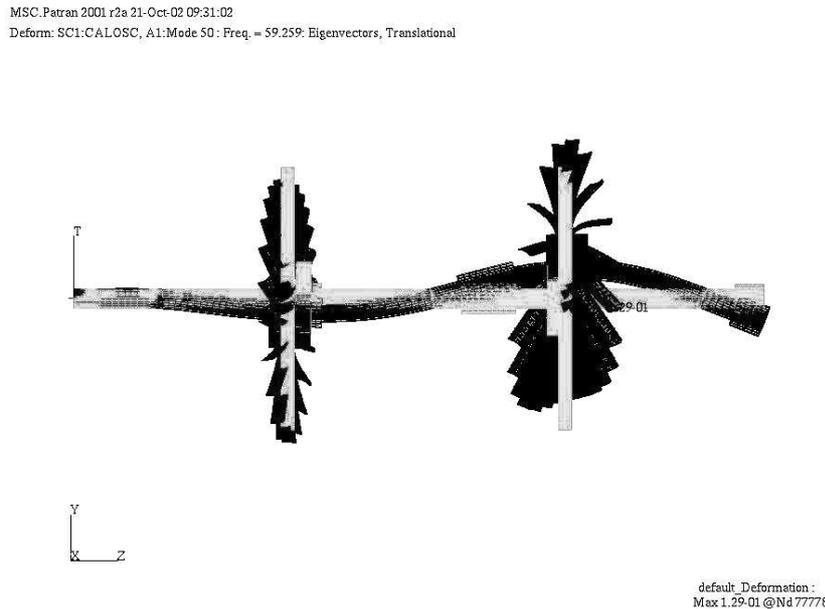


Figure 11. Coupled bending shaft-discs-blades mode; $k = 1$, $f = 59.259\text{Hz}$

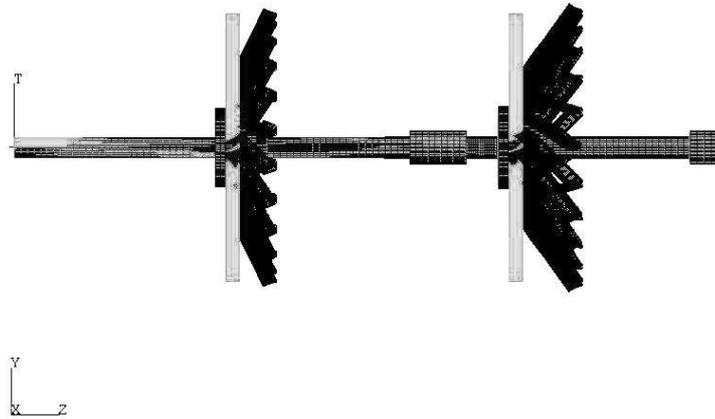
The uncoupled bladed disc's natural frequencies of the three to twelve nodal diameter modes are in the range 45.422–45.447Hz (red) and the coupled natural frequencies of the shaft with two bladed discs are in the range 45.425–45.454Hz (red) for the first and second bladed discs. The coupled natural frequencies of the first and second bladed discs are the same for the considered modes and are greater than those of the uncoupled modes.

Next, the uncoupled shaft-with-two-discs bending mode of one nodal diameter has the frequency 62.079Hz (blue), while the coupled natural frequencies of the shaft with two bladed discs are 59.259Hz for the first and second bladed discs (blue, $k = 1$, see Figure 11). It is seen that two bladed discs vibrate with one predominant. In these modes the shaft bending vibration is visible. This coupled frequency is due to coupling effects with the bending mode of the shaft with two discs.

The coupled modes of the second series of the bladed discs are similar to that in the coupled first series. The torsional uncoupled mode of the shaft with two discs of 122.8Hz (green, see Figure 6) has an influence on the coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter. These frequencies split into values of 65.4Hz for the first blades disc (see Figure 12) and 92.6Hz for the second bladed disc (see Figure 13). The uncoupled bladed disc's natural frequency of the zero nodal diameter mode is 200.04Hz (red).

Now, the uncoupled shaft-with-two-discs bending mode of one nodal diameters is 95.9Hz (blue), while the coupled natural frequencies of the shaft with two bladed discs are 88.7Hz for the first and second bladed discs (blue, $k = 1$, see Figure 14). It can be seen that two bladed discs vibrate with one predominant. In these modes, shaft-bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode.

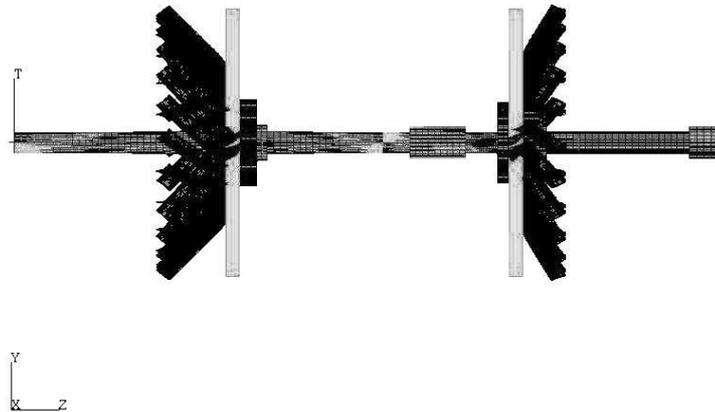
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Figure 12. Coupled shaft-discs-blades mode ($k=0$); series 2, $f = 65.491\text{Hz}$

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default_Deformation:
 Max 1.69-01 @Nd 39965

Figure 13. Coupled shaft-discs-blades mode ($k=0$); series 2, $f = 92.65\text{Hz}$

The uncoupled bladed disc's natural frequency of the one nodal diameter mode is 200.3Hz (red, see Figure 6) and the coupled natural frequencies of the shaft with two bladed discs are 162.95Hz for the first bladed disc ($k=1$, see Figure 15) and 172.86Hz ($k=1$, see Figure 16) for the second bladed disc. In this case the coupling between the shaft bending mode of 178.1Hz and the one nodal diameter modes of

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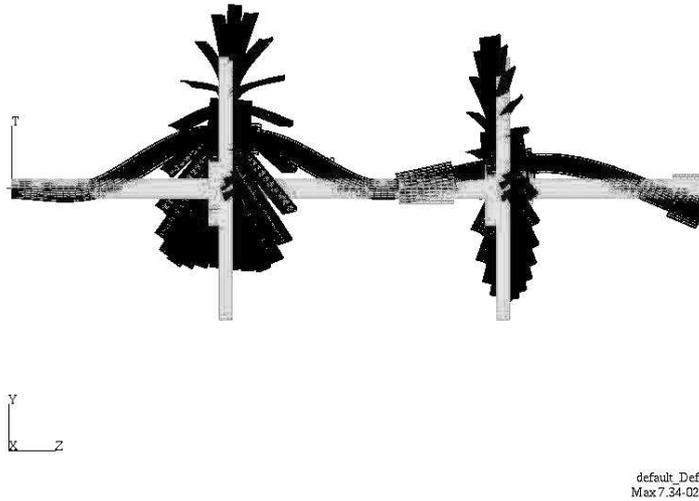


Figure 14. Coupled bending shaft-discs-blades mode ($k = 1$); series 2, $f = 82.708$ Hz

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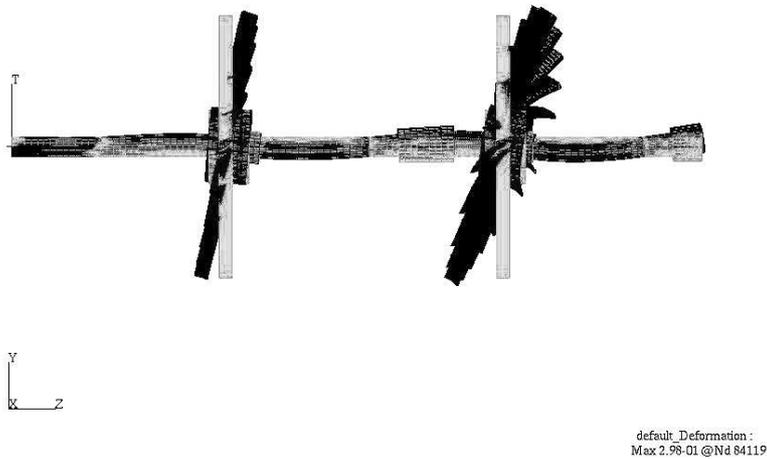


Figure 15. Coupled shaft-discs-blades mode ($k = 1$); series 2, $f = 162.95$ Hz

the bladed discs is noticeable. It can be seen that two bladed discs vibrate with one predominant. In these modes, shaft-bending vibration is visible.

The uncoupled bladed disc's natural frequencies of the two to twelve nodal diameter modes are in the range 200.42–201.23 Hz (red), while the coupled natural frequencies of the shaft with two bladed discs are in the range 200.29–201.25 Hz (red) for the first and second bladed discs. The coupled natural frequencies of the first and

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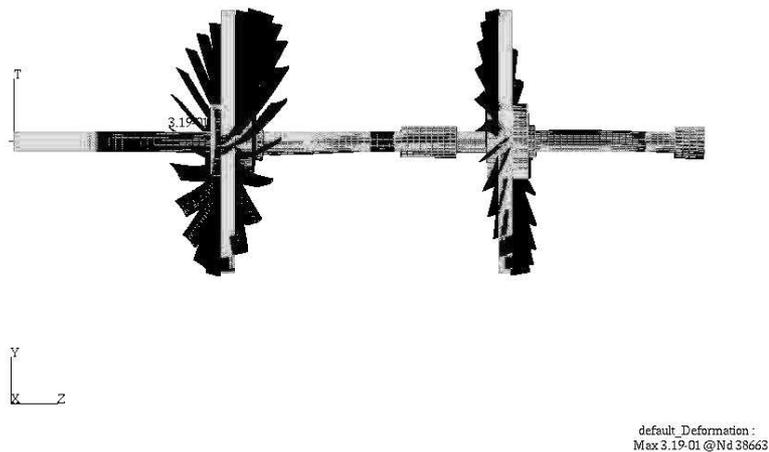


Figure 16. Coupled shaft-discs-blades mode ($k = 1$); series 2, $f = 172.86\text{Hz}$

second bladed discs are the same for the considered modes and are less than those of the uncoupled modes.

The uncoupled shaft-with-two-discs axial mode of the zero nodal diameter is 224.0Hz (black), while the coupled axial natural frequencies of the shaft with two bladed discs are 201.5Hz .

The coupled modes of the third series of the bladed discs are similar to that of the second series in the coupled modes.

The uncoupled bladed disc's natural frequency of the zero nodal diameter mode is 283.8Hz (red), while the coupled natural frequencies of the shaft with two bladed discs are 245.23Hz for the first bladed discs ($k = 0$) and 262.72Hz for the second bladed disc ($k = 0$). The influence of the coupling between the blades/discs and the shaft is visible here, although the uncoupled particular structure frequencies are not close in the considered region.

At the same time, the uncoupled shaft-with-two-discs bending mode of one nodal diameter is 248.0Hz (blue), while the coupled natural frequencies of the shaft with two bladed discs are 269.1Hz for the first and second bladed discs (blue, $k = 1$). In these modes, shaft-bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs.

The uncoupled bladed disc's natural frequency of the one nodal diameter mode is 283.31Hz (red), while the coupled natural frequencies of the shaft with two bladed discs are 281.95Hz for the first bladed disc ($k = 1$) and 289.25Hz for the second bladed disc ($k = 1$, see Figure 17). It can be seen that two bladed discs vibrate with one predominant. As shown in this example, the frequency splits due to coupling effects with the uncoupled shaft mode of 287.5Hz . In these modes, shaft-bending vibration is visible.

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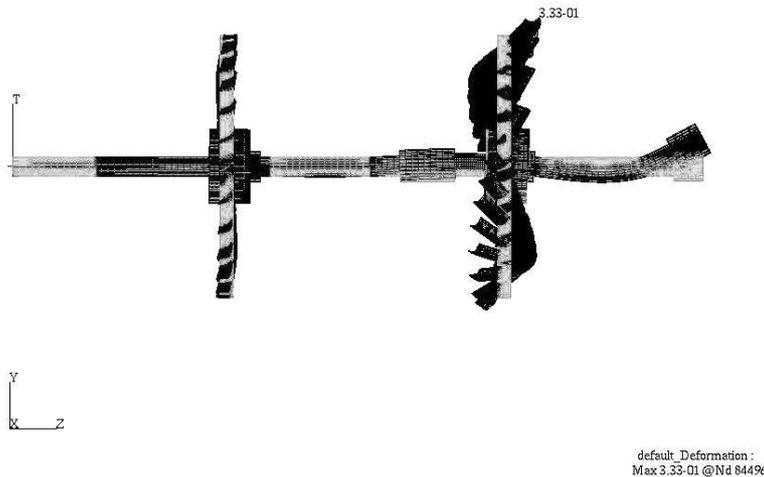


Figure 17. Coupled shaft-discs-blades mode ($k = 1$); series 3, $f = 289.25\text{Hz}$

The uncoupled bladed disc's natural frequencies of the two to twelve nodal diameter modes are in the range 283.31–283.43Hz (red), while the coupled natural frequencies of the shaft with two bladed discs are in the range 283.36–283.47Hz (red) for the first and second bladed discs. The coupled natural frequencies of the first and second bladed discs are the same for the considered modes and are greater than those of the uncoupled modes.

Subsequently, natural frequencies of the rotating two bladed discs on the shaft were calculated ($n = 3898\text{rpm}$, see Figure 18). In this figure, the upper axis indicates the uncoupled natural frequencies of the cantilever blade (see Table 1). The next axis shows natural frequencies of the rotating bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. Next, the natural frequencies of the shaft with two discs are shown. The lowest axis shows the uncoupled natural frequencies of the shaft. Numbers in brackets show the number of nodal diameters of the bladed discs' modes.

The spectrum of natural frequencies of rotating two bladed discs on the shaft consist of the natural frequencies connected with the natural frequencies of the first bladed disc (24 frequencies), the second bladed disc (24 frequencies), and the bending and axial frequencies of the shaft with two discs. The torsional natural frequencies of the shaft with two discs are coupled with the zero nodal diameter frequencies of the bladed discs. Generally, the values of natural frequencies of the rotating system are greater than those of the non-rotating system.

The uncoupled rotating bladed disc's natural frequency of the zero nodal diameter mode is 79.68Hz (red), while the uncoupled torsional natural frequency of the shaft with two discs is 45.5Hz (green). The coupled natural frequencies of the rotating bladed discs on the shaft with zero nodal diameter split into values of 22.4Hz

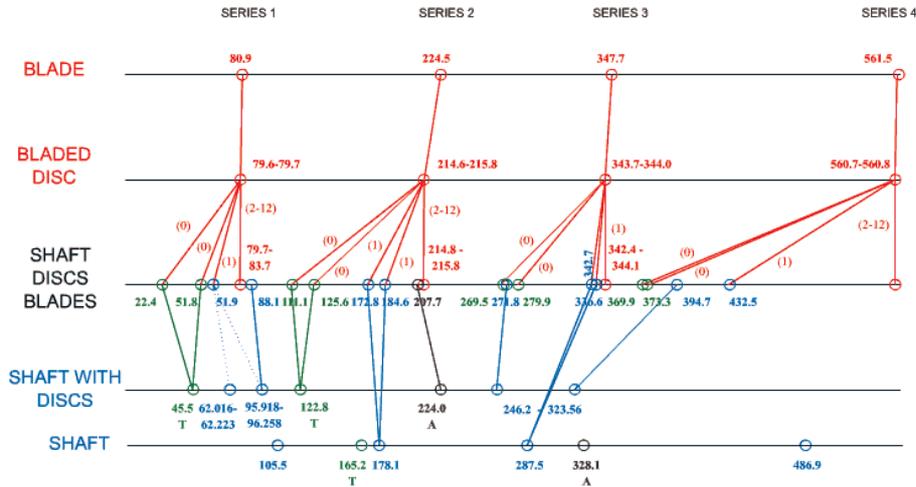


Figure 18. Natural frequencies of the rotating two bladed disc on the shaft ($L = 0.484\text{m}$)

for the first blades disc (for the non-rotating system, Figure 6, 21.526Hz) and 51.9Hz for the second bladed disc (Figure 6, 39.284Hz).

NASTRAN does not currently allow for the gyroscopic effect of the rotating bladed disc on the shaft. So, in our calculations only bladed disc stiffness effects have been considered. This assumption has caused a calculation error of the coupled shaft-discs mode caused by the shaft with two discs. The uncoupled shaft-with-two-discs bending modes of one nodal diameter are in the range 62.016–62.223Hz (blue), while the rotating coupled natural frequency of the shaft with two bladed discs is 51.9Hz (blue) for the first and second bladed discs ($k = 1$). These results are incorrect. The non-rotating coupled natural frequency of the shaft with two bladed discs is 59.2Hz (blue, see Figure 6) for the first and second bladed discs ($k = 1$) and are greater than the corresponding rotating frequencies (51.9Hz).

The uncoupled rotating bladed disc’s natural frequency of the one nodal diameter mode is 79.706Hz (red) and the coupled natural frequencies of the shaft with two bladed discs are 51.9Hz for the first bladed disc ($k = 1$) and 83.725Hz for the second bladed disc ($k = 1$). It can be seen that two bladed discs vibrate with one predominant. Other coupled and uncoupled frequencies are shown in Figure 18. It is difficult to comment on these results at this stage due to omission of the gyroscopic effects.

In order to consider the influence of blade length on the dynamic behavior of the shaft-discs-blades system, the length of the blade was decreased from $L = 0.484\text{m}$ to 0.330m (208048 DOF).

The geometrical cross-section parameters of the blade were taken from the 4th Standard Configuration [12] (see Figure 2).

In Table 5 the first four natural frequencies of the non-rotating cantilever blade are shown.

Thus, the natural frequencies of the non-rotating bladed disc with 24 blades of $L = 0.330\text{m}$ were calculated (see Figure 19).

Table 5. Natural frequencies of cantilever blades $L = 0.330\text{m}$

	0rpm
mode	frequency [Hz]
1 B1	100.04
2 B2	445.90
3 B3	609.22
4 T	826.36

Subsequently, natural frequencies of two bladed discs on the shaft were calculated (see Figure 20).

In this figure, the left axis indicates the uncoupled natural frequencies of the cantilever blade (see Table 5). The next axis shows natural frequencies of the bladed disc. The coupled natural frequencies of the two bladed discs on the shaft are given on the middle axis. Next, the natural frequencies of the shaft with two discs are shown. The right axis indicates the uncoupled natural frequencies of the shaft. The numbers in brackets show the number of nodal diameters of the bladed discs' modes.

The uncoupled bladed disc's natural frequency of the zero nodal diameter mode is 96.98Hz (red), while the uncoupled torsional natural frequency of the shaft with two discs has the frequency 45.5Hz (green). The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter split into values of 29.4Hz for the first bladed disc and 68.5Hz for the second bladed disc.

Then, the uncoupled-shaft-with-two-discs bending mode of one nodal diameter is 62.079Hz (blue), while the coupled natural frequency of the shaft with two bladed discs is 57.7Hz (blue) for the first and second bladed discs ($k = 1$).

The uncoupled shaft-with-two-discs bending mode of one nodal diameter is 95.9Hz (blue), while the coupled natural frequency of the shaft with two bladed discs is 84.9Hz (blue) for the first and second bladed discs ($k = 1$).

In these modes, shaft-bending vibration is visible. This coupled frequency is due to coupling effects with the bending shaft with two discs mode.

The uncoupled bladed disc's natural frequencies of one to twelve nodal diameter modes are in the range 96.9–97.0Hz (red), while the coupled natural frequencies of the shaft with two bladed discs are in the range 97.0–102.9Hz (red) for the first and second bladed discs. The coupled natural frequencies of the first and second bladed discs are the same for the considered modes and are greater than those of the uncoupled modes. In this case the splitting of the coupled frequencies corresponding to one nodal diameter is not observed.

In the next series, the uncoupled bladed disc's natural frequency of the zero nodal diameter mode is 414.07Hz (red), while the uncoupled torsional natural frequency of the shaft with two discs is 122.8Hz (green). The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameter split into values of 120.7Hz for the first bladed disc and 138.6Hz for the second bladed disc.

The uncoupled shaft-with-two-discs axial mode of zero nodal diameter is 224.0Hz (black), while the coupled axial natural frequency of the shaft with two bladed discs is 215.5Hz.

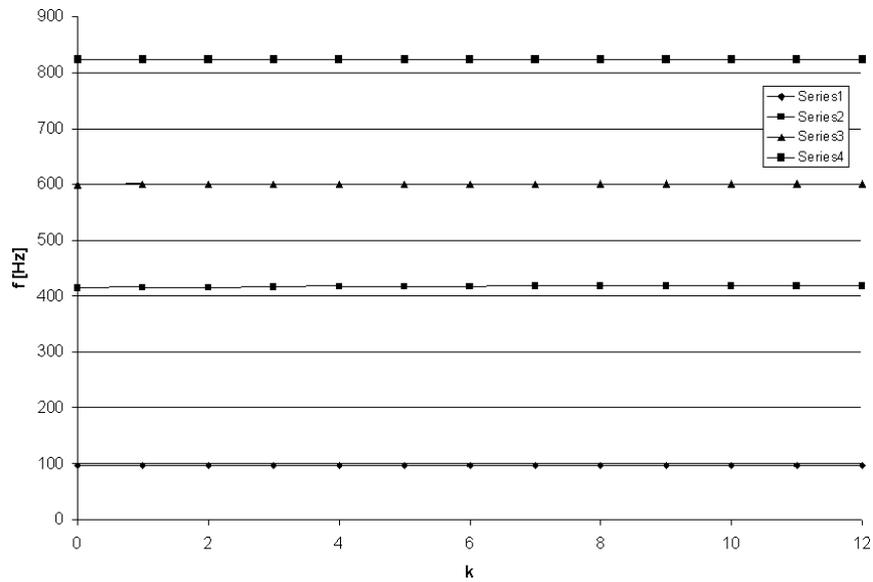


Figure 19. The Interference diagram of non-rotating bladed disc with 24 blades ($L=0.330\text{m}$)

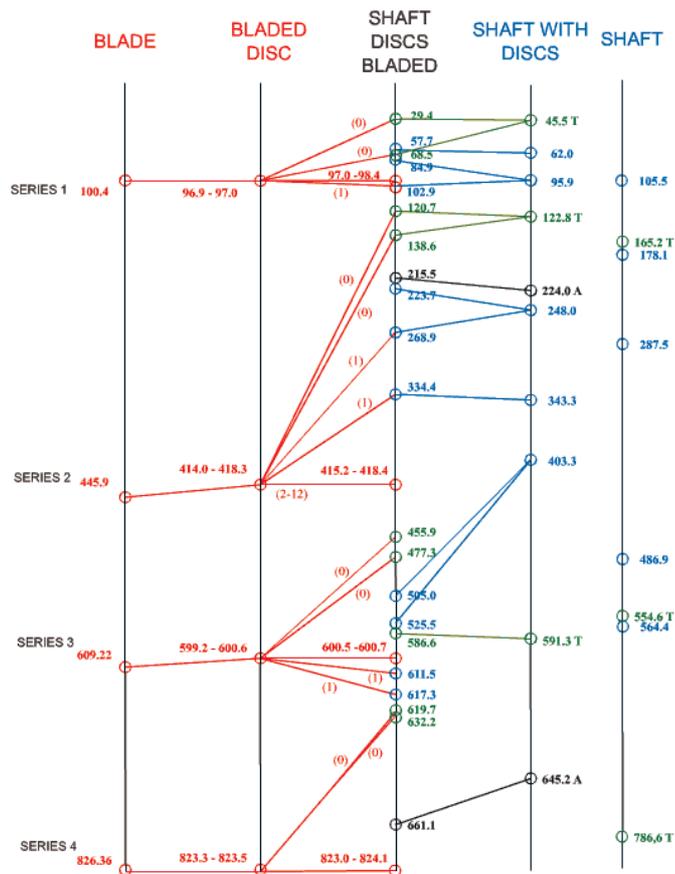


Figure 20. Natural frequencies of the non-rotating two bladed disc on the shaft ($L=0.330\text{m}$)

MSC.Patran 2001 r2a 31-Oct-02 10:56:02
 Deform: SC1:CALOSC, A1:Mode 110: Freq. = 586.62: Eigenvectors, Translational

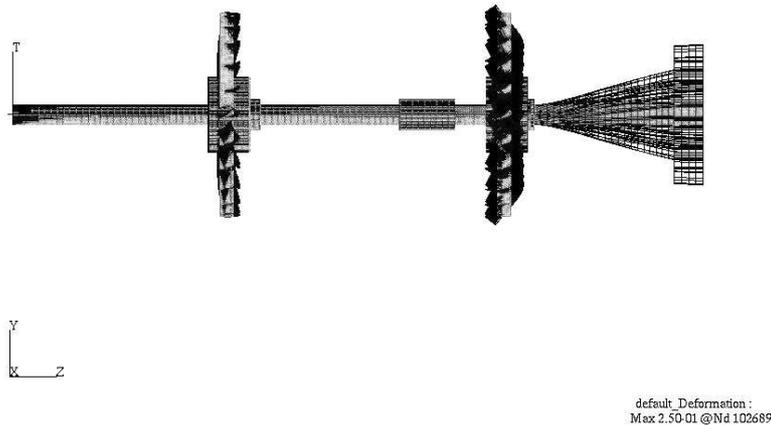


Figure 21. Torsional mode shaft-discs-blades ($f = 586.6\text{Hz}$)

The uncoupled shaft-with-two-discs bending mode of one nodal diameter is 248.0Hz (blue), while the coupled natural frequency of the shaft with two bladed discs is 223.7Hz (blue) for the first and second bladed discs ($k = 1$).

The uncoupled bladed disc's natural frequency of the one nodal diameter mode is 415.11Hz (red), while the uncoupled bending natural frequency of the shaft with two discs is 343.3Hz (blue). The coupled natural frequencies of the bladed discs on the shaft with one nodal diameter split into values of 268.9Hz for the first bladed disc and 334.4Hz for the second bladed disc.

The uncoupled bladed disc's natural frequencies of the two to twelve nodal diameter modes are in the range $414.0\text{--}418.3\text{Hz}$ (red), while the coupled natural frequencies of the shaft with two bladed discs are in the range $415.2\text{--}418.4\text{Hz}$ (red) for the first and second bladed discs. The coupled natural frequencies of the first and second bladed discs are the same for the considered modes and are greater than those of the uncoupled modes.

In the third series, the uncoupled bladed disc's natural frequency of the zero nodal diameter mode is 599.2Hz (red). The coupled natural frequencies of the bladed discs on the shaft with zero nodal diameters split into values of 455.9Hz for the first bladed disc and 477.3Hz for the second bladed disc.

The uncoupled shaft-with-two-discs bending mode of one nodal diameter is 403.0Hz (black), while the coupled bending natural frequencies of the shaft with two bladed discs are 502.0Hz and 525Hz . The uncoupled shaft frequency of 486.9Hz influences these modes.

The uncoupled shaft-with-two-discs torsional mode is 591.3Hz (green), while the coupled torsional natural frequency of the shaft with two bladed discs is 586.6Hz . This shaft torsional mode (see Figure 21) does not influence the zero nodal diameters modes of the bladed discs.

The uncoupled bladed disc's natural frequencies of the two to twelve nodal diameter modes are in the range 599.2–600.6 Hz (red), while the coupled natural frequencies of the shaft with two bladed discs are in the range 600.5–600.7 Hz (red) for the first and second bladed discs. The coupled natural frequencies of the first and second bladed discs are the same for the considered modes and are greater than those of the uncoupled modes.

The uncoupled shaft bending mode is 564.0 Hz (blue), while the coupled natural frequencies of the shaft with two bladed discs are 611.5 Hz (blue) for the first and 617.3 Hz for the second bladed disc ($k = 1$).

4. Conclusions

In this paper natural frequencies of a rotating single blade, a bladed disc, discs on a shaft, a shaft and two bladed discs on a shaft have been examined to discover resonance conditions and coupling. Calculations have shown there to be an influence of the shaft and the shaft with two discs on the natural frequencies of the bladed discs up to one nodal diameter. The torsional frequency of a shaft with two discs is coupled with the zero nodal diameter modes of single bladed discs. When blade dimensions are altered, the range within which strong coupling takes place may be altered. It follows from the calculations that including the shaft in the bladed discs model is important from a designer's point of view and can change the spectrum of frequencies considerably. When flexible bladed discs are mounted on a flexible shaft, the resultant system has vibration characteristics dependent on the coupling between the vibration modes of the individual components. In studies of these vibration characteristics the system cannot be treated as two independent systems, one being flexible bladed discs on a rigid shaft and the other being discs with rigid blades on a flexible shaft.

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