APPLICATION OF A CELLULAR AUTOMATA MODEL TO GRANULAR FLOW

JAN KOZICKI AND JACEK TEJCHMAN

Civil Engineering Department,
Gdansk University of Technology,
Narutowicza 11/12, 80-952 Gdansk, Poland
tejchmk@pg.gda.pl

(Received 6 May 2002)

Abstract: Paper presents the results of simulations of granular flow with a cellular automata model. Flow of granular material behind a moving retaining wall and in a silo with inserts was numerically analysed. Two different migration schemes during flow were assumed. Advantages and limits of the model were outlined.

Keywords: cellular automata, granular flow, silo, passive state, active state

1. Introduction

Cellular automata models (called also lattice grain models or discrete stochastic models) are commonly used to describe gravitational flow of granular material considered as a system consisting of separate particles [1–5]. They describe granular flow as an upward propagation and diffusion of holes through the lattice composed of cells. Each cell may be empty or filled by the particle of the material. The computation process involves searching through all lattice cells for holes. At each time step, a particle flowing downwards due to gravity can move from one cell to an empty neighboring cell or stays at rest. The filling of one cell causes the formation of a new empty one, which is again filled with the randomly chosen particle. Thus, the migration of holes inside the materials occurs. The transition of holes corresponds to the transport of the material in the opposite direction. The evolution rule describing the state change is probabilistic similarly as in a real granular medium where the shape and dimensions of grains, contact points, contact forces between grains, grain roughness are random. The cellular automata models have both advantages and disadvantages relative to other techniques. Among the advantages are the large number of particles, lack of restrictions for deformations, implementation simplicity and small amount of computer time needed to describe flow. The models provide insight into the main physical features of flow on both microscopic and macroscopic level. The main disadvantage is that the models are purely kinematics and no flow dynamics is involved. Nevertheless, the degree of agreement with laboratory tests is surprising in view of the extreme simplicity of models neglecting the dynamics of the flow process. Kozicki and Tejchman [5] have shown that a satisfactory agreement with experimental results of flow patterns during granular silo flow can be obtained by means of a back analysis of experiments. The results were strongly influenced by the migration rule, distribution of probability values, number of cells and cell dimensions [5].

The intention of the paper is to demonstrate the potential of a two-dimensional cellular automata model to simulate granular flow during two different engineering problems, one within soil mechanics (flow of granular material behind a moving retaining wall) and second concerning mechanics of bulk solids (silo flow in silos with inserts). To visualize the process, different horizontal colored layers were introduced.

During the calculations with a retaining wall, the attention was laid on the effect of different wall movements during a passive mode (wall moves away from the granulate) and an active mode (wall moves against the granulate). Three different wall movements occurring in the practice were investigated: uniform horizontal displacement, rotation around the bottom and rotation around the top.

The simulations of silo flow were performed with a silo including 2 different types of inserts used in the industry: a wedge-shaped cone and an internal small hopper. The effect of the location of inserts on the flow pattern was investigated.

2. Migration rules

In the analyses, two types of grain migration were assumed (Figure 1). To simulate both an active mode of a retaining wall and silo flow, a scheme of Figure 1a was applied. This scheme was proved to be the most realistic in calculations of silo flow [5]. In this scheme, an empty cell is filled by particles from 5 cells located in a horizontal layer above. The probability of the particle transport from filled cells to the empty one was assumed to be p_i ($\sum p_i = 1$).

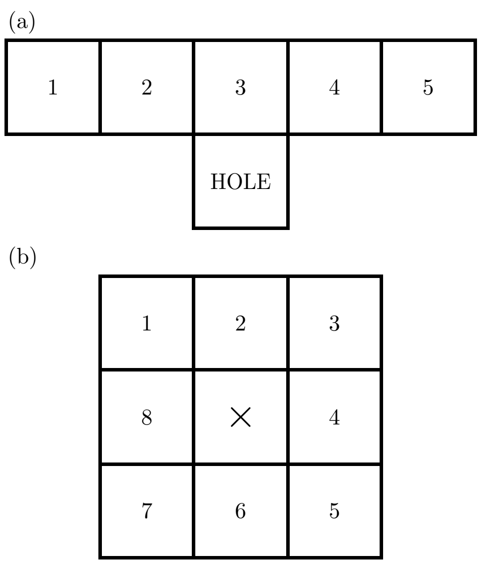

Figure 1. Migration rules for cells (× – conflicting cell)

During an active mode, the movement of the wall was simulated by removing all vertical wall cells every a given number of iterations (*e.g.* 15). In turn, the modeling of discharge in a silo began when the cells along the outlet were released. The holes formed started to move inside the material (opposite to the gravity force) until they came out on the free upper surface.

During simulations of a passive mode of a retaining wall, a completely different migration scheme was used since flow of grains was not caused by gravity (Figure 1b). The movement of the wall was simulated by introducing all vertical wall cells into the material every a given number of iterations (*e.g.* 15). Thus, the flow process began when particles started to move from cells occupied by the wall. They moved next to filled cells causing the occurrence of so-called conflicting cells (cells including two particles). After it, the migration of particles, which earlier occupied the cells, took place.

3. Motion of a retaining wall

A granular specimen included 150 000 cells (500 cells along the length and 300 cells along the height). Figures 2 and 3 present the results of a passive motion of a retaining wall for two different transition probabilities using a migration scheme of

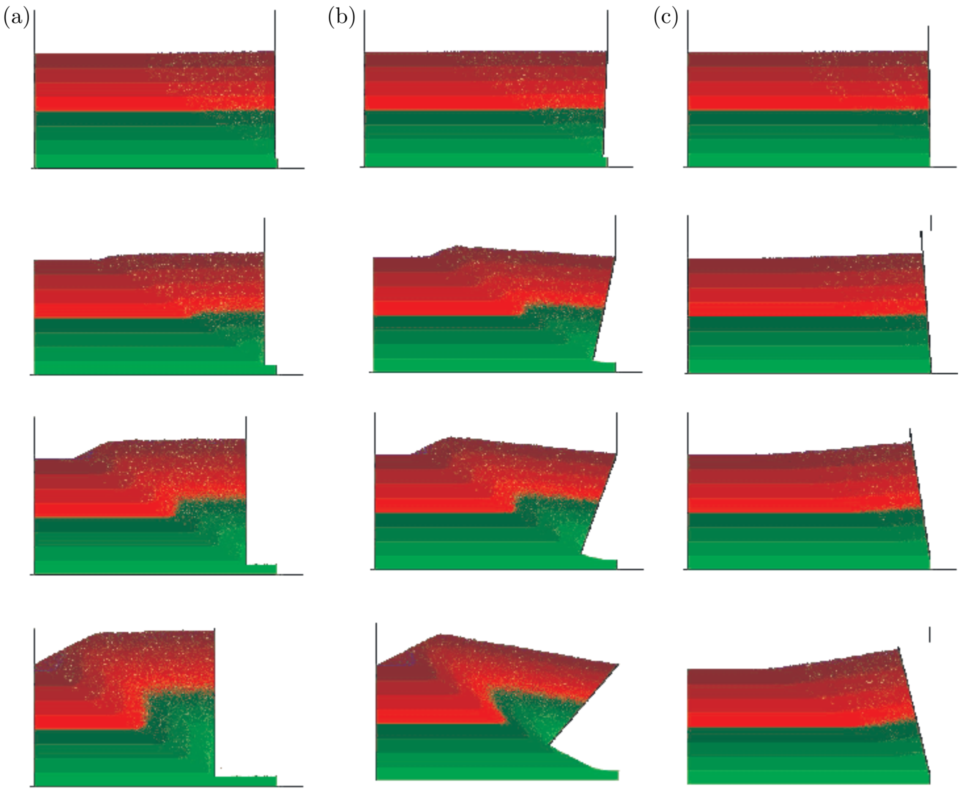

Figure 2. Flow patterns during a passive mode of a retaining wall: (a) translating wall, (b) wall rotating around the top, (c) wall rotating around the bottom (migration scheme of Figure 1b, $p_1 = p_2 = 0.25$, $p_8 = 0.50$, $p_3 = p_4 = p_5 = p_6 = p_7 = 0$)

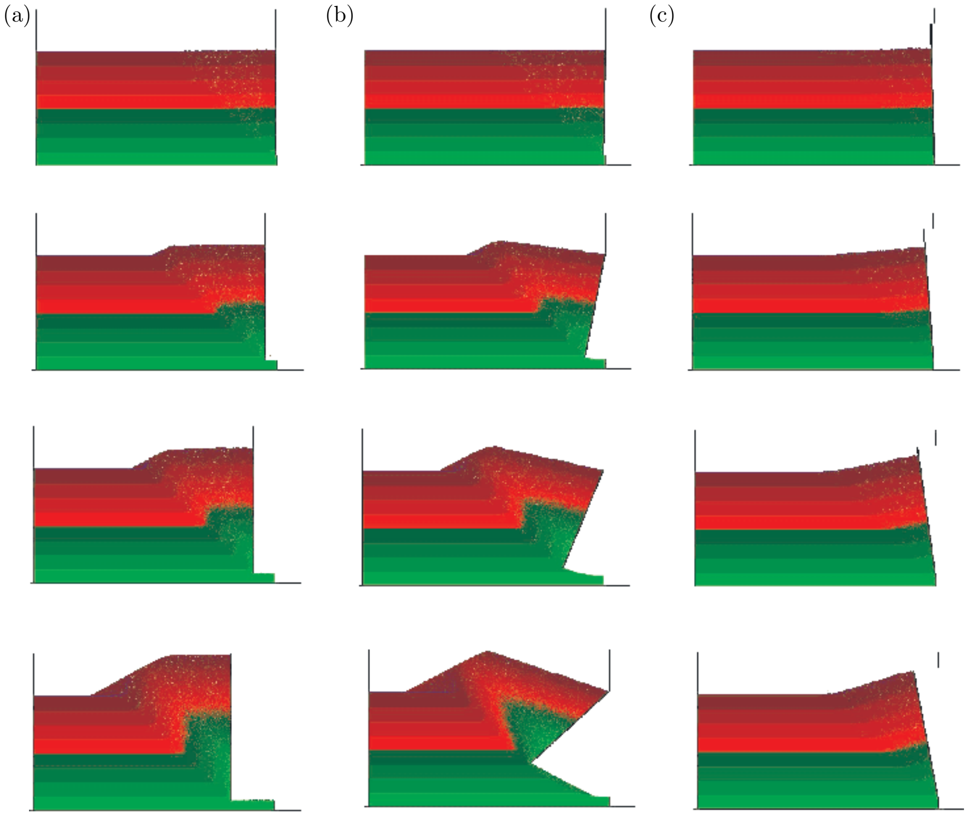

Figure 3. Flow patterns during a passive mode of a retaining wall: (a) translating wall, (b) wall rotating around the top, (c) wall rotating around the bottom (migration scheme of Figure 1b, $p_1 = p_2 = 0.33$, $p_8 = 0.33$, $p_3 = p_4 = p_5 = p_6 = p_7 = 0$)

Figure 1b. In turn, Figure 4 demonstrates the results of an active motion of a retaining wall using a migration scheme of Figure 1a.

In each case, two evident separate regions are always created. Far from the wall, the material is unmovable. In turn, the material behind the wall moves along a shear plane. The inclination of a shear plane from the bottom depends on the type of the wall movement. It is about 35°–40° (Figure 2), 45°–48° (Figure 3) and 45°–55° (Figure 4). By changing probabilistic values, the inclination of a shear plane varies (Figures 2 and 3). Thus, it is possible to achieve realistic inclination angles for both an active and a passive zone. In sand with an angle of internal friction $\phi = 30^\circ$, the inclination of a shear plane is 60° (active case) and 30° (passive case) during an uniform translation of a smooth wall.

4. Silo flow with inserts

Wedge-shaped inserts in silos are among a number of means used to influence the flow pattern and to relieve the feeder [6]. When the insert is located near the transition between the bin and hopper, a significant reduction of wall pressures in the hopper wall is obtained.

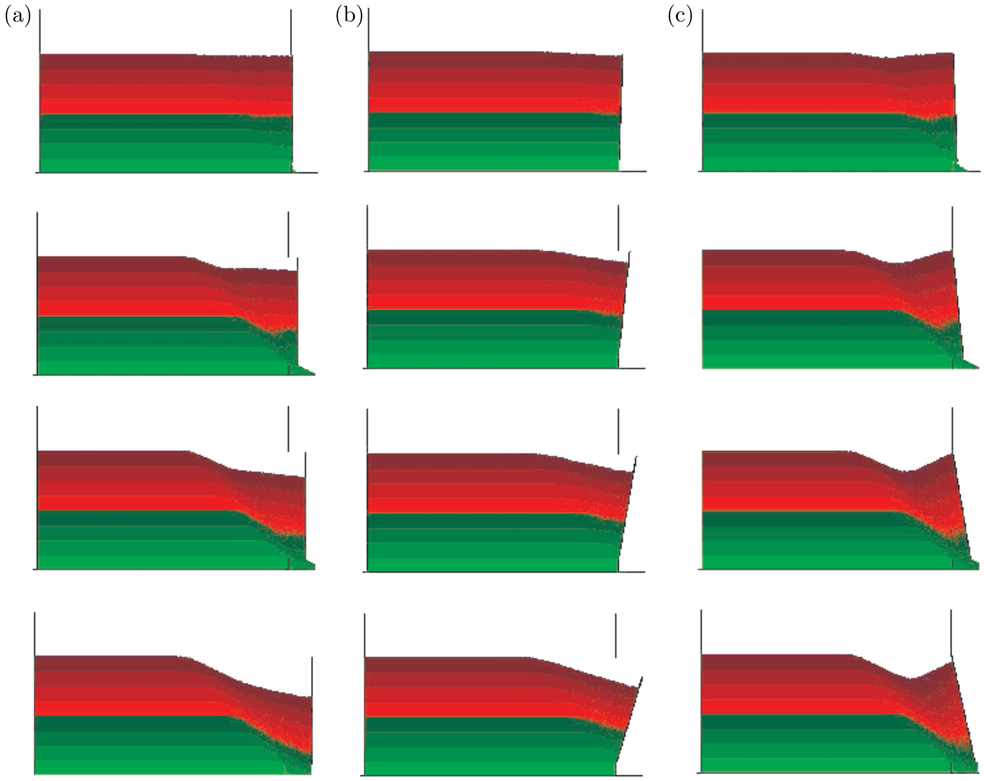

Figure 4. Flow patterns during an active mode of a retaining wall: (a) translating wall, (b) wall rotating around the top, (c) wall rotating around the bottom (migration scheme of Figure 1a, $p_1 = 0.45$, $p_2 = 0.23$, $p_3 = 0.05$, $p_4 = p_5 = 0.14$)

An insert in the form of an internal hopper inside the main hopper (cone-in-cone concept) is applied to obtain mass flow at considerably large hopper inclinations which usually induce funnel flow [7, 8]. The outlet of the internal hopper is usually equal to the outlet of the hopper. The flow pattern is affected mainly by the wall inclination of the internal hopper and the horizontal distance between the main hopper and the internal one.

The calculations were carried out with a wedge-shaped insert located at two different positions along the symmetry axis. During the analysis, 140 000 cells were used. The distribution of probability values was assumed to be symmetric with the largest values at both ends and diminishing towards the center of the layer [5]. Figure 5 demonstrates the flow patterns and distribution of flow rate (number of particles passing through a given cell) during granular flow in a silo with a wedge-shaped insert. A darker region indicates a higher flow rate. The presence of the insert and its position influence significantly the flow pattern and flow rate. The insert decreases the flow rate.

The results with an internal hopper located at the transition between the bin and the main hopper are presented in Figure 6. During this analysis, 200 000 cells were used. Two different wall inclinations of the internal hopper were assumed (Figures 6b and 6c). Flow in a silo without the internal hopper is of a funnel type (Figure 6a). The

(a)

(b)

(c)

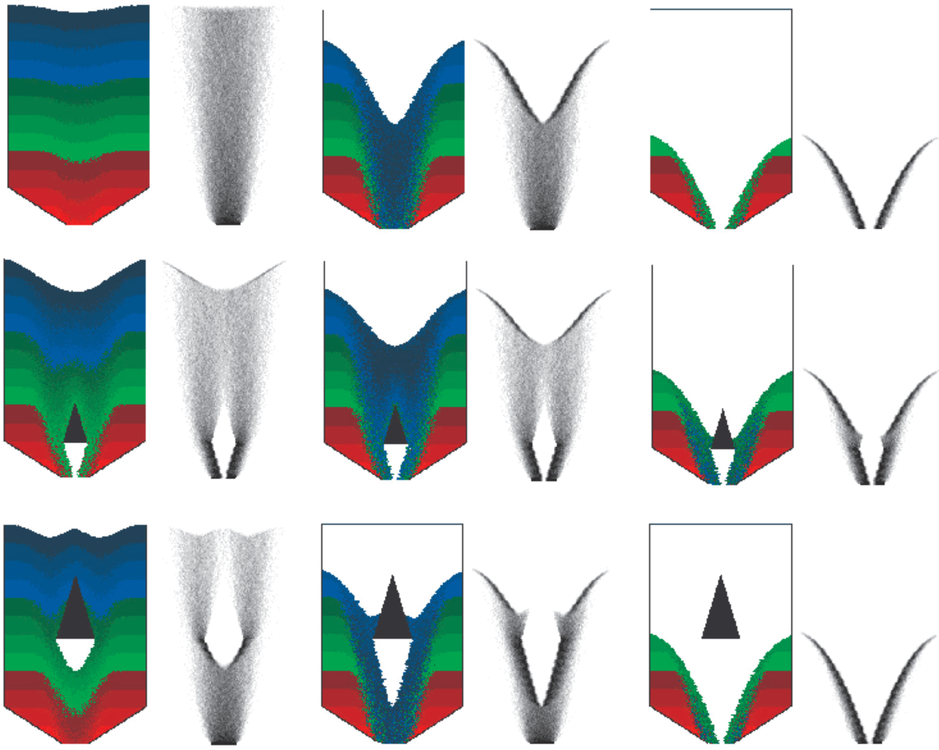

Figure 5. Flow patterns and distribution of flow rate during granular flow: (a) in a silo without insert, (b) and (c) in a silo with insert (migration rule of Figure 1a, $p_1 = p_5 = 0.30$, $p_2 = p_4 = 0.15$, $p_3 = 0.05$)

application of the internal hopper induces mass flow (Figures 6b and 6c) in accordance with experiments [8]. If the wall inclination of the internal hopper is however too small, the material flows in the center too slowly (Figure 6c).

5. Conclusions

Cellular automata simulations are purely kinematic models wherein flow dynamics is not taken into account. However, they are apt to describe consistently the complex behaviour of granular materials during flow on the basis of a back analysis of experiments.

The main parameters governing the motion are the type of the migration rule and transition probabilities.

References

[1] Baxter G W and Behringer R P 1991 *Physica* **D51** 465

[2] Savage S B 1992 *Proc. Int. Conf. on Silos: Silos – Forschung und Praxis*, Karlsruhe, Karlsruhe University, pp. 111–121

[3] Osinov V A 1993 *Continuum Mechanics and Thermodynamics* **6** 51

[4] Martinez J, Masson S and Deserable D 1995 *Proc. Int. Conf. on Silos, Partec95*, Nürnberg, pp. 367–379

[5] Kozicki J and Tejchman J 2001 *Int. J. Storing, Handling and Processing Powder* **13** (3) 267

[6] Strusch J, Schwedes J and Hardow B 1995 *Proc. Int. Conf. on Silos, Partec95*, Nürnberg, pp. 163–182

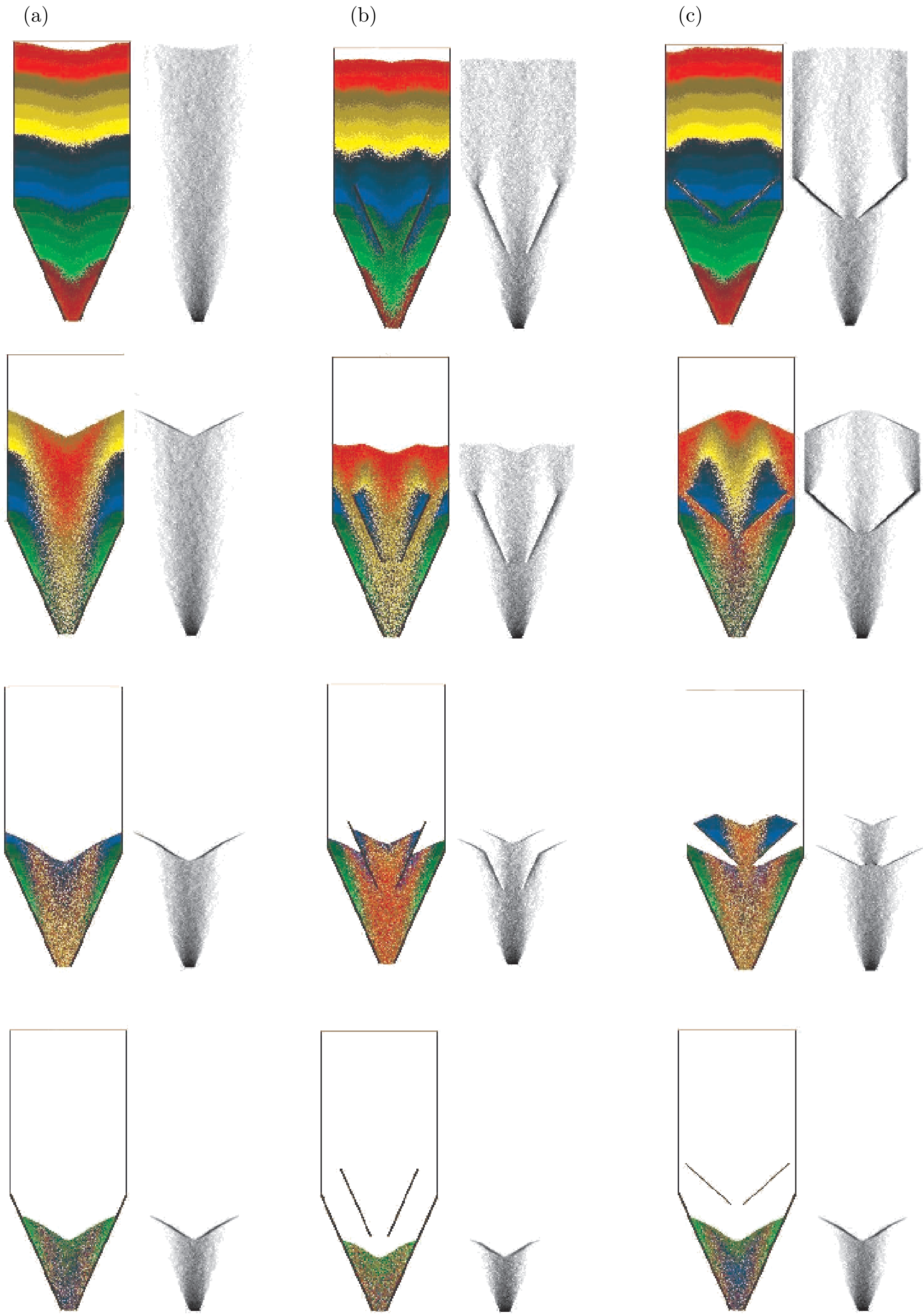

Figure 6. Flow patterns and distribution of flow rate during granular flow:
(a) in a silo without an internal hoper, (b) and (c) in a silo with an internal hoper
(migration rule of Figure 1a, $p_1 = p_5 = 0.30$, $p_2 = p_4 = 0.15$, $p_3 = 0.05$)

[7] Johanson J R 1982 *Bulk Solid Handling* **2** (3)
[8] Eiksa O E, Mosby J and Enstad E E 1995 *Proc. Int. Conf. on Silos, Partec95*, Nürnberg, pp. 417–426