NUMERICAL SIMULATION OF SPONTANEOUSLY CONDENSING FLOWS IN A PLANE TURBINE CASCADE

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(Received 30 July 2001; revised manuscript received 28 December 2001)

Abstract: The low efficiency of wet steam turbine is mainly attributed to wetness losses. To investigate the mechanisms which give rise to these losses, a fully Eulerian model has been developed for calculation of the wet steam flows with spontaneous condensation. In this model, the liquid phase is described with two conservation equations in Eulerian form and coupled with a solver of gas dynamics equations. With such a model, the existing code for simulation of single-phase flows can easily be changed to include wet steam two-phase flows in wet steam turbines. A numerical simulation of condensing flow in a plane turbine cascade is performed, and the numerical results are presented and compared with the experimental results.

Keywords: wet steam, condensing flow, Eulerian/Eulerian model, numerical simulation

1. Introduction

In the low-pressure stages of steam turbines, steam expands across the saturation line. The steam firstly supercools and then suddenly nucleates under some defined supercoolings to become a two-phase mixture. The wet steam flow in turbine lowers the performance of stages, and its effects on efficiency are known as wetness losses. Though the well-known Baumann's law has been employed since 1910 to estimate the wetness losses in the steam turbine design, the deep-concealed mechanisms responsible for these losses are still ambiguous and need more efforts to investigate. With a dominant role played by steam turbines in the area of power generation, any progress in understanding such mechanisms will lead to improved design of steam turbines and, as a result, yield handsome economic dividends. Nowadays, CFD has been widely used in the engineering design, and therefore, to develop a numerical method for simulation of wet steam flow is practically important.

However, it is not easy to investigate the wet steam flows numerically when considering the facts that there are a great number of water droplets dispersed in wet steam and the droplets are very different in size. Bakhtar *et al.* has contributed much to the accurate simulation of wet steam flows. All of the published paper by Bakhtar *et al.* [1–3] concentrate on a mixed Eulerian/Lagrangian method in which the conservation equations of gas dynamics were solved in Eulerian formulation while the liquid phase was treated in Lagrangian coordinates. Other authors such as Young [4], Skillings and Jackson [5], Guha and Young [6] adopted the same mixed Eulerian/Lagrangian method to study the condensing flows. In this paper, a fully Eulerian method resolving the wet steam flows with spontaneous condensation is presented. The most important character of the method is to describe the liquid phase in wet steam with two conservation equations in Eulerian form. The knowledge of the flow path of water droplets is of less consequence and the wet steam flows can be solved using the same numerical arrangements as those in simulation of single-phase flows. Also, it is more straightforward to extend to 3D calculation of wet steam flows. A numerical simulation of condensing flow in a plane cascade is performed; the numerical results are presented and compared with experimental measurements.

2. Governing equations of wet steam flow

Considering the two coupled systems of steam and water droplets in wet steam, the governing equations for steam phase are the general gas dynamics equations with mass transfer, momentum transfer and energy transfer between phases. In Cartesian coordinates, the conservative equations can be written as:

$$\frac{\partial U_g}{\partial t}+\frac{\partial E_g}{\partial x}+\frac{\partial F_g}{\partial y}=\frac{\partial E_\nu}{\partial x}+\frac{\partial F_\nu}{\partial y}+S_g, \tag{1}$$

$$U_g=\begin{bmatrix}\rho_g\\ \rho_g u\\ \rho_g\nu\\ E_g\end{bmatrix},\quad E_g=\begin{bmatrix}\rho_g u\\ \rho_g u^2+p\\ \rho_g u\nu\\ (E_g+p)u\end{bmatrix},\quad F_g=\begin{bmatrix}\rho_g\nu\\ \rho_g u\nu\\ \rho_g\nu^2+p\\ (E_g+p)\nu\end{bmatrix},$$

$$E_\nu=\begin{bmatrix}0\\ \tau_{xx}\\ \tau_{xy}\\ q_x\end{bmatrix},\quad E_\nu=\begin{bmatrix}0\\ \tau_{yx}\\ \tau_{yy}\\ q_y\end{bmatrix},\quad S_g=\begin{bmatrix}-\rho\dot{m}\\ -\rho u\dot{m}\\ -\rho\nu\dot{m}\\ \rho\dot{m}(h_{fg}-h_t)\end{bmatrix},$$

where ρ_g is the density of the vapour; u and ν are velocity components in x and y directions respectively; p is the pressure; E is the total energy; and the subscript $_g$ denotes the vapour phase. The source term S_g counts in transfers of mass, momentum and energy between the phases. In the source term, $\dot{m}$ is the condensed mass per unit time; ρ is the density of wet steam; h_{fg} is the latent heat and $\rho\dot{m}(h_{fg}-h_t)$ is the latent heat released to the vapour during the condensing process. The condensed mass per unit time can be expressed as

$$\dot{m}=(i-Y)J\rho_l\frac{4\pi r_c^3}{3}+4\pi r^2\frac{dr}{dt}\rho_l N. \tag{2}$$

Here, J is the nucleation rate; dr/dt is the growth rate; ρ_l is the density of water; r_c is the critical radius; N is the water droplet number contained in unit mass of wet steam and r is the droplet radius. This formula can be deduced from the fact that the condensed mass involves the mass coming from the nucleation at the critical radius and from droplets growth on surfaces of existing droplets.

The liquid phase can be treated as a continuum due to the very large number of minute water droplets contained in the wet steam flows. Under the zero slip hypothesis between the steam and water droplets, the momentum equations for these droplets are unnecessary. Here, two extra conservation equations are added to characterize the liquid phase. One describes the water droplets number distribution and the other describes the wetness distribution. In Cartesian coordinates these two equations can be written as follows:

$$\frac{\partial U_l}{\partial t}+\frac{\partial E_l}{\partial x}+\frac{\partial F_l}{\partial y}=S_l, \tag{3}$$

$$U_l=\begin{bmatrix}\rho N\\ \rho Y\end{bmatrix},\quad E_l=\begin{bmatrix}\rho uN\\ \rho uY\end{bmatrix},\quad F_l=\begin{bmatrix}\rho \nu N\\ \rho \nu Y\end{bmatrix},\quad S_l=\begin{bmatrix}\rho_g J\\ \rho\dot{m}\end{bmatrix}.$$

Here Y is the wetness. According to the definition of wetness Y, the following expression can be obtained:

$$r^3=3Y/(4\pi\rho_l N). \tag{4}$$

This expression can be used to calculate the water droplet radius after the parameters N and Y are resolved at each time step in the time-marching method.

Before the calculation can be started, the nucleation rate J and water droplet growth rate dr/dt must be determined. Though the classical nucleation theory and water droplet growth theory have been widely disputed, they are still the most competent selection in the calculation of wet steam flows. The expressions for the nucleating rate J and water droplet growth rate dr/dt adopted here are those described by Guha and Young [6]:

$$\begin{aligned}
J&=\frac{1}{1+\phi}q_c\sqrt{\frac{2\sigma}{\pi m_m^3}}\frac{\rho_g}{\rho_l}\exp\left(-\frac{4\pi r_c^2\sigma}{3kT_g}\right),\\
\phi&=\frac{2(\gamma-1)}{\gamma+1}\frac{h_{fg}}{R_gT_g}\left(\frac{h_{fg}}{R_gT_g}-\frac{1}{2}\right),\\
r_c&=\frac{2\sigma T_s}{\rho_l h_{fg}\Delta T},
\end{aligned} \tag{5}$$

$$\begin{aligned}
\frac{dr}{dt}&=\frac{\lambda_g\Delta T}{\rho_l r h_{fg}\left(\dfrac{1}{1+4K_n}+3.78(1-\nu)\dfrac{K_n}{P_{rg}}\right)},\\
\nu&=\frac{R_gT_s}{h_{fg}}\left[\alpha-\frac{1}{2}-\frac{2-q_c}{2q_c}\left(\frac{\gamma+1}{2\gamma}\right)c_p\frac{T_s}{h_{fg}}\right],\\
\Delta T&=T_s-T_g.
\end{aligned} \tag{6}$$

In the wet steam flows, the sudden condensation can lead to a very sharp variation of flow parameters. For example, in a very short distance, the water droplet number may increase from zero to a very large figure, say 10^{18} or more. This makes it not easy to get a convergent numerical result. However, the development of TVD schemes provides the possibility of handling such flow fields since they were originally proposed to capture shock waves. In the present work, Harten's TVD scheme [7] is adopted.

3. Numerical results and discussion

In this paper the wet steam flow was investigated in a cascade of turbine rotor tip section, for which Bakhtar *et al.* conducted a series of 2D experiments in a blow-down facility [8].

Two numerical simulations were performed. One calculation aimed at investigating the superheated flow, and the other, the wet steam flow. The flow conditions are shown in Table 1.

Table 1. Flow conditions

	P_0	$T_0-T_s(P_0)$	P_0/P_{s2}
Superheated flow	0.997 Bar	419.3 K	2.36
Condensing flow	1.015 Bar	370.1 K	2.33

The flow inlet angle between the stream and the tangential direction was 128° in both superheated flow and wet steam flow. An 'H' type grid was used and the grid number was 491×91.

Figure 1 shows the surface pressure distributions for both superheated flow and wet steam flow. The numerical results agree well with those of the experimental measurements. Compared to the pressure distribution in superheated flow, the surface pressure in wet steam flow is significantly different at the suction surface from 40% of the axial length to the trailing edge. A pressure knee is also observed on the suction surface in wet steam flow, and this contributes to the rapid condensation.

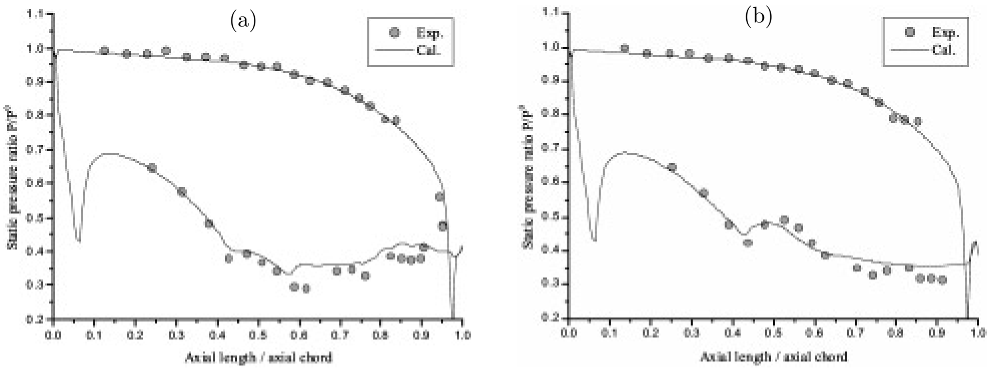

Figure 1. Surface pressure distributions: (a) superheated flow; (b) wet steam flow

In Figure 2 the Mach number distributions for both superheated flow and wet steam flow are presented. Besides the knee in the Mach number contours of wet steam flow at about 40% axial length, the structure of trailing edge shock waves in wet steam flow is different from that in the superheated flow. Bakhtar has illustrated it as a λ shock. The most marked difference between the superheated flow and wet steam flow may exist in the wakes. Referring to Figure 3, the patterns of velocity vectors near the suction surface in wet steam flow are different from those in superheated flow. In the wet steam flow vortices are observed along the suction surface. At the trailing edge these vortices separate from the blade and move with the stream into the wake leading to different structures of wakes as shown in Figure 2. Thus, besides the wetness

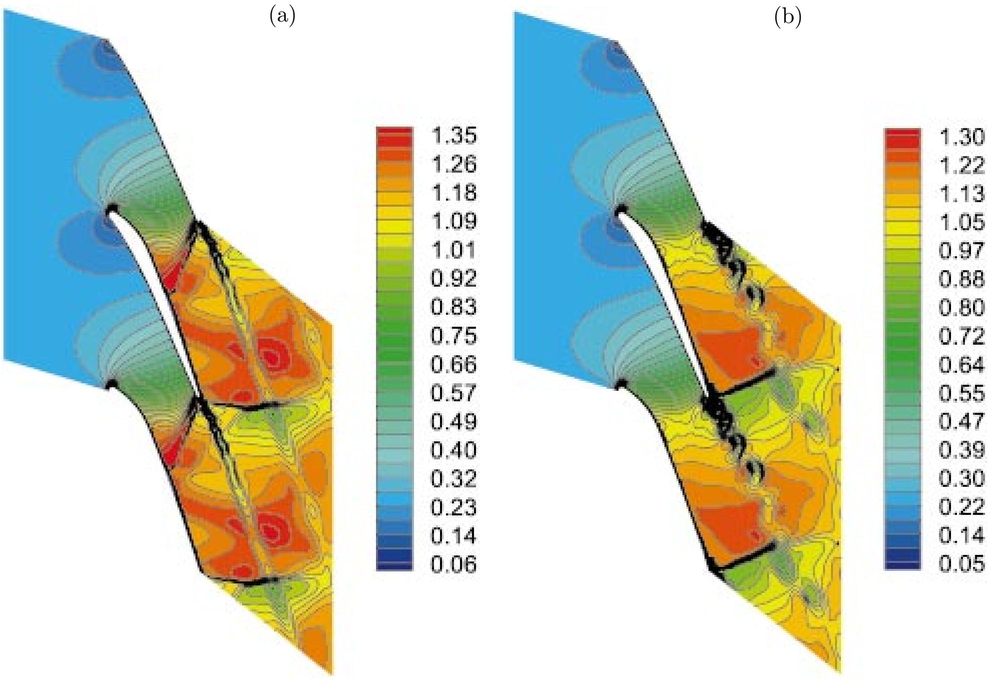

Figure 2. Mach number distributions: (a) superheated flow; (b) wet steam flow

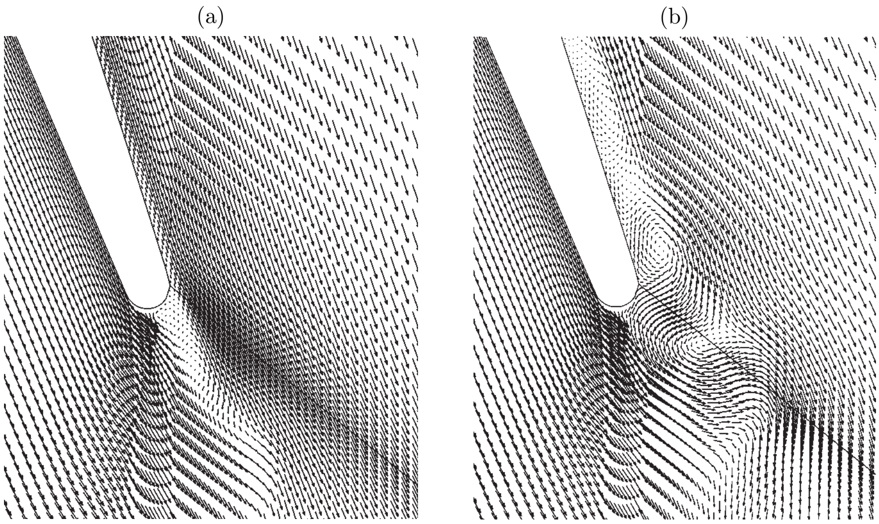

Figure 3. Velocity vectors on the suction surface near the trailing edge:
(a) superheated flow; (b) wet steam flow

losses the condensing flow produces additional aerodynamic losses by changing the flow pattern behind the rapid condensation zone. It can also be predicted that the changes of flow pattern will have effects on the next cascade. Therefore, to design wet steam stages with a wet steam flow solver is practically necessary. Figure 4 shows a comparison between the entropy distributions in superheated flow and wet steam flow. It indicates that in the wet steam flow the rapid condensation occurring at the cascade

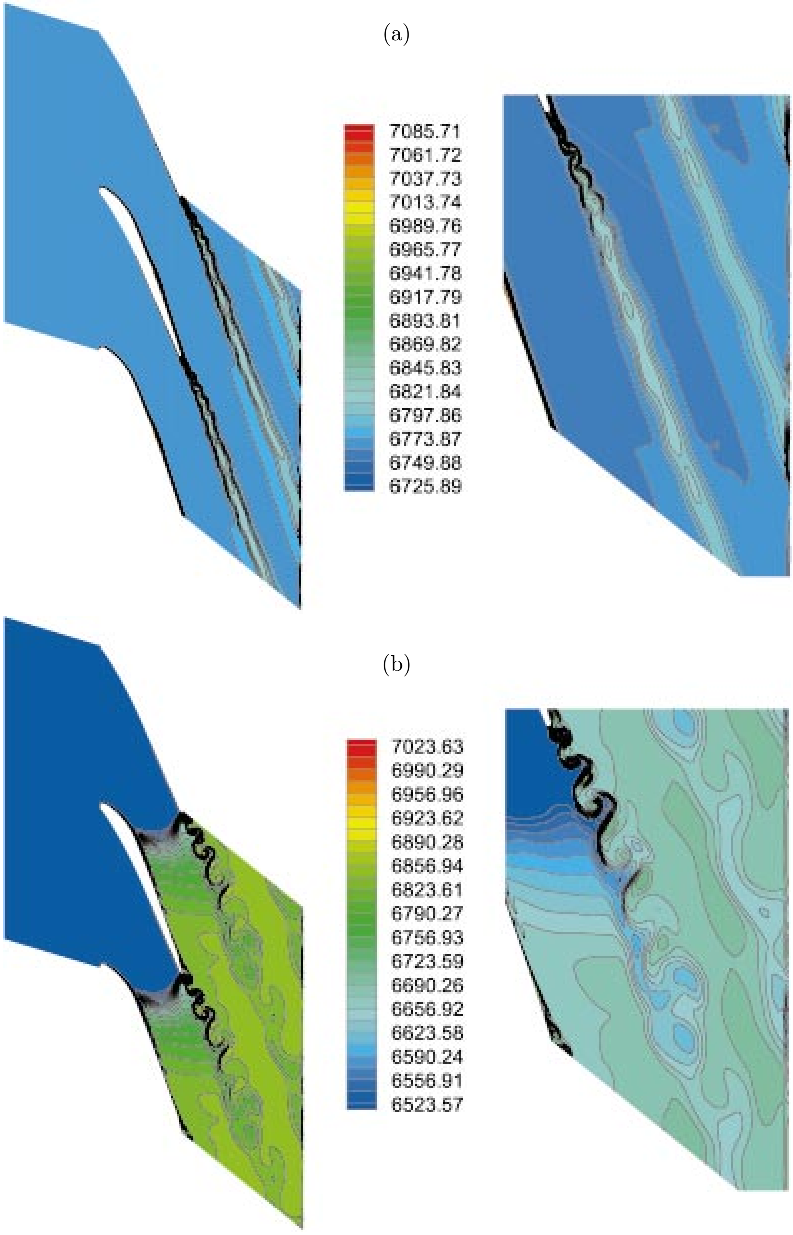

Figure 4. Entropy distributions: (a) superheated flow (b) wet steam flow

throat leads to a very fast increase of the entropy, compared to the superheated flow. The total entropy increase in wet steam flow is also larger than that in superheated flow.

The distribution of the degree of supercooling in wet steam flow is shown in Figure 5. The supercooling degree increases during the course of expansion. It reaches the maximum at the throat. Then the steam suddenly condenses. In the meantime, the supercooling degree drops rapidly and the steam returns to the equilibrium state.

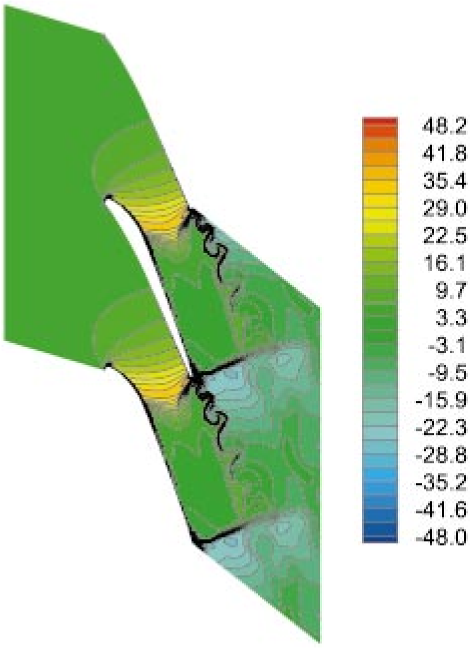

Figure 5. Supercooling degree distribution

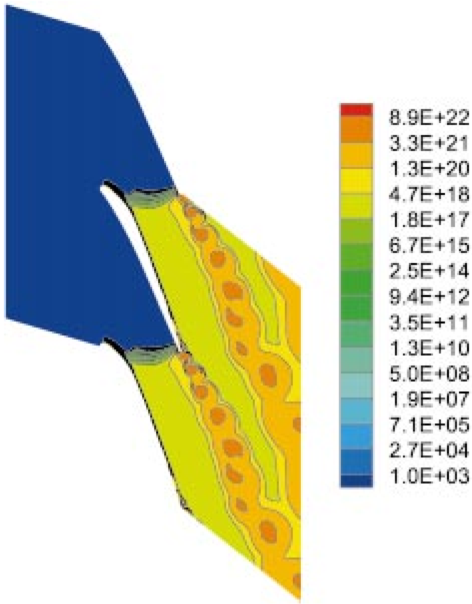

Figure 6. Water droplet number distribution

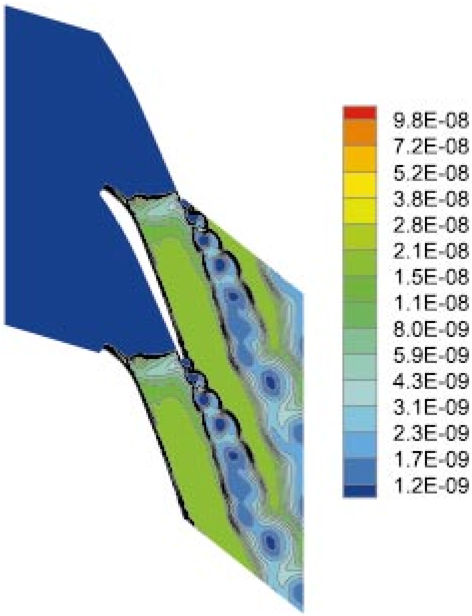

Figure 7. Water droplet radius distribution

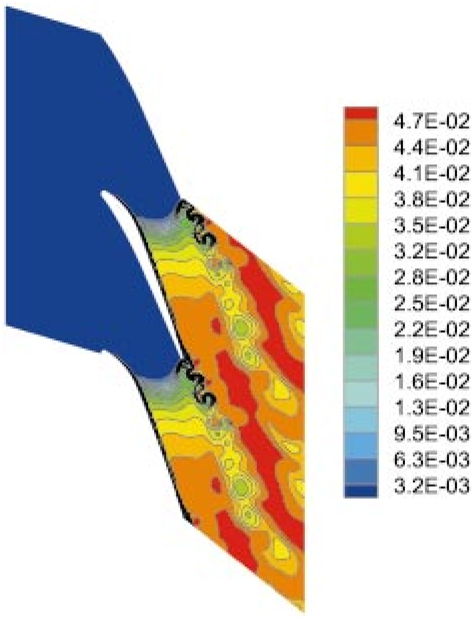

Figure 8. Wetness distribution

Referring to Figure 6, in the rapid condensation zone, the water droplet number increases very quickly from nearly zero to a very large figure. It can also be seen that there are more droplets in the wake than in the main stream. Figure 7 shows the distribution of the water droplet radius. It should be noted that the radius of droplets in the wake is smaller than that in the main stream. This is just contrary to the distribution of the water droplet number. In Figure 8 the wetness distribution is presented. The wetness in the rapid condensation zone increases quickly to a meaningful value. Though the number of water droplets in the wake is more than that in the main stream, the smaller droplet size in the wake leads to the fact that the wetness in the wake is smaller than that in the main stream.

4. Conclusions

A numerical simulation of the wet steam flow in a plane turbine cascade was performed with an Eulerian/Eulerian model. This model was designed to investigate the two-phase wet steam flows with spontaneous condensation.

The numerical simulation showed significant differences between the superheated flow and condensing flow. Compared to the superheated flow, the wet steam flow gives a pressure knee on the suction surface where the rapid condensation occurs. The condensation of steam produces wetness losses. It also brings vortices along the suction surface of the blade. In this case, the existence of vortices significantly changes the flow pattern of the wake region in wet steam flow and leads to additional aerodynamic losses. The investigation shows that it is really important to design a wet steam turbine stage with a wet steam flow solver.

Acknowledgements

The first author is very grateful for the support of the Doctorate Fundation of Xi'an Jiaotong University (Fundation No. DFXJU 2000-18).

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