AN ANALYSIS OF INFLUENCE OF SMA MATERIAL ON DYNAMIC PROPERTIES OF A SPRING TAKING INTO ACCOUNT CONSTRUCTIONAL FRICTION

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Abstract: In this paper, an analysis of forced transverse harmonic vibration with constant changes of the exciting force in a dynamic system consisting of two beams is presented together with a model of a leaf spring. In this junction both beams are made of shape memory material (SMA). The constructional friction is also taken into account. For the purpose of the analysis an equivalent linearization of the elasticity-damping features of the beams was conducted. As an effect of linearization a junction was acquired with both stiffness of elements and viscosity damping. The obtained differential equations were solved using the asymptotic Bogolubov-Krylov-Mitropolski method. The results of the conducted simulation are presented on graphs, showing the influence of unitary pressure on the system's damping properties.

Keywords: control, SMA, constructional friction, connection, linearization

1. Introduction

Movement in real mechanical systems is bound with vibration damping which is a manifestation of dissipation of mechanical damping (aero- and hydrodynamic damping), *i.e.*:

1. friction in mobile junctions (bearings, ways, *etc.*),
2. friction in immobile junctions (riveted, forced, wedged, *etc.*) called construction friction.

The nature of constructional friction is considered taking into account the elastic deformation of the system under stress. A slip of the surfaces of joined materials occurs and friction forces appear, while the system is stressed. Those forces cause amplitude decrease due to the dissipation of mechanical energy.

Researches on constructional friction are related to simplified models of joints and are done with the following assumptions, given in [1]:

- the material the joints are made of is ideally elastic,
- Coulomb's law describes the intensity of friction forces on the surface of the slip of joined elements,
- unitary pressure in the joint has uniform distribution.

As regards quality, the shape memory alloys (SMA) are a new group of construction materials. Counted among SMA are two-ingredient alloys consisting of metals (one of which stands right to chrome in the periodic table, an the other one stands left to it), and alloys of noble metals. In certain conditions, alloys of copper and uranium also show shape memory features. Alloys used in practice include "Nitinol" – a group of alloys with 53%−57% mass of nickel and three-ingredient Cu-Zn-Al [2].

Among the shape memory effect we distinguish:

- one-way shape memory effect,
- two-way shape memory effect,
- effect of pseudo-elasticity.

Shape memory alloys have a spectrum of unique features, not appearing in other materials. The most important are:

- Young's modulus depends on temperature and deformation,
- inner friction depends on amplitude and the temperature at which the deformation is performed,
- depending on the temperature, the yield point changes,
- depending on the temperature, the electric resistance changes.

Those unique features of SMA are connected with the reverse martensite transformation, which occurs in temperatures close to environmental temperature.

Most of the papers describing constructional friction, are devoted to the determination of the static hysteresis loop, basing on the effects occurring in the slip zones and their experimental verification.

Among works not following the classic assumptions of constructional friction we can mention [3], in which the author conducted an analysis of vibrations with non-uniform distribution of the unitary pressure, and [4], in which the authors have taken into account the elastic interaction with the base, assuming that the friction factor has a viscosity-coulomb characteristic. In [5] an analysis of the dynamic of an inseparable sleeve-shaft joint, where the shaft is made of SMA material, is conducted. On the other hand, in [6] the author examined the influence of inertia momenta of the sleeve-shaft joint on the vibration's amplitude. In [7] the problems of damping vibrations of a set of two beams were examined using an analytic method taking into account constructional friction.

2. Building the mathematical model

For the purpose of the analysis we assume that the joint is inseparable and consist of two beams, thus acquiring a model of a leaf spring which is very often used in vehicles as an element of the kinematical chain. A scheme of the analyzed system is presented in Figure 1.

For the purpose of the analysis we assume the following:

1. the pressure has a uniform distribution,

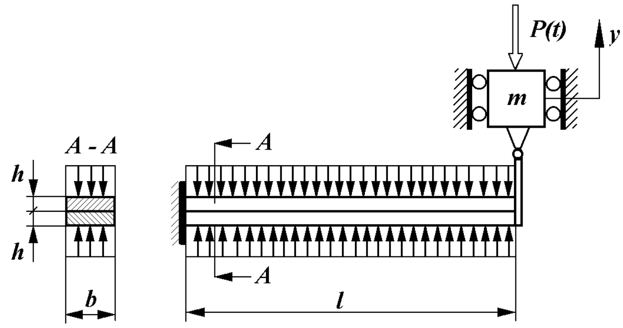

Figure 1. A scheme of the investigated system dependence for pseudo-elastic materials

2. the coefficient of dry friction has a constant value along the whole length,
3. both beams are made of a material with shape memory (SMA),
4. the constructional friction between beams is taken into account.

2.1. The linearization of elasticity and damping properties

In order to carry out the linearization we consider free vibrations of the system of two beams, the dimensions of which are given in Figure 1. To carry out the linearization we modify the pseudo-elastic characteristic of the SMA material presented in Figure 2. It is done by extracting from it the linear elastic characteristic. The scheme of the modification is presented in Figure 3.

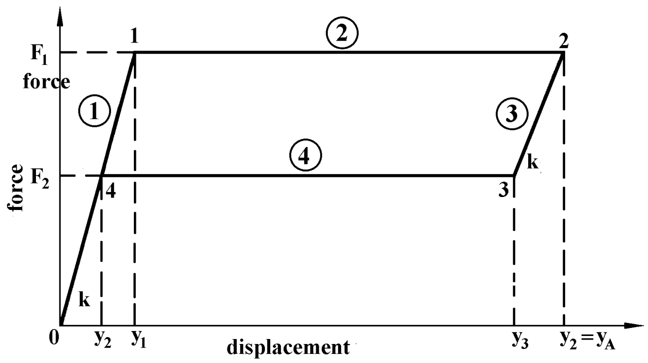

Figure 2. Simplified stress-deflection

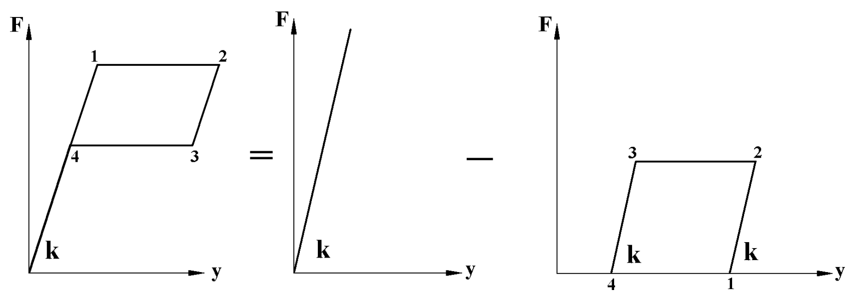

Figure 3. A scheme of modification of the shape memory material's (SMA) characteristic

As an effect of the modification we consider the sleeve's differential equation of motion in the following form:

$$m\ddot{y}+k_b y=\nu F(y). \tag{1}$$

Thus we obtained a harmonic oscillator's differential equation of motion with nonlinear disturbance which is described by the following relationship:

$$\nu F(y)=\operatorname{sign}\dot{y}\begin{cases}k(y_3-y_4) & \text{for } y_3<y<y_4,\\ k(y-y_4) & \text{for } y_4<y<y_3,\\ 0 & \text{for } -y_1<y<y_4,\\ k(y-y_1) & \text{for } y<-y_1,\end{cases} \tag{2}$$

where m is the body's mass, $k=\frac{24EI}{l_p^4}$ – the beam's stiffness for deflecting, E – the beam's Young modulus, I – the second moment of area, l_p – the length of the joint, ν – the amplitudes at the beginning and of the phase transition.

To evaluate the equivalent stiffness and equivalent damping coefficients we used the asymptotic Bogolubov-Krylov-Mitropolski method assuming the first approximation of the following form:

$$y=y_A\cos(\Theta+\xi), \tag{3}$$

where y_A and ξ are determined by a system of differential equations of the first approximation:

$$\begin{aligned}\frac{y_A}{dt}&=\nu A_1(t,y_A,\xi),\\ \frac{d\xi}{dt}&=\omega_0+\nu B_1(t,y_A,\xi).\end{aligned} \tag{4}$$

Basing on [8], the equations of the first approximation, in the case of free vibrations can be written as follows:

$$\begin{aligned}\frac{dy_A}{dt}&=-\frac{\nu}{2\pi m\omega_0}\int_0^{2\pi}F_0(y_A,\vartheta)\sin\vartheta d\vartheta,\\ \frac{d\xi}{dt}&=\omega_0-\frac{\nu}{2\pi m\omega_0 y_A}\int_0^{2\pi}F_0(y_A,\vartheta)\cos\vartheta d\vartheta,\end{aligned} \tag{5}$$

where

$$\begin{aligned}F_0(y_A,\vartheta)&=F(y_A\cos\vartheta,-y_A\omega_0\sin\vartheta),\\ \vartheta&=\Theta+\xi.\end{aligned} \tag{6}$$

The equations describing the equivalent damping coefficient and equivalent frequency of vibrations become the following:

$$\begin{aligned}h_{eq}(y_A)&=-\frac{\nu}{2\pi m\omega_0 y_A}\int_0^{2\pi}F_0(y_A,\vartheta),\\ \omega_{eq}^2(y_A)&=\omega_0^2-\frac{\nu}{\pi m y_A}F(y_A<\vartheta)\cos\vartheta d\vartheta.\end{aligned} \tag{7}$$

Using the given Equations (7), we determine the dependences describing the equivalent stiffness and resistance coefficients to have the following form:

$$\begin{aligned} c_{eq}(y_A) &= 2h_{eq}(y_A)m = -\frac{\nu}{2\pi m\omega_0 y_A}\int_0^{2\pi} F_0(y_A,\vartheta)\sin\vartheta d\vartheta, \\ k_{eq}(y_A) &= \omega_{eq}^2(y_A)m = k - \frac{\nu}{\pi y_A}\int_0^{2\pi} F_0(y_A,\vartheta)\cos\vartheta d\vartheta. \end{aligned} \tag{8}$$

Taking into account the equations for non-linear disturbance (2), we obtain:

$$c_{eq}(y_A) = -\frac{\nu}{\pi\omega_0 y_A}\int_0^{2\pi} f_0(y_A,\vartheta)\sin\vartheta d\vartheta =$$

$$= \frac{2}{\pi\omega_0 y_A}\left\{\int_0^{\beta_3} k(y_3-y_4)\sin\vartheta d\vartheta + \int_{\beta_3}^{\beta_4} k(y-y_4)\sin\vartheta d\vartheta + \int_{-\beta_1}^{\pi} k(y+y_1)\sin\vartheta d\vartheta\right\}, \tag{9}$$

and

$$k_{eq}(y_A) = k - \frac{\nu}{\pi y_A}\int_0^{2\pi} F_0(y_A,\theta)\cos\theta d\theta =$$

$$= k - \frac{2}{\pi y_A}\left\{\int_0^{\beta_3} k(y_3-y_4)\cos\vartheta d\vartheta + \int_{\beta_3}^{\beta_4} k(y-y_A)\sin\vartheta d\vartheta + \int_{-\beta_1}^{\pi} k(y-y_1)\cos\vartheta d\vartheta\right\}. \tag{10}$$

After integrating, we obtain the following expressions:

$$c_{eq}(y_A) = \frac{\omega_0}{\pi}\left[\cos\beta_3 - \cos\beta_4 + \cos\beta_1 - \frac{1}{2}(\cos^2\beta_3 - \cos^2\beta_4 + \cos^2\beta_1 + 1)\right], \tag{11}$$

and

$$k_{eq}(y_A) = k + \frac{k}{\pi}\left[\frac{3}{2}(\sin 2\beta_3 - \sin 2\beta_4 - \sin 2\beta_1) + (\beta_3 - \beta_4 - \beta_1)\right], \tag{12}$$

where we have taken into account, that:

$$\cos\beta_i = \frac{y_i}{y_A}, \tag{13}$$

and $y_2 = y_A$ is the vibration's amplitude.

As an effect the linearization we obtain a substitute beam made of a viscous-elastic material (Kelvin-Voigt model) with k_{eq} stiffness and c_{eq} viscosity resistance factor. The substitute beam's Young's modulus is described by the following dependance:

$$E_{eq} = \frac{kl_p}{m}\left[1 + \frac{1}{\pi}\left[\frac{3}{2}(\sin 2\beta_3 - \sin 2\beta_4 - \sin 2\beta_1) + (\beta_3 - \beta_4 - \beta_1)\right]\right]. \tag{14}$$

2.2. Building the mathematical model

With the above assumptions concerning the constructional friction, we can furthermore assume that the friction on the shaft-sleeve contact surface is a dry friction. The force referred to unitary length is described by the Coulomb's law and is thus proportional to pressure:

$$q=\mu\cdot p\cdot b, \tag{15}$$

where μ is the friction coefficient, p is the unitary pressure, and b is the beam's width.

The tangential force referred to beam's length unit is:

$$t=\frac{3}{4h}\alpha F, \tag{16}$$

where h is the beam's thickness, and α – the load coefficient at any moment of the movement.

To construct the differential equation of motion for the system presented in Figure 1, it is necessary to determine a general dependence describing the deflection of the system's end as a function of the load.

2.2.1. Phase I – the load $(0<\alpha_1<\alpha_{1M})$

- *no slip*

In this phase the tangential force t has a constant value along the whole length of the beams. As long as $t<q$ there is no slip between the beams and the beams' deflection is described with the equation:

$$y_{11}=\frac{l_p^3}{24E_{eq}I}\alpha F=\frac{\alpha F}{k_{eq}}, \tag{17}$$

where $k_{eq}=\frac{24E_{eq}}{l_p^3}$ is the beams' stiffness for deflecting.

In the state when the tangential forces are equal to the friction forces $(t=q)$ the load factor is:

$$\alpha_{01}=\frac{4qh}{3}=\frac{4\mu pbh}{3}, \tag{18}$$

and then the deflection is:

$$y_{10}=\frac{4\mu pbh}{3k_{eq}}. \tag{19}$$

- *during the slip*

When the translation is bigger than that given by formula (19), the slip will appear simultaneously along the whole length of the beams. We determine the deflection by analyzing the deflection of one of the beams where the loading force is $\alpha F/2$ and the friction forces are distributed over the contact with the second beam, that is:

$$y_{12}=\alpha_1 F\frac{l_p^3}{6E_{eq}I}-qh\frac{l_p^3}{6E_{eq}I}=(4\alpha_1-3\alpha_{01})\frac{F}{k_{eq}}. \tag{20}$$

The maximum deflection, at the moment when $(1=\alpha_1=\alpha_{1M})$, has the value of:

$$y_{1M}=\frac{F}{k_{eq}}(4\alpha_{1M}-3\alpha_{01}). \tag{21}$$

2.2.2. Phase II – unloading $(\alpha_{1M}<\alpha_2<\alpha_{2M})$

- *no slip*

The tangential stress, in this phase of the movement, in the beams' plain has the following value:

$$t=q-\frac{3}{4h}(\alpha_{1M}-\alpha_2)F. \tag{22}$$

The deflection of the system of beams' is described by the dependence given below:

$$y_{21} = y_{1M} - \frac{l_p^3}{24E_{eq}I}(\alpha_{1M} - \alpha_2)F = \frac{F}{k_{eq}}(3\alpha_{1M} - 3\alpha_{01} + \alpha_2). \tag{23}$$

During further unloading with the stressing force value of $\alpha_{02}F$ the tangential forces equal the friction forces of the opposite sign – like in the first phase, that is:

$$q - \frac{3(\alpha_{1M} - \alpha_{02})}{4h} = -q, \tag{24}$$

thus:

$$\alpha_{02} = \alpha_{1M} - 2\alpha_{01}. \tag{25}$$

The deflection of the end of the system of two beams at this moment is described as follows:

$$y_{20} = \frac{l_p^3}{24E_{eq}I}(4\alpha_{1M} - 5\alpha_{01})F = \frac{F}{k_{eq}}(4\alpha_{1M} - 5\alpha_{01}). \tag{26}$$

- *during the slip*

When the translation is bigger than that given by formula (26), the slip will appear simultaneously along the whole length of the beams. We determine the deflection by analyzing the deflection of one of the beams where the loading force is $\alpha F/2$ and the friction q forces are distributed over the contact with the second beam, that is:

$$y_{22} = y_{20} - \frac{l_p^3}{6E_{eq}I}(\alpha_{02} - \alpha_2)F = \frac{F}{k_{eq}}(3\alpha_{01} - 4\alpha_2). \tag{27}$$

The maximum deflection, at the moment when $(1 = \alpha_2 = \alpha_{2M})$, has the value of:

$$y_{2M} = -\frac{l_p^3}{6E_{eq}I}(\alpha_{02} - \alpha_2)F = -\frac{F}{k_{eq}}(4\alpha_{2M} - 3\alpha_{01}). \tag{28}$$

2.2.3. Phase III – loading $(\alpha_{2M} < \alpha_3 < \alpha_{3M})$

- *no slip*

The tangential stress in this phase of motion on the beams' surface of contact has the value:

$$t = q - \frac{3}{4h}(\alpha_{1M} - \alpha_2)F. \tag{29}$$

The deflection of the system of beams is described by the dependence given below:

$$y_{31} = y_{2M} + \frac{l_p^3}{24E_{eq}I}(\alpha_{2M} - \alpha_3) = \frac{F}{k_{eq}}(3\alpha_{1M} - 3\alpha_{01} + \alpha_2). \tag{30}$$

During further unloading with the stressing force value of $\alpha_{03}F$ the tangential forces equal the friction forces of the opposite sign – like in the first phase, that is:

$$q - \frac{3(\alpha_{1M} - \alpha_{20})}{4h} = q, \tag{31}$$

thus:

$$\alpha_{03} = \alpha_{2M} - \alpha_{02}. \tag{32}$$

The deflection of the end of the system of two beams at this moment is described as follows:

$$y_{30} = \frac{l_p^3}{24E_{eq}I}(4\alpha_{2M} - \alpha_{01})F = \frac{F}{k_{eq}}(4\alpha_{2M} - 5\alpha_{01}). \tag{33}$$

• *during the slip*

After the transition is bigger than by (33), the slip will appear simultaneously along the whole length of the beams. We determine the deflection by analyzing the deflection of one of the beams where the loading force is $\alpha F/2$ and the friction q forces are distributed over the contact with the second beam, that is:

$$y_{32} = y_{30} + \frac{l_p^3}{6E_{eq}I}(\alpha_{03} - \alpha_3)F = \frac{F}{k_{eq}}(\alpha_{01} - \alpha_3). \tag{34}$$

The maximum deflection, at the moment when $(\alpha_3 = \alpha_{3M})$, has the value of:

$$y_{3M} = \frac{l_p^3}{6E_{eq}I}(\alpha_{03} - \alpha_2)F = \frac{F}{k_{eq}}(4\alpha_{3M} - \alpha + 01). \tag{35}$$

Using that method we can determine the deflection in the fourth, fifth and further phases of the movement.

Generalizing the above, we can describe any phase of the movement.

The expressions describing the deflection in each phase of the load, starting with the second phase can by generalized. Having in mind that in the second, fourth *etc.* phase of the movement the rate at which the deflection changes is negative $(\operatorname{sign} y = -1)$, while in the first, third *etc.* phase of the movement it is positive $(\operatorname{sign} y = 1)$ the deflection, basing on Equations (17), (23) and (30), in phases when there is no slip, can be presented as follows:

$$y_{(i+1)1} = \left[-y_i + \frac{F}{k_{eq}}(\alpha_i - \alpha_{i+1})\right] \operatorname{sign} \dot{y}. \tag{36}$$

Corresponding with that, the deflection in phases when the friction equals the tangential forces $(t = q)$, using Equations (19), (26) and (33) can be presented as:

$$y_{i0} = \frac{F}{k_{eq}}(4\alpha_{iM} - 5\alpha_{i1}) \operatorname{sign} \dot{y}. \tag{37}$$

Finally, deflection during the slip, basing on Equations (20), (27) and (34) can be presented as:

$$y_{(i+1)2} = \frac{F}{k_{eq}}(\alpha_{i+1} - 3\alpha_{i0}) \operatorname{sign} \dot{y}. \tag{38}$$

Basing on dependences (36)–(38) the deflection at any moment of the movement can be described as follows:

$$y_{i+1} = \begin{cases} \left[-y_1 + \frac{F}{k_{eq}}(\alpha_{iM} - \alpha_{i+1})\right] \operatorname{sign} \dot{y} & \text{for } \alpha_{iM} > \alpha > 2\alpha_{i0}, \\ \frac{F}{k_{eq}}(4\alpha_{i+1} - \alpha_{i0}) \operatorname{sign} \dot{y} & \text{for } 2\alpha_{i0} > \alpha > \alpha_{(i+1)M}. \end{cases} \tag{39}$$

Making an inverse transformation of Equation (39), we get the dependence of loading as a function of the deflection of the springs end in the following form:

$$F(y,\dot{y},t) = \begin{cases} F_{iM} - k_{eq}(y_A \operatorname{sign} \dot{y} + y_i) & \text{for } y_{iM} > y > y_{i0}, \\ \frac{1}{4}(3F_{iM} + k_{eq} y_i \operatorname{sign} \dot{y}) & \text{for } y_{i0} > y > -y_A. \end{cases} \tag{40}$$

Basing on relation (4) we presented the course of the loading force in Figure 4.

Using the obtained dependence between the loading force and the deflection of the end of the spring we can modify the motion equation into the following form:

$$m\ddot{y} + F(y,\dot{y},t) = P(t), \tag{41}$$

where function $F(y,\dot{y},t)$ is given in expression (40).

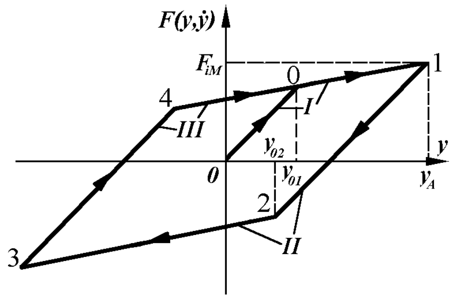

Figure 4. The course of loading force as a function of the deflection

Having all that in mind we can now move on to analyzing Equation (41) in the conditions of harmonic forcing with constant rate of increase of frequency and force, that is:

$$P(t) = P_0 \sin\Theta,$$
$$\Theta = \frac{\varepsilon t^2}{2} + \varphi_0, \tag{42}$$
$$\omega(t) = \frac{d\Theta}{dt} = \varepsilon t,$$

where P_0 is the the amplitude of the forcing force, ε – the angular acceleration, Θ – the phase of the movement, $\omega(t)$ – the forcing force frequency.

Analyzing Equation (40) we can notice that there is a component proportional to the deflection in an explicit form. To extract this component we modify the elastic-damping characteristic of the joint. A scheme of the modification is presented in Figure 5.

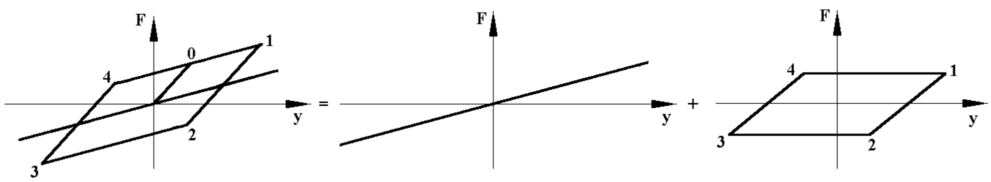

Figure 5. A scheme of the joint's characteristic's modification

As a result of the modification the system's differential equation of motion takes the following form:

$$m\ddot{y} + k_{eq} y = -\nu F_0(y\dot{y},t) + P(t), \tag{43}$$

where the non-linear disturbance is described by the dependence:

$$\nu F(y,\dot{y},t) = \begin{cases} \left(\frac{3}{4}F_{iM} - 3k_{eq}y\right)\operatorname{sign}\dot{y} & \text{for } -y_A \operatorname{sign}\dot{y} > y > -(y_A - 2y_{0i})\operatorname{sign}\dot{y}, \\ \frac{3}{4}F_{iM}\operatorname{sign}\dot{y} & \text{for } -(y_A - 2y_{0i})\operatorname{sign}\dot{y} > y > y_A \operatorname{sign}\dot{y}. \end{cases} \tag{44}$$

3. Solving the equation of motion

We will solve Equation (43) using the asymptotic Bogolubov-Krylov-Mitropolski method shown in [8], assuming that the first approximation has the form of:

$$y = y_A \cos(\Theta + \xi), \tag{45}$$

where y_A and ξ are determined by a system of differential equations of the first approximation:

$$\begin{aligned}\frac{dy_A}{dt}&=\nu A_1(t,y_A,\xi),\\ \frac{d\xi}{dt}&=\omega_0-\omega(t)+\nu B_1(t,y_A,\xi),\end{aligned}\tag{46}$$

where, using the notation:

$$\begin{aligned}\vartheta&=\Theta+\xi,\\ F_0(y_A,\vartheta)&=F_n(y_A\cos\vartheta d\vartheta,-y_A\dot{\vartheta}\sin\vartheta d\vartheta),\end{aligned}\tag{47}$$

the equations of the first approximation describing the formulas of amplitude derivative and phase derivative, in the case when slip occurs, assume the following form:

$$\frac{dy_A}{dt}=-\frac{\nu}{2\pi m\omega_0}\int_0^{2\pi}F_0(y_A,\theta)\sin\theta d\theta+h_{eq}y_A-\frac{P_0}{m[\omega_0-\omega(t)]}\cos\xi,\tag{48}$$

$$\frac{d\xi}{dt}=\omega_0-\omega(t)-\frac{\nu}{2\pi m\omega_0 y_A}\int_0^{2\pi}F_0(y_A,\theta)\cos\theta d\theta+\frac{P_0}{m[\omega_0-\omega(t)]}\sin\xi,\tag{49}$$

where

$$\nu F_0(y_A,\vartheta)=\begin{cases}\left(\frac{3}{4}F_{iM}-3k_{eq}y_A\cos\vartheta\right)\operatorname{sign}\dot{y} & \text{for } -y_A\operatorname{sign}\dot{y}>y>-(y_A-2y_{01})\operatorname{sign}\dot{y},\\ \frac{3}{4}F_{iM}\operatorname{sign}\dot{y} & \text{for } -(y_A-2y_{01})\operatorname{sign}\dot{y}>y>y_A\operatorname{sign}\dot{y}.\end{cases}$$

After integration of Equations (47) and (48) we eventually get the differential equations of the first approximation:

- *in the case when the slip occurs:*

$$\begin{aligned}\frac{dy_A}{dt}=&\frac{1}{\pi m\omega_0}\frac{3}{2}\left[(F_{iM}-2k_{eq}y_A)(1-\cos\varphi)+k_{eq}y_A\sin^2\varphi\right]\\ &+h_{eq}y_A-\frac{P_0}{m[\omega_0+\omega(t)]}\cos\xi,\end{aligned}\tag{50}$$

$$\begin{aligned}\frac{d\xi}{dt}=&\omega_0-\omega(t)-\frac{1}{\pi m\omega_0\frac{3}{2}}\left[\left(\frac{F_{iM}}{y_A}-4k_{eq}\right)\sin\varphi+k_{eq}\left(\varphi+\frac{\sin 2\varphi}{2}\right)\right]\\ &+\frac{P_0}{my_A[\omega_0+\omega(t)]}\sin\xi.\end{aligned}\tag{51}$$

- *in the case when there is no slip:*

$$\frac{dy_A}{dt}=h_{eq}y_A-\frac{P_0}{m[\omega_0-\omega(t)]}\cos\xi,\tag{52}$$

$$\frac{d\xi}{dt}=\omega_0-\omega(t)-\frac{1}{m\omega_0}\frac{3}{2}k_{eq}+\frac{P_0}{my_A[\omega_0+\omega(t)]}\sin\xi.\tag{53}$$

Using the obtained Equations (50)–(53) we conducted a numeric simulation, investigating the influence of the friction factor μ on the course of vibration amplitude. The results are shown in Figure 6.

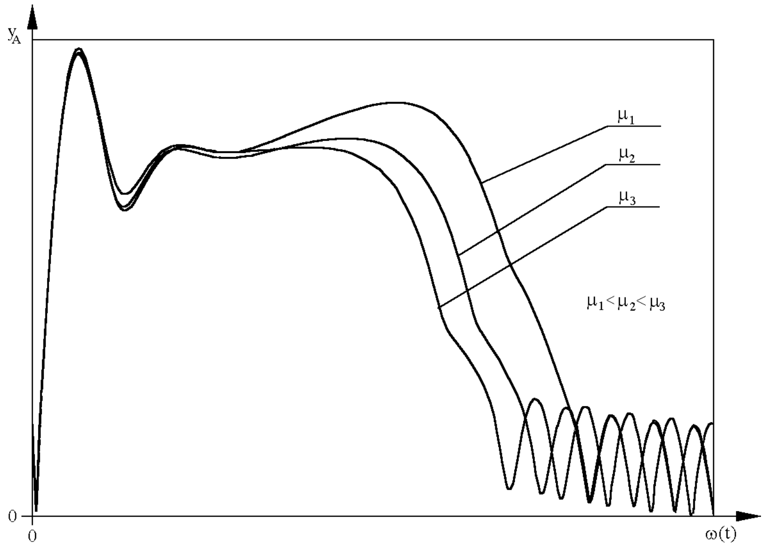

Figure 6. The influence of unitary pressure on the course of harmonically forced vibration's amplitude

From the numerical analysis, shown in Figure 6, it arises that using materials with shape memory in a joint of two beams, when the amplitude is bigger than y_1 results in more intensive damping; when $y_A < \min(y_1, y_0)$ we observe undamped harmonic vibrations. Increasing the friction factor μ results in a decrease of the vibrations' amplitude and narrows the range in which resonance vibrations take place.

4. Conclusions

1. Using SMA material to build the leaf spring increases the effectiveness of damping in the joint.
2. An increase of the friction factor μ effects in a decrease of the vibrations' amplitudes and narrows the range where big amplitudes occur.
3. Taking into account the constructional friction and the SMA material's pseudo-elastic properties in the analyzed joint, results in origination of zones in which damped and undamped vibrations take place, which is a result of both pseudo-elasticity and constructional friction.

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