# DYNAMICS OF WHEEL-TYRE SUBJECTED TO MOVING OSCILLATING FORCE

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**Abstract:** The subject of the paper is analysis of wheel of a moving railway vehicle which is subjected to a moving oscillating force. Rail ring is treated as a beam of small curvature connected to wheel axle with a Winkler foundation. Bernoulli-Euler and Timoshenko beam model is used. Results are gained using Fourier transformation. Space and space-time graphs, showing wave propagation in subcritical and supercritical zones of excitation, concerning resonance of transverse vibrations, are included.

**Keywords:** railway vehicles, waves, oscillating force, railway wheels

## 1. Introduction

Due to the increase of the speed of rail vehicles the dynamic behaviour of train/track interaction is getting more and more important. In the modelling of interaction between wheel and rail, the influence of contact force oscillation cannot be neglected. This behaviour of the contact force can be caused by the periodic spacing of sleepers or corrugation of wheels and rails, as well as polygonalisation of railway wheels. The problem of the non-constant contact forces was studied in [1]. Our study will give an insight of wave propagation phenomena in the wheel-tyre subjected to moving oscillating force.

## 2. Equations of motion

In the general case the equations that describe the displacements of curved beam are given as follows:

$$\begin{aligned}\frac{EI}{R^4}(w''+w)''+\frac{EA}{R^2}(w+\eta')+\rho A\frac{\partial^2 w}{\partial t^2}=0,\\ \frac{EI}{R^4}(w''+w)'-\frac{EA}{R^2}(w+\eta')+\rho A\frac{\partial^2 \eta}{\partial t^2}=0,\end{aligned}\tag{1}$$

$()'=\frac{\partial()}{\partial s}$, where $s$ is arc co-ordinate. The remaining notations are: $EI$ – bending stiffness of rim, $R$ – average wheel radius, $w$ – transverse displacement, $\eta$ – external damping coefficient, $A$ – area of beam cross section, $\rho$ – mass density, $t$ – time.

The normal force and bending moment are described by the following expressions:

$$\begin{aligned}N=EA\varepsilon=\frac{EA}{R}(w+\eta'),\\ M=-\frac{EI}{R^2}(w''+w).\end{aligned}\tag{2}$$

According to the research presented in paper [2], a straight symmetric beam on a visco-elastic foundation is so close in behaviour to a curved beam, that this similarity can be used for technical purposes as well. The above assumption was found to be valid for stationary vibrations of rail wheel [3].

The transverse beam displacement of the wheel rim as a beam can be expressed by formula [2]:

$$\frac{d^6N}{ds^6}+\frac{1}{R^2}\frac{d^4}{ds^4}+\lambda^4\frac{d^2N}{ds^2}=0,\tag{3}$$

where $\lambda=\frac{\beta}{EI}$, and $\beta$ is the Winkler foundation coefficient.

We are looking for the solution of Equation (3) which has the following form:

$$N(s)=C\exp(rs).\tag{4}$$

After substituting Equation (4) into Equation (3) we obtain the characteristic equation with the following non-zero roots:

$$r_{1,\dots,4}=\pm\sqrt{-\frac{1}{2R^2}\left(1\pm i\sqrt{\lambda R\sqrt{2}-1}\right)},\quad \text{where}\quad \lambda R\sqrt{2}>1.$$

Solution of the Equation (3) will be defined as:

$$N(s)=\exp\left(+\frac{\partial s}{L}\right)\left[N_1\cos\left(\frac{s}{L}\right)+N_2\sin\left(\frac{s}{L}\right)\right]+\exp\left(-\frac{\partial s}{L}\right)\left[N_3\cos\left(\frac{s}{L}\right)+N_4\sin\left(\frac{s}{L}\right)\right],$$

where $L$ – characteristic length, $N_{1,2,3,4}$ – parameters calculated from the following boundary conditions.

The additional two terms are neglected due to the given radius of the wheel $R$ and perfect circularity. In such a case the boundary conditions are as follows:

$$\begin{aligned}\left.\frac{dw}{ds}\right|_{s=0}=0,\qquad \left.\frac{dw}{ds}\right|_{s=R\pi}=0,\\ -EI\left.\frac{d^3w}{ds^3}\right|_{s=0}=\frac{F}{2},\qquad \left.\frac{d^3w}{ds^3}\right|_{s=R\pi}=0,\end{aligned}\tag{5}$$

where $F$ – external load.

In the case when the curvature of the beam is neglected, the equation of rim transverse can be written in the form shown in [4–6]:

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}+\eta\frac{\partial w}{\partial t}+qw=\delta(x-vt)(F_0+F_1\cos\omega t),\qquad(6)$$

where $x$ – Cartesian co-ordinate, $q$ – Winkler foundation coefficient, $\delta()$ – Dirac's distribution, $F_0$ – constant force component in moving reference frame, $\delta(x-vt)F_1\cos\omega t$ – force oscillating with frequency $\omega$ and moving with velocity $v$.

In the coordinate frame moving with constant velocity the Equation (6) can be written as follows:

$$EI\frac{\partial^4 w}{\partial \bar{x}^4}+\rho A\left(\frac{\partial^2 w}{\partial t^2}+2v\frac{\partial^2 w}{\partial \bar{x}\partial t}+v^2\frac{\partial^2 w}{\partial \bar{x}^2}\right)+\eta\left(\frac{\partial w}{\partial t}+v\frac{\partial v}{\partial t}\right)+qw=\delta(x)(F_0+F_1\cos\omega t).\qquad(7)$$

After introducing dimensionless parameters and variables Equation (7) takes a form:

$$\frac{\partial^4 W}{\partial X^4}+4V^2\frac{\partial^2 W}{\partial X^2}-8V\frac{\partial W}{\partial X\partial\tau}+4\frac{\partial^2 W}{\partial\tau^2}-8\alpha\frac{\partial W}{\partial X}+8\alpha\frac{\partial W}{\partial\tau}4W=8\delta(X)\cos\Omega\tau,\qquad(8)$$

where: $W=w\left(\frac{8EIa_0^3}{F_0}\right)$, $a_0=\sqrt[4]{\frac{q}{4EI}}$, $X=(x-vt)a_0$, $\tau=t\sqrt{\frac{q}{\rho A}}$, $V=v\sqrt[4]{\frac{(\rho A)^2}{4qEI}}$, $\Omega=\omega\sqrt{\frac{\rho A}{q}}$, and $\alpha=\frac{1}{2}\eta/\sqrt{2q\rho A}$.

Applying Fourier's transformation to Equation (8), as was done in [5] we gain in denominator of transform the following characteristic equation:

$$\left[S^4-4V^2S^2+8V\Omega S+4(1-\Omega^2)\right]\left[S^4-4V^2S^2-8V\Omega S+4(1-\Omega^2)\right]=0.\qquad(9)$$

Equation (9) was solved numerically. The result of excitation by a periodic structure of length $L$ is given in Figure 1a. The experimental results of sleeper excitation presented in [1] confirm the qualitative form of the resonance (Figure 1b). The results for values of dimensionless velocity $V$ between 0 and 0.3 according to the speed of fast trains will be also presented. The range of dimensionless frequencies is limited to $\Omega\in\langle 0,2\rangle$ if the Bernoulli-Euler model is to be used.

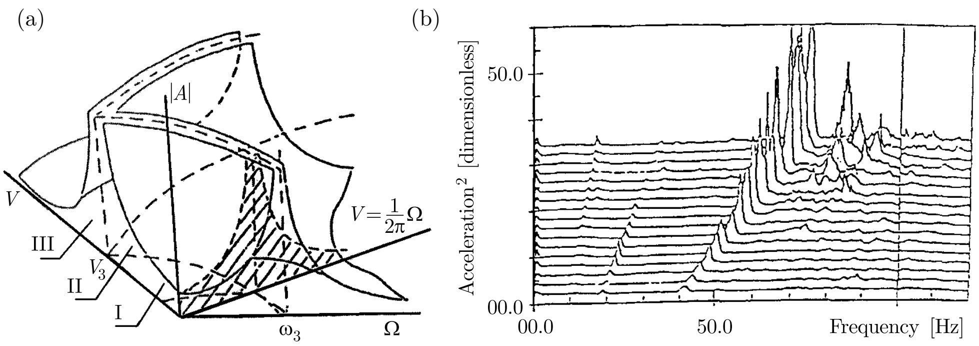


**Figure 1.** (a) Regions of solutions of characteristic equation: I – 4 real compound roots, II – 2 compound and 3 real roots, III – 4 real roots; (b) results of experimental verification

## 3. Results of the analysis

In the process of moving wheel and oscillating force there is a difference between the distance covered by waves travelling forward and backward until they reach the point of the wheel/rail contact. Such a difference is increasing with increase of velocity. The distance covered by waves moving forward and backward in case of moving wheel in comparison with the distance when the wheel is not in movement is shown in Figure 2.

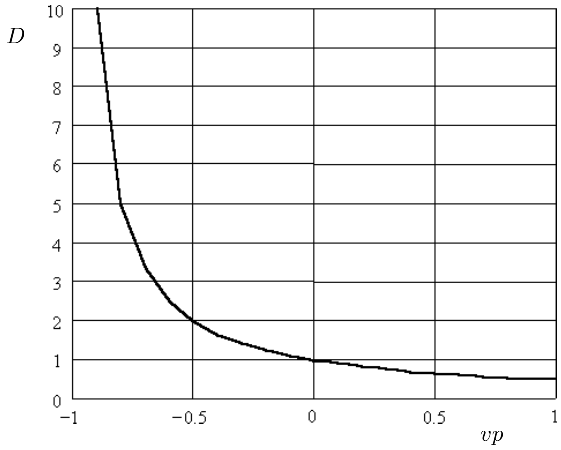


**Figure 2.** Distance covered by wave generated at the point of contact between wheel and rail as a function of movement velocity normalised to wave velocity (for waves propagating backwards velocity is negative), $D$ – distance normalised to distance covered when wheel is not moving, $vp$ – movement velocity normalised to wave propagation velocity

Superposition of waves propagating in opposite directions in wheel rim was shown in Figures 3 and 4. If the train's speed is greater than 0 the symmetry of displacements is present in pre-resonant region and the asymmetry in post-resonance region. If the dimensionless velocity is $V = 0.3$, resonance occurs for excitation frequency $\Omega = 0.808$.

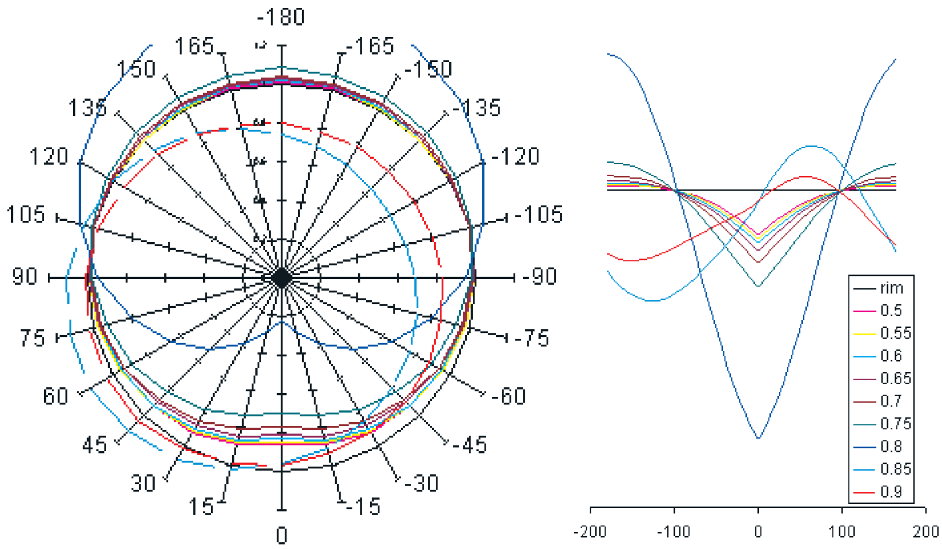


**Figure 3.** Superposition of waves when $V = 0.3$ and $\Omega = 0.5,\dots,0.9$

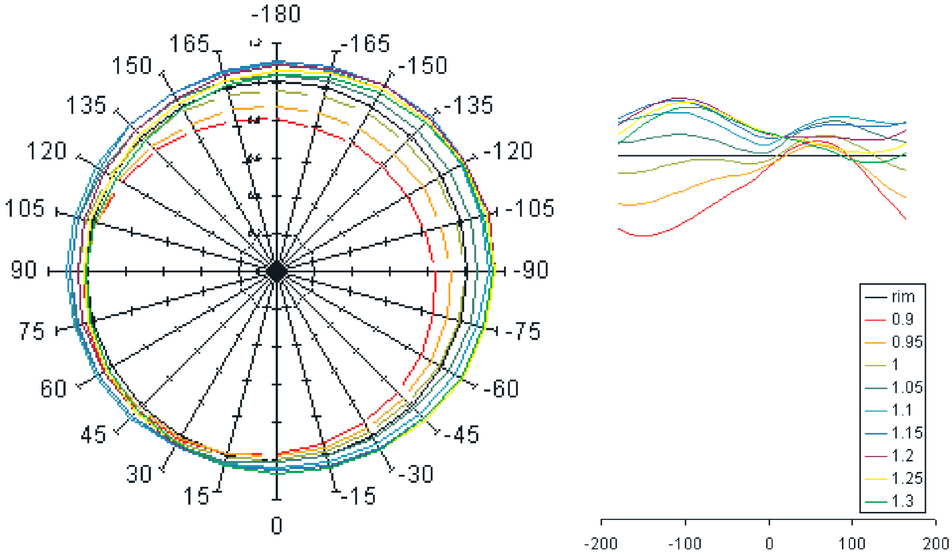


**Figure 4.** Superposition of waves when $V=0.3$ and $\Omega=0.9,\ldots,1.3$

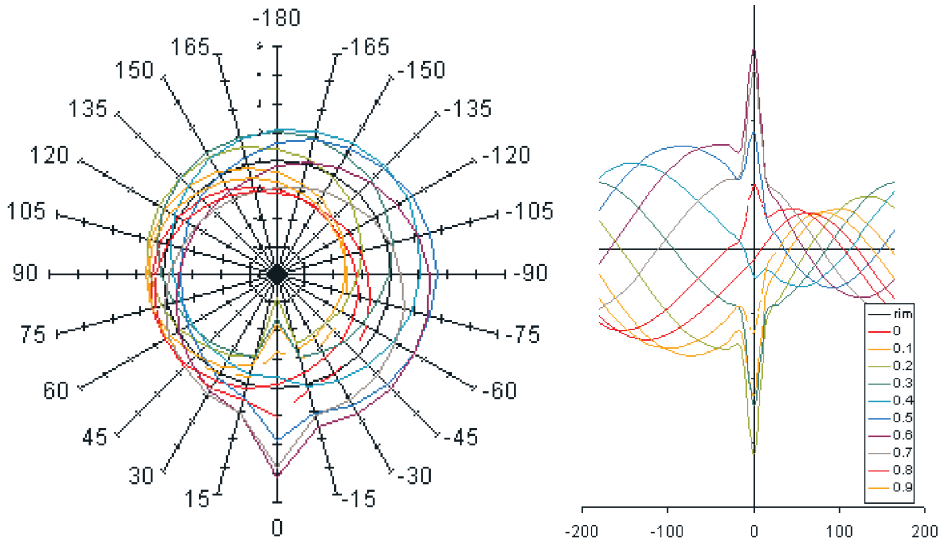


**Figure 5.** Vibration forms for selected values of time if $V=0.3$ and $\Omega=0.85$

The behaviour of the wheel rim when dimensionless velocity is 0.3 and frequency of excitation is 0.85 or 1.5 is shown in Figures 5 and 6. In Figure 7 waves propagation in opposite direction from source of vibration occurs when dimensionless velocity is 0.85, 1.0 or 1.5. The change of direction of waves propagation moving forward from the source can be seen for excitation frequency equal to $\Omega=1.0$. The propagation is characteristic for wave propagating back of the source. If the excitation frequency is lower than $\Omega=1.0$ waves propagate to the source, if the frequency value is higher waves propagate from the source. If excitation frequency is $\Omega=1.0$ sharp, the length and propagation velocity of wave back from source is infinite (see Figure 7b).

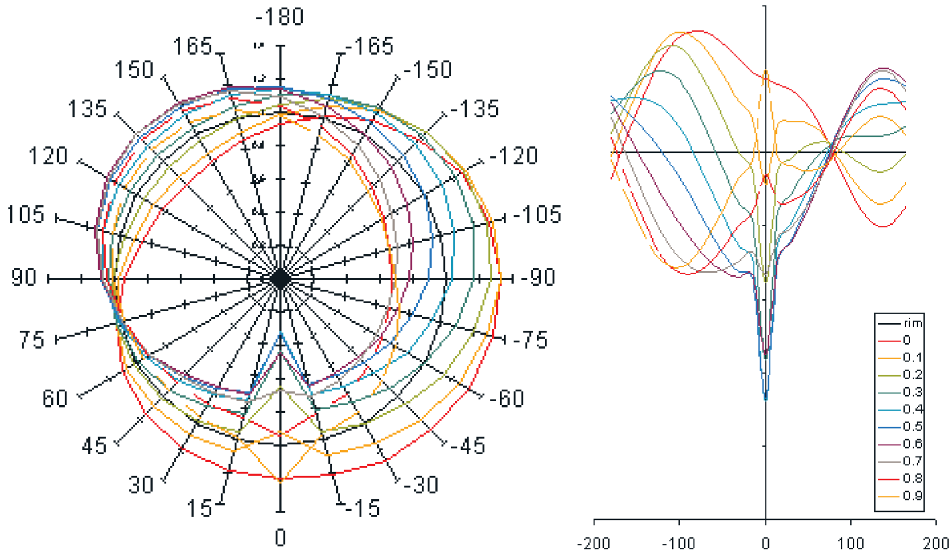


**Figure 6.** Vibration forms for selected values of time if $V=0.3$ and $\Omega=1.5$

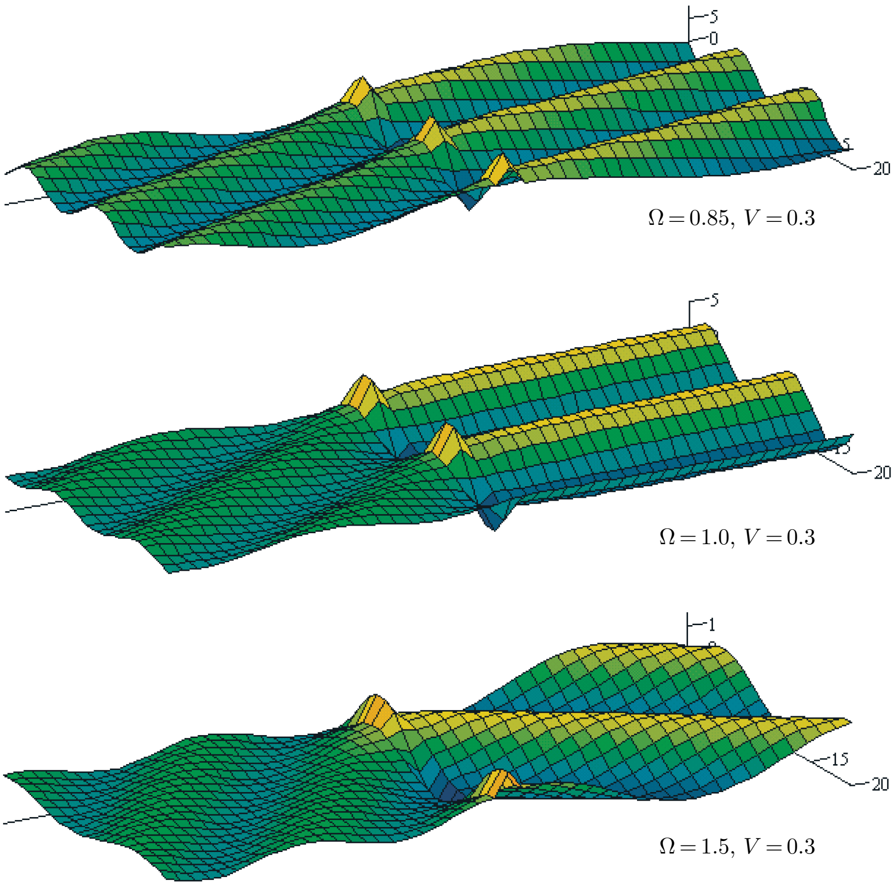


**Figure 7.** Propagation of waves from source of excitation

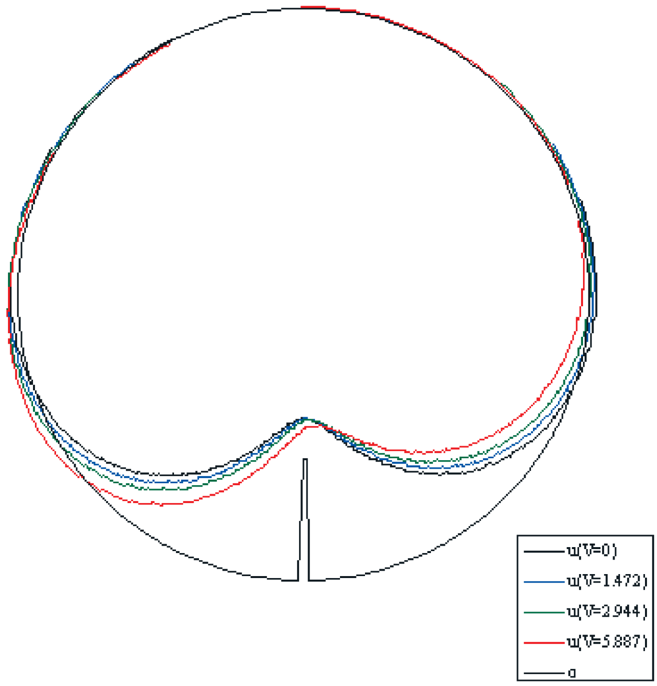


**Figure 8.** Displacement of the rim as a function of circumferential coordinate for chosen values of velocity

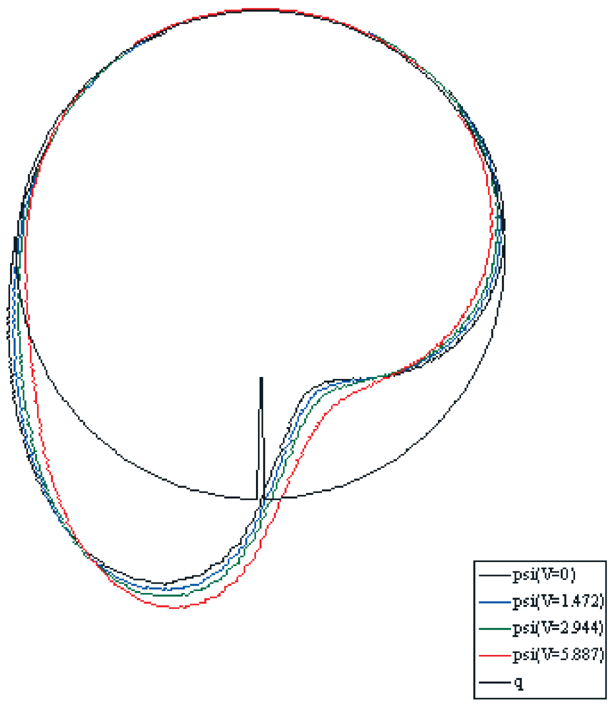


**Figure 9.** Angle of distortion of the rim as a function of circumferential coordinate for chosen values of velocity

In the case of the Timoshenko beam model subjected to the stationary force $\delta(x-vt)F_0$ the displacement and the angle of distortion of the rim as a function of circumferential coordinate for chosen values of velocity is presented in Figures 8 and 9.

## 4. Conclusions

The results show symmetry of displacement in sub-resonant region, which is not present in post-resonant one. In the range of frequencies between boundary of first region and line $\Omega=1$, the propagation of waves in the direction to the source of disturbances is possible. The investigations made in the paper could be the foundation used for optimal design of railroad wheels and tracks, especially in the case of high-speed motion of vehicles. Optimisation for track design will concern sleepers' spacing and pad properties. In the future the research model will be extended to viscoelastic structures with damping and curvature influence (in the case of small wheels).

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