CHAOTIC MOTION OF A MULTI-ARTICULATED TOWER IMMERSED IN WAVING FLUID: NUMERICAL EXPERIMENT

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Abstract: The offshore structures, such as multi-articulated towers, manipulators working in waving fluid are more and more popular structures. Commonly they are modelled as an inverted pendulum. The motion of such tower immersed in waving fluid is under consideration. It is assumed that the tower is hinged to the seabed by the hinge with friction. The joint of tower elements is the hinge with friction, as well. Motion of tower loaded by regular wave fluid force is considered. The numerical experiment shows that the multi-potential systems may move chaotically for some parameters of the tower and the fluid.

Keywords: multi-articulated tower, inverted pendulum, multi-potential system, equilibrium positions, multi-periodic motion, chaotic motion

1. Introduction

The multi-articulated towers are becoming more and more popular in the offshore structures engineering. They are used for mooring the tank in the neighborhood of oil platforms. This removes the probable platform damage due to the tank motion.

Moreover, using the multi-articulated towers is economically useful due to reduced mass of such tower.

The manipulators working immersed in water are modelled as the multi-articulated towers, as well. They are loaded by regular waves, but the response of such a tower can be multi-periodic, or chaotic (see [1]). The multi-articulated towers are modelled as an inverted pendulum (compare with [2]), moored to the seabed by the hinge with one degree of freedom. Each element of the tower has one degree of freedom. Therefore, the multi-articulated tower with n elements has n degrees of freedom. For the considered system (the tower/fluid) the motion equations represent an initial problem, *e.g.* the system of n ordinary differential equations with n unknowns and the initial conditions for the variables and their differentials with respect to time, in the initial time. The double-articulated tower is the subject of the present work.

2. Double-articulated tower

The considered tower consists of two elements (Figure 1). The motion of the tower goes in (x,y) plane. Therefore, the tower has two degrees of freedom: the plane angles between every element axes and the vertical axis. The deflections of the elements are very small, comparing to their displacement, so the elements are treated as rigid bodies. The tower is moored to the seabed by the hinge with friction. The elements are connected to one another by hinges with friction, as well.

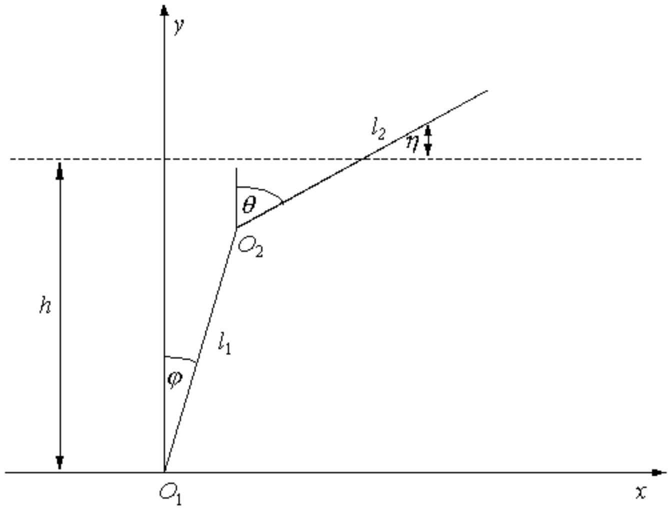

Figure 1. The model of the double-articulate tower

The following external forces acting on the tower elements are included in consideration: the gravity force, the buoyancy force, the wave forces (drag and inertia forces), the forces due to the added mass effect and the friction forces in the hinges. To obtain the wave forces the Morison equation has been used. The lower element is fully immersed in fluid, the upper element is partially immersed. Therefore, the length of the immersed part is treated as a function of both the elements position and the wave elevation. The wave elevation is governed by:

$$\eta = A\cos(kx_s - \sigma t). \tag{1}$$

Using the Equation (1) in the equation of the tower geometry:

$$l_s = \frac{\eta + h - l_1\cos\varphi}{\cos\theta} \tag{2}$$

gives the non-linear equation with respect to the length of the immersed part of the upper element, in implicit form:

$$l_s\cos\theta = h - l_1\cos\varphi + A\cos[k(l_1\sin\varphi + l_s\sin\theta) - \sigma t], \tag{3}$$

where h – fluid depth, A – wave amplitude, T – wave period ($\sigma = 2\pi/T$), k – wave constant, l_1 – the length of the lower element.

3. The forces and elements acting on the element of the tower

The considered forces acting on each tower element are presented in Figure 2. The forces and the moments acting on the lower element are marked with subscript equal to 1, while the forces and moments acting on the upper element are with subscript 2. The moments acting on the lower element are taken with respect to the point O_1, and for the upper element moments are written with respect to the point O_2.

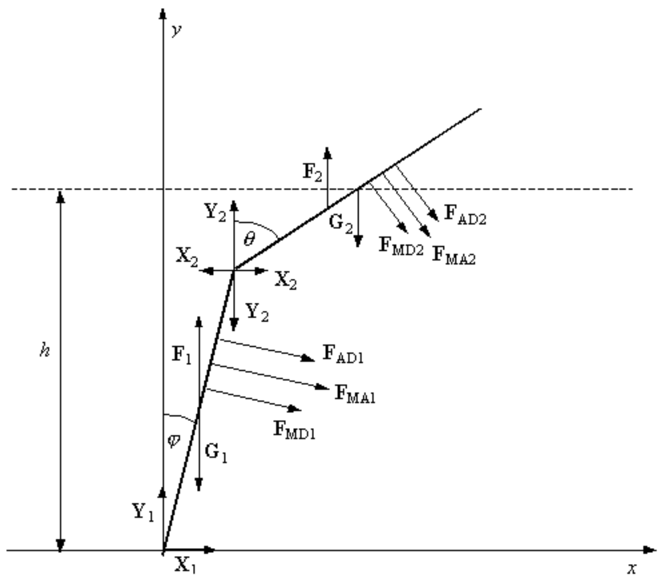

Figure 2. The forces acting on the double-articulated tower

The buoyancy force and the moment due to the buoyancy force are:

$$\mathbf{F}_1 = \rho' g\pi \frac{D^2}{4} l_1 \mathbf{j}, \tag{4}$$

$$\mathbf{F}_2(t,\varphi,\theta) = \rho' g\pi \frac{D^2}{4} l_s(t,\varphi,\theta)\mathbf{j}, \tag{5}$$

$$\mathbf{M}_{F1} = \rho' g\pi \frac{D^2}{4}\frac{l_1^2}{2}\sin\varphi\mathbf{k}, \tag{6}$$

$$\mathbf{M}_{F2}(t,\varphi,\theta) = \rho' g\pi \frac{D^2}{4}\frac{l_s^2(t,\varphi,\theta)}{2}\sin\theta\mathbf{k}, \tag{7}$$

where ρ' is the mass density of the fluid.

The gravity force and the moment due to gravity force:

$$\mathbf{G}_1 = \rho_1 g\pi \frac{D^2}{4} l_1 \mathbf{j}, \tag{8}$$

$$\mathbf{G}_2(t,\varphi,\theta) = \rho_2 g\pi \frac{D^2}{4} l_2 \mathbf{j}, \tag{9}$$

$$\mathbf{M}_{G1} = \rho_1 g\pi \frac{D^2}{4}\frac{l_1^2}{2}\sin\varphi\mathbf{k}, \tag{10}$$

$$\mathbf{M}_{G2}(\theta)=\rho_2 g\pi\frac{D^2}{4}\frac{l_2^2}{2}\sin\theta\mathbf{k}, \tag{11}$$

where ρ_1 is the mass density of the lower element, ρ_2 – the mass density of the upper element, l_2 – the length of the upper element, D – the diameter of both elements.

The wave forces:

– the drag force (and moment)

$$\mathbf{F}_{MD1}=C_D\rho'\frac{D}{2}\mathbf{n}_1\int_0^{l_1}|v_{n1}(t,\varphi,\dot{\varphi})|v_{n1}(t,\varphi,\dot{\varphi})ds, \tag{12}$$

$$\mathbf{F}_{MD2}=C_D\rho'\frac{D}{2}\mathbf{n}_2\int_0^{l_s(t,\varphi,\theta)}|v_{n2}(t,\varphi,\theta,\dot{\varphi},\dot{\theta})|v_{n2}(t,\varphi,\theta,\dot{\varphi},\dot{\theta})ds, \tag{13}$$

$$\mathbf{M}_{MD1}=-C_D\rho'\frac{D}{2}\mathbf{k}\int_0^{l_1}|v_{n1}(t,\varphi,\dot{\varphi})|v_{n1}(t,\varphi,\dot{\varphi})sds, \tag{14}$$

$$\mathbf{M}_{MD2}=-C_D\rho'\frac{D}{2}\mathbf{k}\int_0^{l_s(t,\varphi,\theta)}|v_{n2}(t,\varphi,\theta,\dot{\varphi},\dot{\theta})|v_{n2}(t,\varphi,\theta,\dot{\varphi},\dot{\theta})sds, \tag{15}$$

– the inertia force (and moment)

$$\mathbf{F}_{MA1}=C_M\rho'\frac{D^2}{4}\mathbf{n}_1\int_0^{l_1}a_{n1}(t,\varphi)ds, \tag{16}$$

$$\mathbf{F}_{MA2}=C_M\rho'\frac{D^2}{4}\mathbf{n}_2\int_0^{l_s(t,\varphi,\theta)}a_{n2}(t,\varphi,\theta)ds, \tag{17}$$

$$\mathbf{M}_{MA1}=-C_M\rho'\frac{D^2}{4}\mathbf{k}\int_0^{l_1}a_{n1}(t,\varphi)sds, \tag{18}$$

$$\mathbf{M}_{MA2}=-C_M\rho'\frac{D^2}{4}\mathbf{k}\int_0^{l_s(t,\varphi,\theta)}a_{n2}(t,\varphi,\theta)sds, \tag{19}$$

where C_D, C_M are the drag and inertia coefficients, v_{n1} – the relative velocity of the lower element and fluid particles, v_{n2} – the relative velocity of the upper element and fluid particles, a_{n1} – the acceleration of fluid particles in normal direction of the lower element, a_{n2} – the acceleration of fluid particles in normal direction of the upper element.

The force and the moment due to the added mass effect:

$$\mathbf{F}_{AD1}=-(C_M-1)\rho'\pi\frac{D^2}{4}\frac{l_1^2}{2}\ddot{\varphi}\mathbf{n}_1, \tag{20}$$

$$\mathbf{F}_{AD2}=-(C_M-1)\rho'\pi\frac{D^2}{4}\frac{l_s^2(t,\phi,\theta)}{2}\ddot{\theta}\mathbf{k}, \tag{21}$$

$$\mathbf{M}_{AD1}=-(C_M-1)\rho'\pi\frac{D^2}{4}\frac{l_1^3}{3}\ddot{\varphi}\mathbf{n}_1, \tag{22}$$

$$\mathbf{M}_{AD2} = -(C_M - 1)\rho'\pi\frac{D^2}{4}\frac{l_s^3(t,\phi,\theta)}{3}\ddot{\theta}\mathbf{k}. \tag{23}$$

It is assumed, that friction occurs in each hinge. To obtain the friction moments in the hinges the kinetic friction law is used:

$$\mathbf{M}_{FR1} = \rho_F\sqrt{X_1^2 + Y_1^2}, \tag{24}$$

$$\mathbf{M}_{FR2} = \rho_F\sqrt{X_2^2 + Y_2^2}, \tag{25}$$

where ρ_F is the kinetic radius of friction, X_1, Y_1 are the friction forces in hinge O_1, X_2, Y_2 are the friction forces in the hinge between the tower elements *i.e.* in the point O_2.

4. The motion equations

The Newton-Euler motion equations for rigid bodies are used to obtain the motion equation of the double articulated tower immersed in the waving fluid. Therefore, the motion of the tower is described by the initial problem:

$$\begin{aligned} a_1\ddot{\varphi} =& (a_2 + a_3)\ddot{\theta}\cos(\theta-\varphi) + a_4(\dot{\theta})^2\sin(\theta-\varphi) + (a_5 + a_6)\sin\varphi \\ &+ a_7\cos(\theta-\varphi) + a_8 - M_{FR1}\,\mathrm{sign}(\dot{\varphi}), \\ b_1\ddot{\theta} =& b_2[\ddot{\varphi}\cos(\theta-\varphi) + (\dot{\varphi})^2\sin(\theta-\varphi)] + b_3\sin\theta + b_4 - M_{FR2}\,\mathrm{sign}(\dot{\theta}) \end{aligned} \tag{26}$$

with the initial conditions:

$$\varphi = \varphi_0 \quad \text{for} \quad t = 0, \tag{27}$$

$$\theta = \theta_0 \quad \text{for} \quad t = 0, \tag{28}$$

$$\dot{\varphi} = \dot{\varphi}_0 \quad \text{for} \quad t = 0, \tag{29}$$

$$\dot{\theta} = \dot{\theta}_0 \quad \text{for} \quad t = 0, \tag{30}$$

where $a_1 = I_{S1} + m_1 l_1 l_{C1}^2 + 2m_2 l_2 l_1 l_{C1} + (C_M - 1)\rho'\pi D^2 l_1^3/12$, $a_2 = -2m_2 l_2 l_{C1} l_{C2}$, $a_3 = -2(C_M - 1)\rho'\pi D^2 l_s^2(t,\varphi,\theta)/8$, $a_4 = 2m_2 l_2 l_{C1} l_{C2}$, $a_5 = (G_1 + 2G_2)l_{C1}$, $a_6 = 2F_2 l_{C1}$, $a_7 = 2(F_{MD2} + F_{MA2})l_{C1}$, $a_8 = M_{MD1} + M_{MA1} - M_{FR1}\,\mathrm{sign}(\dot{\varphi})$, $b_1 = I_{S2} + m_2 l_2 l_{C2}^2 + (C_M - 1)\rho'\pi D^2 l_s^3(t,\varphi,\theta)/12$, $b_2 = -m_2 l_2 l_1 l_{C2}$, $b_3 = G_2 l_{C2}$, $b_4 = M_{MD2} + M_{MA2} - M_{FR2}\,\mathrm{sign}(\dot{\theta})$, $m_1 = \rho_1\pi D^2/4$, $m_2 = \rho_2\pi D^2/4$.

The system of differential equations given above is nonlinear in implicit form. To solve such system of equations the implementations and modification of proper numerical method to solve the systems of ordinar differential equation should be constructed.

5. The numerical experiment

To perform the numerical experiment a computer program has been created following the algorithm presented in [3].

The aim of this paper is to show the chaotic motion of the multi-articulated tower loaded by regular waving fluid. It is well known that the chaotic motion occurs in multi-potential systems. The considered double-articulated tower may possess one stable equilibrium position, *i.e.* both elements have stable position at angle position equal to zero. The other case is for tower with three equilibrium positions: one of the elements has three equilibrium positions (two stable ones and one unstable) and the other element has one stable equilibrium position at the position angle equal to zero.

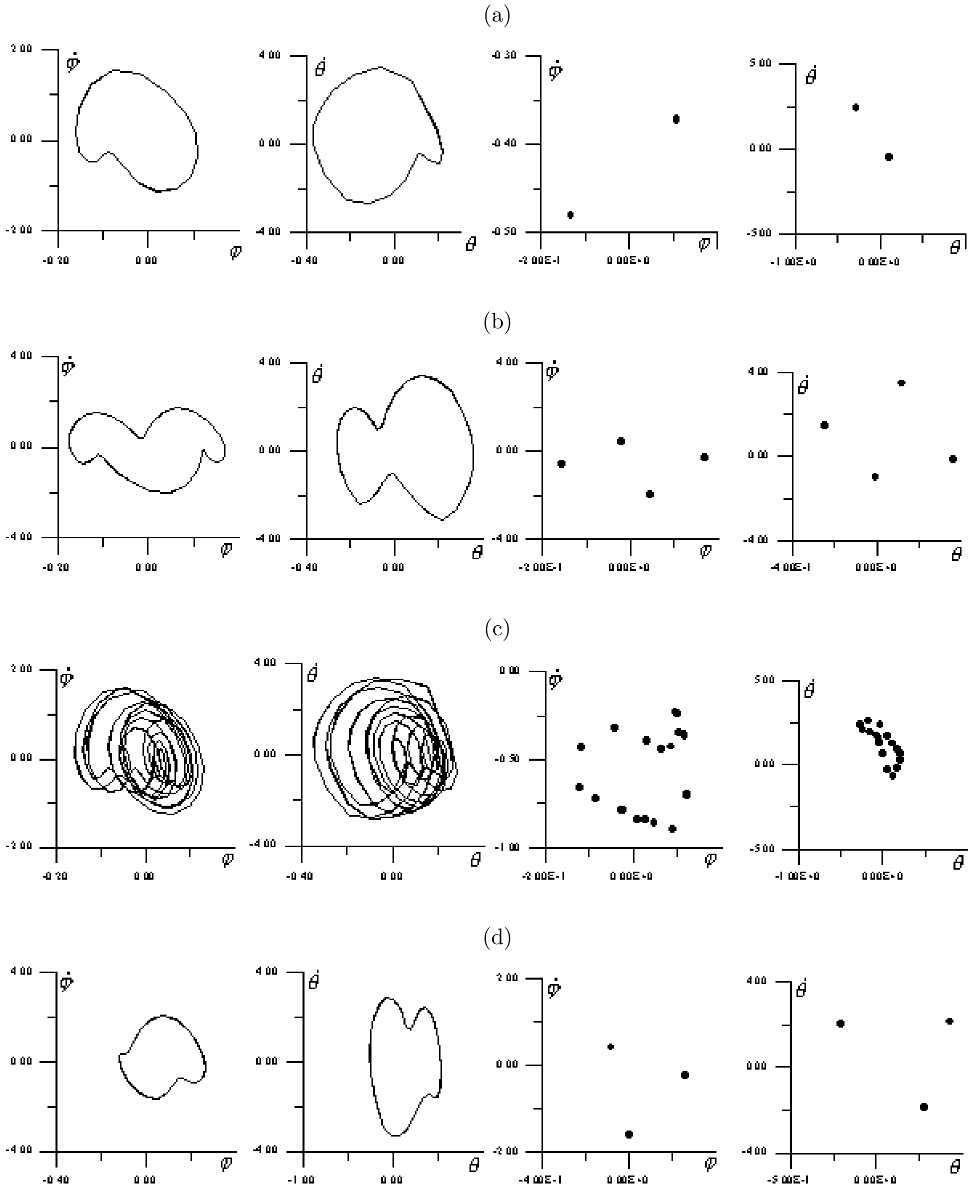

Figure 3. The phase planes and the Poincaré cross-sections for two degrees of freedom of the double-articulated tower under wave loading with period (a) $T=3.8$s, (b) $T=3.5272$s, (c) $T=3.1$s, (d) $T=2.8$s

It is also possible that the tower has 9 equilibrium positions, each element has three equilibrium positions (one unstable and two stable ones).

Example. The double-articulated tower with 9 equilibrium positions is considered. The tower parameters are: $\rho_1 = 1313.47\,\text{kg/m}^3$, $\rho_2 = 648.66\,\text{kg/m}^3$, $l_1 = 1\,\text{m}$, $l_2 = 1\,\text{m}$, $h = 1.3\,\text{m}$.

The influence of the wave period changes on the tower response has been examined. For quite a long period of fluid waving the period of tower response is equal to the period of the wave. For wave with period $T = 3.8$s (Figure 3a) both elements of the tower move between proper positions of the stable equilibrium positions. Moreover, the period of the tower motion is two times longer than the wave period. The decreasing of the wave period ($T = 3.5272$s) yields doubling of the tower motion period (Figure 3b). Figure 3c consists of the phase planes and Poincaré cross-sections of the response of the tower loaded by wave with period $T = 3.1$s. The period of the tower motion is 16 times longer than the period of the loading. The observed 'route to chaos' suggests that the double-articulated tower may perform the chaotic motion. Other calculations for double-articulated tower show the considered tower responses with periods 5, 7, and 9 times longer than the wave period. While the tower is loaded by wave with period $T = 2.8$s the period of the tower motion is 3 times longer than the period of loading (see Figure 3d). The conclusion of the Sharkovsky theorem (see [4]) is that the period of the tower motion can be any multiple of the loading period.

6. Conclusions

The dynamic response of the double-articulated tower under wave loading is presented in this work. The motion of the considered tower is described by a system of ordinary differential equation with initial conditions. Numerical experiments show that the systems (the double-articulated tower)/(waving fluid) with the tower multiperiodic or chaotic motion do exist.

References

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