A DESIGN OF PIEZOELECTRIC VIBRATION ABSORBER WITH THE CAPACITIVE ADJUSTMENT

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(Received 24 April 2002)

Abstract: In the paper the problem of vibration damping via piezoelectric absorbers capacitively shunted is studied. The considered system is a simply supported beam with the absorber composed of piezoceramic patches joined by means of brackets in a uniform distance from the beam surface, symmetrically on both sides of the beam. Due to the applied damping strategy piezoceramic actuators are shunted with a changeable capacitor in the external circuit to tune the resonance frequencies of the system. The analysis is completed with results of numerical simulations.

Keywords: vibrations, semi-active damping, piezoelectric absorber, capacitive shunting

1. Introduction

Piezoelectric distributed actuators and sensors have become popular for control purposes of continuous structures. Of particular interest of many applications is minimising vibration amplitude, especially in resonant regions. Beside active damping concepts, the passive damping obtained by capacitively shunted piezoelectric elements is being developed (*cf.* [1, 2]). The main effect of shunting is to change electrically the dynamic stiffness of the system and therefore, to modify its natural frequencies. The comparison of active and passive damping of thermally induced vibrations of beams with piezoelectric layers can be found in [3]. Suppression of circular plate vibrations by using annular piezoelectric layers with capacitive circuit is analysed in [4].

The presented paper develops the design of semi-active absorbers of beam or one-dimensional plate transverse vibrations. In the proposed device configuration rectangular piezoelectric patches (actuators) are mounted in a uniform distance from the beam's upper and lower surfaces by means of brackets, which are joined with the beam perpendicularly to its longitudinal axis. The capacitive shunting is used to alter dynamic characteristics of the structure. The action of the piezoelectric absorber is substituted by bending moments distributed along the lines of brackets.

The appropriate shunt capacitance results in changing the chosen resonant frequencies of the structure within the tuning frequency bands related to the short-circuit and open-circuit of the absorber. Therefore, the piezoelectric absorber can be used to diminish vibration at a specific excitation frequency range. The numerical results show the influence of the shunting capacity and of the distance between the piezoceramic actuator and the beam surface on the dynamic characteristics of the system.

2. Model of the system and fundamental relations

The considered system is a visco-elastic simply supported beam of the length l, width b and thickness t_b. The piezoelectric absorber is located between the co-ordinates x_2 and x_3. The device is composed of rectangular piezoceramic PZT (lead zirconate titanate) elements of the width b, thickness t_p, and length $l_p = x_3 - x_2$, which is far smaller comparing with that of the beam. They are mounted on the beam's both sides by brackets in the same relatively small distance e from the beam surface. It is assumed that the brackets provide approximately an articulated joint along the piezoelement edges perpendicular to the beam's longitudinal axis. Both piezoceramic patches (actuators) are polarised in the thickness direction and their surface electrodes are connected with the external capacity C_{sh}. The beam transverse vibrations are excited by a harmonic load distributed across the beam at the co-ordinate x_1 with the intensity $q(t)$. The system is schematically shown in Figure 1.

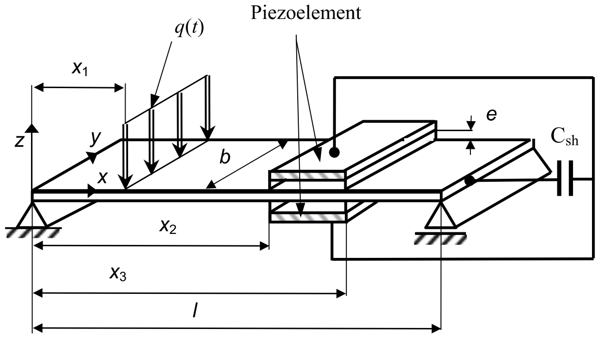

Figure 1. Geometry of the beam with the piezoelectric absorber

2.1. Modelling of the absorber/beam interaction

The one-dimensional piezoelectric effect is described by the following constitutive relations:

$$\sigma_1 = E_p \varepsilon_1 - e_{31} E_3, \tag{1}$$

$$D_3 = e_{31} \varepsilon_1 + \epsilon_{33} E_3, \tag{2}$$

where σ_1 – stress, ε_1 – strain, E_p – Young modulus, D_3 – electric displacement, E_3 – electric field intensity, e_{31} – piezoelectric constant, ϵ_{33} – dielectric permittivity, the subscribes 1 and 3 indicate the directions which coincide with the x and z axes, respectively.

Assuming an extremely high stiffness of the brackets, which stay perpendicular to the deformed beam's longitudinal axis, the actuator strain ε_1 is related to the beam transverse displacement w according to the simple geometric formula:

$$\varepsilon_1 = -\left(\frac{t_b}{2} + e\right)\frac{\partial^2 w}{\partial x^2}, \tag{3}$$

where e – distance of the actuator from the beam surface (see Figure 1).

The charge Q stored on the surface electrodes is obtained after integrating the electric displacement, Equation (2), and substituting the electric field/voltage relation:

$$Q = e_{31}\int_A \varepsilon_1 dA - C_p V, \tag{4}$$

where C_p – capacity of piezoelectric element, $C_p = A\epsilon_{33}/t_p$, V – voltage, $V = -E_3 t_p$, A – effective surface electrode area.

Using the standard relation between the voltage and capacity C_{sh} in the shunt circuit:

$$Q = C_{sh} V, \tag{5}$$

the voltage $V(t)$ produced by the deformed piezoelement is described as:

$$V = \frac{e_{31}}{C_p + C_{sh}}\int_A \varepsilon_1 dA. \tag{6}$$

Taking into account the joint conditions of the actuator a unidirectional strain state is assumed. In the case of a small mass of the element in relation to the beam, it is reasonable to neglect the longitudinal inertia forces and to determine the average strain ε_p by strains of the actuator edges as follows:

$$\varepsilon_p = \varepsilon_1(x_3) - \varepsilon_1(x_2). \tag{7}$$

The stress in the actuator cross-section is given by the constitutive relation Equation (1). After eliminating the voltage from Equation (1) by means of Equation (6) and performing the indicated integration, the stresses in the piezoelements can be expressed as:

$$\sigma_p = \varepsilon_p\left(E_p + \frac{e_{31}^2 b(x_3 - x_2)}{t_p(C_p + C_{sh})}\right). \tag{8}$$

The longitudinal forces acting on the brackets create a pure bending of the beam by moments distributed with the constant intensity m_a:

$$m_a = \sigma_p t_p\left(\frac{t_b}{2} + e\right). \tag{9}$$

Substituting Equation (8) with Equation (7) into Equation (9) and taking into account Equation (3) the bending moment intensity generated by the absorber can be rewritten in the form:

$$m_a = -t_p\left[E_p + \frac{e_{31}^2 b(x_3 - x_2)}{t_p(C_p + C_{sh})}\right]\left(\frac{t_b}{2} + e\right)^2\left(\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=x_3} - \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=x_2}\right). \tag{10}$$

The moments produced by the absorber depend on the geometrical parameters, electromechanical properties of piezoelectric material, beam curvature in the cross-sections corresponding with the brackets' location and also the shunting capacity C_{sh}.

Therefore, the dynamic response of the system can be modified by changing the capacitance in the external circuit. This ability of the piezoelectric absorber is the base of the concept of the semi-active reduction of transverse vibrations.

2.2. Equation of motion and solution

For analysis the beam is divided into four parts due to the acting force cross-section and location of the absorber (see Figure 1). The equation of free vibrations for each segment of the visco-elastic beam can be written as:

$$E_b^* J \frac{\partial^4 w}{\partial x^4} + \rho_k t_b b \frac{\partial^2 w}{\partial t^2} = 0, \tag{11}$$

where J – cross-sectional moment of inertia, ρ_k – equivalent mass density of the classical beam sections ($k = b$) and the section with the absorber ($k = d$), respectively, $\rho_d = \frac{\rho_b t_b + 2\rho_p t_p}{t_b + 2t_p}$, E_b^* – complex form of the Young modulus referring to the Kelvin-Voigt model of visco-elastic material:

$$E_b^* = E_b(1 + i\omega\tau_0) \tag{12}$$

with τ_0 – retardation time, E_b, – elastic Young modulus, $i = \sqrt{-1}$.

The governing equations of the particular beam sections satisfy the boundary conditions corresponding to the simply supported edges at $x = 0$ and $x = l$, the continuity conditions of deflection, slope, bending moment and transverse force between sections ($x = x_1, x_2, x_3$).

The continuity of transverse force at $x = x_1$, where the external load is applied, has the form:

$$\left.\frac{\partial^3 w}{\partial x^3}\right|_{x=x_1^-} = \left.\frac{\partial^3 w}{\partial x^3}\right|_{x=x_1^+} + \frac{bq(t)}{E_b^* J}\,. \tag{13}$$

The beam vibrations are coupled with deformations of the piezoelements and their interaction in the form of the generated moments m_a. The continuity conditions of bending moment at the cross-sections related to the absorber edges ($x = x_2, x_3$) are as follows:

$$\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=x_2^-} = \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=x_2^+} - \frac{bm_a}{E_b^* J} \quad \text{and} \quad \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=x_3^-} - \frac{bm_a}{E_b^* J} = \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=x_3^+}. \tag{14}$$

Other boundary and continuity conditions have a well-known form.

In the considered case the steady-state response is analysed, hence the solutions of equations of motion are harmonic with the same angular velocity ω as the excitation:

$$w_j(x,t) = W_j(x)\exp(i\omega t), \quad j = 1,2,3,4, \tag{15}$$

where the subscript j indicates the beam section.

The functions $W_j(x)$ are the solutions in the spatial domain of the boundary problem formulated by the described above boundary and continuity conditions (*cf.* [5]):

$$W_j(x) = A_j \exp(kx) + B_j \exp(-kx) + C_j \exp(ikx) + D_j \exp(-ikx), \tag{16}$$

where k denotes the wave number defined separately for the beam section with the absorber ($j = 3$) and other sections ($j = 1,2,4$). The constants A_j, B_j, C_j, D_j are

Table 1. Material parameters

Parameter		Beam	Actuator
E_b, E_p	Nm^{-2}	$2.11\cdot 10^{11}$	$6.3\cdot 10^{10}$
ρ_b, ρ_p	kgm^{-3}	7800	7280
τ_0	s	$5\cdot 10^{-6}$	—
$d_{31}=E_p e_{31}$	mV^{-1}	—	$1.9\cdot 10^{-10}$

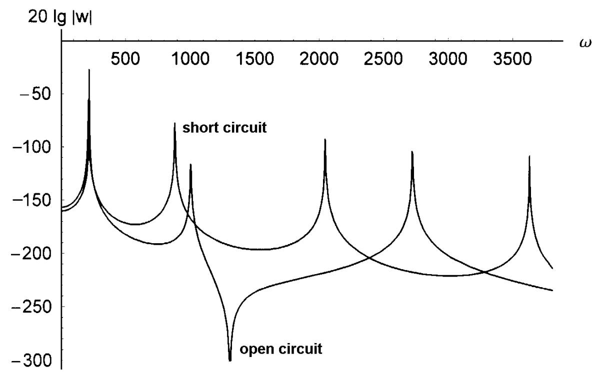

Figure 2. Near field beam deflection response, $x=325$mm

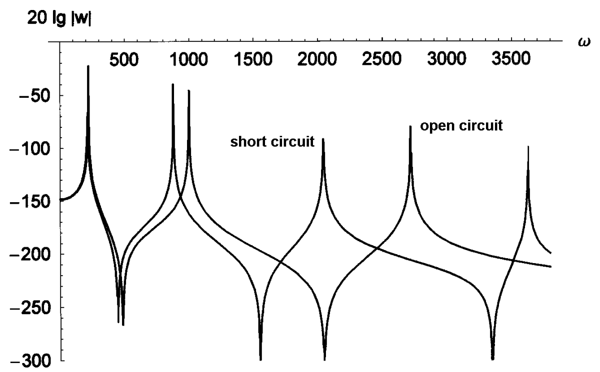

Figure 3. Far field beam deflection response, $x=55$mm

obtained from the system of algebraic equations given by the boundary and continuity conditions.

3. Results

Calculations are performed for the simply supported visco-elastic beam of the length $l=380$mm, width $b=40$mm and thickness $t_b=2$mm. The piezoelectric absorber position is determined by the co-ordinates $x_2=320$mm and $x_3=358$mm. Active elements are the rectangular patches of dimensions $38\times 40\times 0.5$mm and made of the piezoceramic PZT (G-1195) material. They are mounted between the brackets in the distance $e=1$mm from the beam surface. Material parameters of the beam and piezoceramic elements used in calculations are listed in Table 1. The external loading is the harmonic force of amplitude 10N distributed across the beam at the co-ordinate $x_1=75$mm.

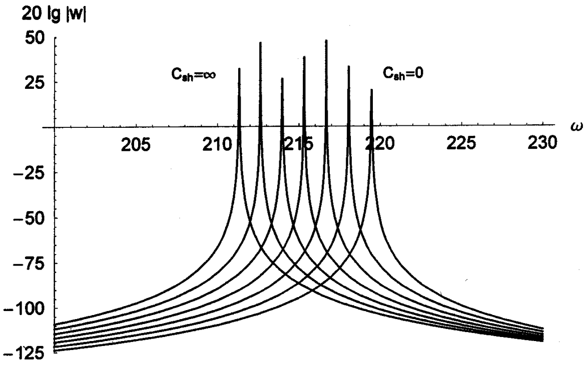

Figure 4. Effect of the capacitance shunting on the first mode response

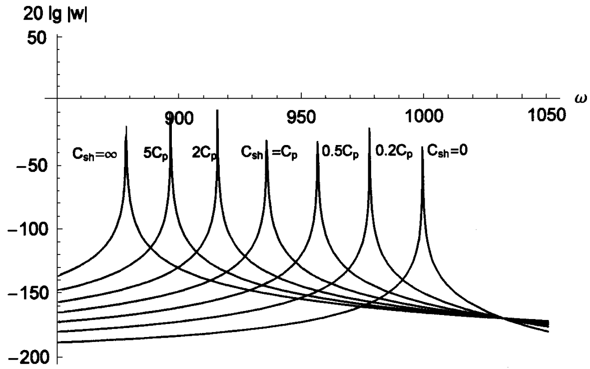

Figure 5. Effect of the capacitance shunting on the second mode response

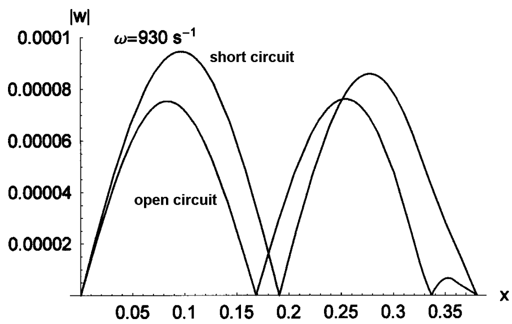

Figure 6. Spatial beam response in the second resonance region

In Figures 2 and 3 the amplitude-frequency characteristics of transverse vibrations at the near field point $x=325$mm and far field point $x=55$mm are presented. The plots illustrate the influence of the shunting capacity for its extreme values $C_{sh}=0$ (open circuit) and $C_{sh}\to\infty$ (short circuit *i.e.* when C_{sh} is very large compared to C_p) on the dynamic response. The effect of variations in values of the shunting capacity C_{sh} on the first and the second resonance frequencies calculated at the near field point is demonstrated in Figure 4 and Figure 5, respectively. As an example, the de-

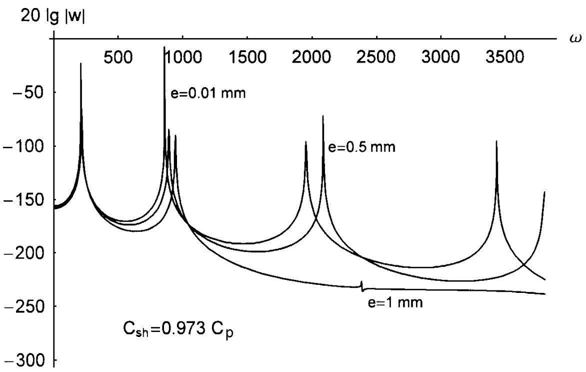

Figure 7. Effect of the absorber distance parameter e on the beam response

flection mode referring to the second resonance frequency region is shown in Figure 6. In the case of the open circuit the disturbance created by the absorber device can clearly be seen. The presented characteristics prove that the resonance frequencies become lower with increasing the shunt capacitance. The range of changes is greater for higher vibration modes. As it is shown in Figure 7 the resonance frequencies of the system also depend on the distance e between the piezoelement and the beam. The technically acceptable increasing of the parameter e results in the dynamic stiffening of the system, thus its resonance frequencies raise.

4. Final remarks

The analysis and results of numerical simulation confirm the concept of semi-active vibration control based on the capacitively shunted piezoceramic elements. The dynamic response of the flexible system can be modified by a selection of the shunting capacity and thus, the influence on the natural frequencies of the structure. In the proposed absorber design the tuning effect can also be corrected in terms of the resonant amplitudes and the frequency band by changing the actuator-structure distance.

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