PERCEPTION-BASED REASONING: EVALUATION SYSTEMS

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Abstract: A perception-based interpretation of evaluation systems is proposed in this paper. Thus, a perception-based approach to create intelligent systems is considered. The evaluation systems can be employed *e.g.* in order to assess student exams, as well as to other applications. Evaluation marks used in these systems are given as perceptions expressed by words. The words play the role of labels of perceptions, and are represented by fuzzy sets. This means that the idea of perception-based systems, introduced by Zadeh, is applied. Various algorithms of overall assessment are suggested in this paper. Overall evaluation is produced as an aggregation of component evaluation marks. Systems of this kind can be obtained using fuzzy neurons, so fuzzy neural networks are also mentioned as a method of perception-based reasoning. The usefulness in artificial intelligence of both fuzzy sets and neural networks, and especially a combination of these, is shown.

Keywords: fuzzy sets, perception-based systems, fuzzy neurons, neural networks, artificial intelligence

1. Introduction

The main purpose of this paper is to illustrate that an intelligent system that can imitate a reasoning process usually performed by humans should be a perception-based system. The idea of perception-based systems was proposed by Zadeh. The so-called computational theory of perceptions (CTP), developed by Zadeh, is based on his methodology of computing with words [1, 2]. In CTP, words play the role of labels of perceptions. The assumption is that perceptions are described in a natural or synthetic language. Computing with words (CW), as the name suggests, involves manipulation of words rather than numbers. CW is a derivative of fuzzy logic [3], which is a well-known theory introduced by Zadeh about thirty years ago [4]. This theory is based on his ideas of fuzzy sets [5]. CW is a methodology for computing and reasoning which is close to human intuition, and may thus be applied in Artificial Intelligence (AI). Humans mostly employ words in thinking and reasoning.

In CTP, reasoning with perception is a process of arriving at an answer, a, to a specific question, q, given a collection of perceptions, as propositions expressed in a natural language (for details, see *e.g.* [6]). In this paper, a very simple example of a system based on perceptions is presented as an illustration of applications of fuzzy sets in Artificial Intelligence. The proposed system is a perception-based evaluation system that produces an overall assessment based on component evaluation marks given as perceptions expressed by words.

The system intended to evaluate student exams is described in Sections 2 and 3. An illustrative example is shown in Section 4, and a realization by a fuzzy neuron is depicted in Section 5, Figure 10. Other applications are mentioned in Section 6, where some conclusions are presented.

2. A perception-based evaluation system

Let us consider the problem of evaluation of student exams. In order to pass an exam, students usually have to solve several tasks (or give answers to several questions). The teacher has to assess results of the exam, proposing a proper evaluation of each student's job, which is composed of the several tasks.

A teacher (human) can employ various strategies to arrive at an assessment. However, we can observe that it is very easy for him to assess each particular task solved by a student, using a perception. This means that the teacher can easily evaluate a student's answer to a particular question (or his solution of a task) as *good*, *very good*, *sufficient*, *insufficient*. In order to give such an evaluation, the teacher does not need to apply any score system. As a human, and an expert of the subject of the exam, he knows how to provide the assessment of this kind without any computations, only based on his perception of what is good, sufficient, *etc.* This is similar to the example of parking a car, mentioned by Zadeh [1]. People can park a car without any measurements and any computations. They do it routinely, based on their perceptions.

As described above, a teacher uses his perception, expressed by words (*insufficient*, *sufficient*, *good*, *very good*) to evaluate results of solving a particular task (or answering a question) by a student. The teacher (human) understands very well the meaning of *sufficient*, *good*, *etc.* These words resemble his perception about the quality of student's answer to the task.

The words, used as the labels of the perceptions, may then be translated into synonymous words that correspond to evaluation marks. For example, the meaning of the word *sufficient* can be understood as *satisfactory*. However, for simplicity, the same word (in this case, the word *sufficient*) is used in this paper as the label of the perception and the meaning of the evaluation mark. Similarly, the word *good* concerns the perception of the quality of students' job as well as the mark expressed by the same word.

The teacher's perception with regard to the student task evaluation is strictly related with fuzzy sets, in particular, with fuzzy numbers [5–7]. For example, let us consider his assessment as *good*. It is obvious that, according to Polish system of evaluation of students, the perception of assessment expressed as good corresponds to number 4. Similarly, the perception-based rates *insufficient*, *sufficient*, and *very good* are associated with numbers 2, 3, and 5, respectively. Of course, as a matter of fact, these numbers should be viewed as fuzzy numbers. The crisp numbers 2, 3, 4, 5 can be treated as labels of the fuzzy numbers. These labels, like the words *insufficient*, *sufficient*, *etc.*, are labels of perceptions.

In the case of an assessment expressed by the word *good* (associated with number 4), the perception of this evaluation can be represented by the fuzzy set illustrated in Figure 1. This fuzzy set can be viewed as the fuzzy number described as "about 4" (see *e.g.* [8]). The perception of *good* is that number 4 fully belongs to the fuzzy set "about 4", but the numbers a little less than 4 and a little greater than 4 also belong, though partially, to this fuzzy set. This means that the membership values, in the latter cases, are less than 1, while the membership value of the crisp number 4 equals 1. This membership function can be defined as a triangular function, portrayed in Figure 1. The membership values of numbers equal or less than 3.5 and equal or greater than 4.5 equal 0, since these numbers do not belong to the fuzzy number "about 4".

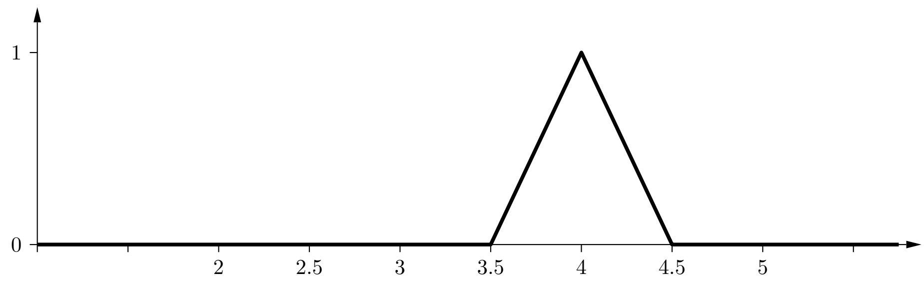

Figure 1. Fuzzy number "about 4"

In a similar way, the fuzzy number "about 3", which represents the perception of evaluation as *sufficient*, can be defined by a triangular membership function. An illustration is shown in Figure 2.

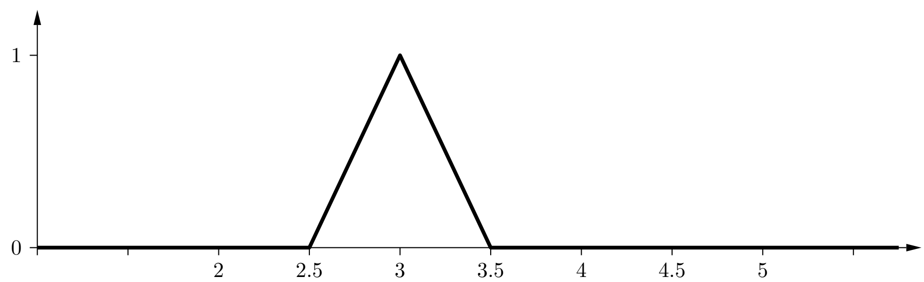

Figure 2. Fuzzy number "about 3"

We can analogously express the perceptions of assessments *insufficient* and *very good* that correspond to the fuzzy numbers "about 2" and "about 5", respectively. However, it is obvious that our perceptions of what the marks *insufficient*, *sufficient*, *good*, *very good* mean can lead to different representations of these perceptions by fuzzy numbers (or fuzzy sets in general). Thus, instead of the fuzzy numbers "about 2" and "about 5", we can express our perceptions of the marks *insufficient* and *very good* by use of the membership functions shown in Figures 3 and 4, respectively. These fuzzy sets reflect the situations when a student does not solve a task (or does not give an answer to a question) even partially, so his job is evaluated as *insufficient* (without any doubts). Similarly, if a student has solved a task much better than it was required in order to obtain the mark *very good*, his job would still be evaluated as *very good*. Thus, values of the membership functions equal 1 not only for the crisp values 2 and 5 but also for the values less than 2 and greater than 5, respectively.

Of course, we can represent a perception of the assessment *very good* in a different way, including the evaluation mark *excellent*, which means more than *very good*. However, for simplicity, this mark is not taken into account in this paper.

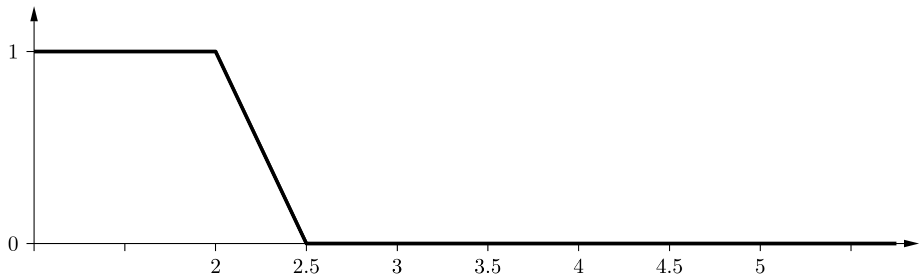

Figure 3. Fuzzy set that represents the perception of the mark *insufficient*

The fuzzy numbers "about 3", "about 4", *etc.*, can be labeled by other words, for example "around 3", "around 4", respectively, and considered as fuzzy granules [9].

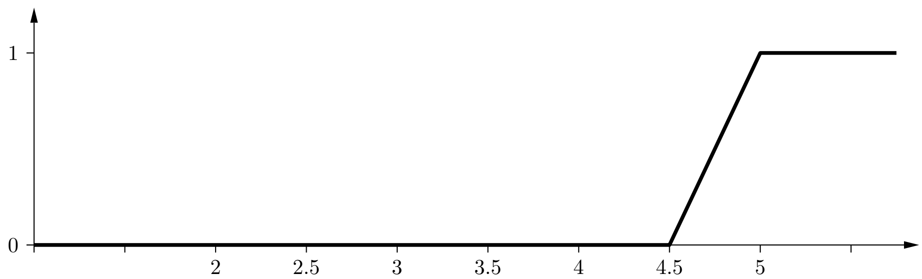

Figure 4. Fuzzy set that represents the perception of the mark *very good*

The fuzzy numbers illustrated in Figures 1 and 2 are represented by triangular membership functions. The values 4 and 3, respectively, are called the mean values of the fuzzy numbers. Of course, other shapes of membership functions can be used to define the fuzzy numbers. For example, Gaussian or trapezoidal functions are very often applied (see *e.g.* [6–8]). However, it is worth emphasizing that trapezoidal functions represent fuzzy intervals rather than fuzzy numbers. The fuzzy sets shown in Figures 3 and 4 are not fuzzy numbers, either.

Trapezoidal functions are very useful in assessment systems, especially in the situation when a teacher wants to distinguish several slightly different rates of his evaluation within one mark, *e.g. good*. He can evaluate a student's task, for example, as *weakly good*, *certainly good*, and *strongly good*. Similarly, he may apply his perception-based assessments as *weakly sufficient*, *certainly sufficient*, *strongly sufficient*, and *weakly very good*, *certainly very good*, *strongly very good*, within his evaluation marks as *sufficient* and *very good*, respectively. This kind of differentiation within the mark *insufficient* is not necessary, however it can be employed.

Figure 5 portrays the membership functions of the *weakly sufficient*, *certainly sufficient*, and *strongly sufficient* fuzzy sets, which are included in the fuzzy set which corresponds to the assessment expressed as *sufficient*. In this case, a trapezoidal membership function is used to represent the fuzzy set labeled as *sufficient*. As mentioned earlier, a trapezoidal fuzzy set is a fuzzy interval (not a fuzzy number). However, sometimes it is considered as a fuzzy number [9, 10]. Using this interpretation, the fuzzy interval in Figure 5 can be viewed as the fuzzy number "approximately 3". On the other hand, according to the definition of fuzzy numbers [7], a fuzzy set that have membership values equal 1 not only at one point (called the mean value) is not a fuzzy number. Thus, in Figure 5, the perception-based mark *sufficient* is represented by the fuzzy interval, and the crisp number, 3, is a center of this interval. Triangular membership functions, in Figure 5, are applied in order to define the fuzzy subsets associated with the perception-based assessments: *weakly sufficient*, *certainly sufficient*, and *strongly sufficient*.

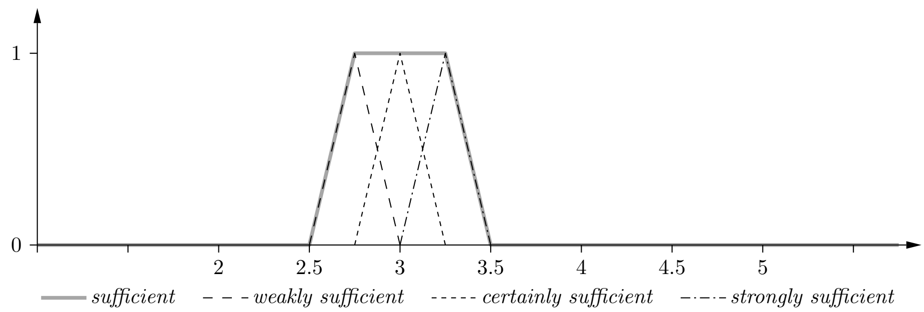

Figure 5. Fuzzy sets that represent the perception of assessment expressed as *sufficient*

Similarly, we can illustrate the trapezoidal fuzzy sets associated with the perception-based evaluation marks *good*, *very good*, as well as *insufficient*, and the triangular fuzzy subsets that correspond to the rates *weakly*, *certainly*, *strongly*. However, in the cases of the marks *insufficient* and *very good*, instead of the trapezoidal functions, the membership functions similar to those portrayed in Figures 3 and 4 can be employed, respectively. Analogously, the triangular membership functions associated with the rates *weakly* with regard to the mark *insufficient* and the assessment *strongly very good* should be replaced by the proper membership functions of the shapes illustrated in Figures 3 and 4, respectively. It is obvious that the fuzzy sets defined by those membership functions should be completely included in the fuzzy sets that correspond to the perceptions of the marks *insufficient* and *very good*, respectively.

Figure 6 shows all the trapezoidal fuzzy sets that represent the evaluation marks *insufficient*, *sufficient*, *good*, and *very good*. For simplicity, the perceptions of all these marks are reflected by the trapezoidal membership functions. However, the interpretation of the marks *insufficient*, and *very good*, analogous to those portrayed in Figures 3 and 4 can be used, as mentioned above. The fuzzy subsets (depicted in Figure 5) are not shown in Figure 6, to make this illustration clearer. For the same reason, the lines representing zero values of the membership functions are omitted in this figure.

Apart from the trapezoindal membership functions, triangular fuzzy sets which correspond to the perception-based assessments associated with fuzzy numbers "about 2.5", "about 3.5", "about 4.5" are portrayed in Figure 6. The marks represented by these fuzzy numbers are used, when an evaluation of a student's task does not suit to any of the previously distinguished fuzzy sets (evaluation marks).

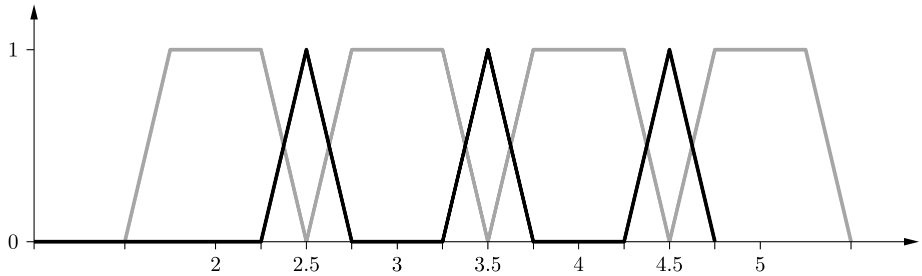

Figure 6. Fuzzy sets representing perception-based evaluation marks

The differentiation of assessment rates within one evaluation mark is very useful from the viewpoint of an overall opinion of a student's exam result. This allows to provide fairer assessment than in the case when only the evaluation marks *insufficient*, *sufficient*, *good* and *very good* are used. The method of reasoning used in order to obtain an overall opinion is presented in the next section.

3. Aggregation of component assessments

Now, let us assume that a teacher has to evaluate results of an exam which is composed of n tasks (or questions), and the number of students trying to pass this exam equals m. Let q_i, where $i=1,\ldots,n$, denote the tasks (questions) which are the components of the exam. The teacher checks the answers (solutions) provided by students, and for each q_i, $i=1,\ldots,n$, gives his perception-based assessment, expressed by words, *e.g. weakly sufficient*, *certainly good*, *strongly very good* (see Section 2). These evaluation marks are represented by fuzzy sets (fuzzy numbers). The words play the role of labels of the perceptions, as well as the labels of the fuzzy sets.

If n is small, for example, only 4 or 5 tasks of the exam, it is very easy for a human (teacher) to infer his opinion about the result of the exam based on component assessments with regard to particular students. The overall evaluation can be easily inferred using the human's perception. In this way, the teacher does not need to calculate the average value (arithmetic average) of the numbers associated with the evaluation marks, as we usually do. He can only use his perception and operate on the words, *i.e.* the labels of the perceptions of the component assessments, in his reasoning process.

However, the bigger n is, the more difficult a reasoning process of this kind is to be performed by a human. Thus, as n increases, it becomes more reasonable to apply computation (to calculate the average value of the marks), instead of relying on human perception only. Additionally, it is preferable that the calculations are performed by a computer, especially when the number of students, m, is large. On the other hand, in the case of exam tasks, there is no difference whether n is small or big, if the overall evaluation is determined by a computer. Thus, if the perception-based reasoning was implemented as a computer program, the overall assessment of an exam would be easily inferred, no matter how many tasks were included in the exam.

Since human perceptions of the evaluation marks are represented by fuzzy sets, the perception-based reasoning can be performed by a computer, employing the methods of manipulation of fuzzy sets, especially fuzzy numbers. In this way, the perception-based reasoning of humans can be imitated by a computer program, where the perceptions are reflected by the fuzzy sets.

A typical fuzzy set theory approach to obtain an overall result based on particular fuzzy sets, is aggregation of the fuzzy sets. There are many aggregation operators [11] that can be employed in order to get an overall fuzzy set. A very popular operator, commonly used in the process of approximate reasoning in fuzzy systems [6, 8, 12], is the max operator. In this case, the membership function of the overall fuzzy set is determined as the maximum of the membership functions of the particular fuzzy sets. The maximum operator, applied to membership functions, realizes the union operation of fuzzy sets.

According to our intuition (and perception), component assessments should be aggregated (as fuzzy sets corresponding to the evaluation marks), and the aggregation may be performed as the union of the fuzzy sets, but an average value (crisp or fuzzy) ought also to be determined.

Since the overall assessment must be expressed by a crisp number, such as 2, 2.5, 3, 3.5, 4, 4.5, 5, we can employ a defuzzification method known as center-average-defuzzification, which is used in fuzzy systems (see *e.g.* [6, 8, 12]). In this way, a crisp value (close to a crisp overall evaluation mark) is obtained as an average of the centers of the membership functions of the component fuzzy sets. It is obvious that the result is the same as obtained by use of the classical mathematical formula that is applied to calculate the arithmetic average of the crisp evaluation marks.

The same result can be determined by means of fuzzy arithmetic (see *e.g.* [9, 13, 14]). The extension principle, introduced by Zadeh [5], allows to extend algebraic operations, such as addition, multiplication, *etc.* from crisp numbers to fuzzy numbers (details can also be found in [7, 8, 10, 15]).

It is not important now to introduce mathematical formulas which may be applied in a computer program in which the perception-based approach has been employed to infer an overall assessment of a student's exam. The interpretation of the problem is significant. This means that using soft computing techniques, especially computing with words, and the perception-based theory introduced by Zadeh [1–5], we can create intelligent systems in the form of computer programs that imitate the perception-based reasoning performed by humans.

4. An illustrative example

In order to illustrate the perception-based reasoning, described in the previous sections, let us present the following example. Students have to pass an exam composed of $n=6$ tasks (or questions). A teacher evaluates these tasks, q_i, $i=1,\dots,n$, using the marks *weakly sufficient*, *certainly good*, *strongly very good*, *etc.*, based on his

Table 1. An example of perception-based exam evaluation

tasks	Student 1	Student 2	…	Student m
q_1	*weakly sufficient*	*strongly good*		*certainly very good*
q_2	*strongly good*	*weakly very good*		*strongly good*
q_3	*certainly sufficient*	*strongly sufficient*		*weakly very good*
q_4	*weakly good*	*certainly very good*		*strongly good*
q_5	*certainly good*	*weakly good*		*strongly very good*
q_6	*strongly sufficient*	*strongly good*		*certainly good*
EM	3.5	4		4.5

perceptions. An example of his assessments is portrayed in Table 1. The last row of the table shows overall evaluation marks of the exam (EM).

The EM values, expressed as crisp marks in Table 1, can easily be inferred by a teacher, based only on his perceptions with regard to the component assessments of the tasks $q_1,\ldots,q_6$ for particular students. The teacher, in his perception-based reasoning, operates on the words (not crisp numbers) that represent his perceptions about the component evaluation marks, such as *weakly sufficient*, *strongly good*, *etc.* In this way, he tries to infer the overall assessment in the form of his perceptions labeled by words, such as *insufficient*, *sufficient*, *good*, and *very good*, associated with crisp numbers 2, 3, 4, and 5, respectively. When it is difficult to evaluate a student's exam using only these crisp marks, the teacher can employ the additional crisp marks 2.5, 3.5, 4.5 to provide an assessments "in between". Thus, $EM=3.5$ expresses the evaluation "better than *sufficient* and worse than *good*." Similarly, $EM=4.5$ means "better than *good* and worse than *very good*."

Analogously to the perception-based reasoning performed by a teacher, as the example in Table 1 shows, reasoning of this kind can be performed by a computer. As mentioned in Section 3, in this case, a computer program operates on fuzzy sets (fuzzy numbers) that correspond to perceptions labeled by words. Component assessments expressed by words, as depicted in Table 1, can be entered using *e.g.* string-type variables in the computer program and then associated with the corresponding fuzzy sets.

As explained in Section 2, component evaluation marks, such as *weakly sufficient*, *etc.*, can be represented by triangular fuzzy numbers, which are illustrated in Figure 5. The component assessments of the tasks solved by Student 1 in Table 1, represented in this way, are shown in Figure 7. For simplicity, the lines representing zero values of the membership functions are omitted in this figure.

If we look at Figure 7, it seems to be obvious that the overall assessment (EM) is 3.5, as shown in Table 1. It is our perception that the evaluation, inferred from the fuzzy numbers depicted in Figure 7, should be represented by a fuzzy number "about 3.5". Hence, the crisp evaluation mark is 3.5.

The result deduced from Figure 7 can be determined by a computer program, using either the extension principle and fuzzy arithmetic or aggregation and defuzzification, as mentioned in Section 3.

The aggregation of the component fuzzy sets as their union, realized by the maximum of their membership functions, gives the result illustrated in Figure 8.

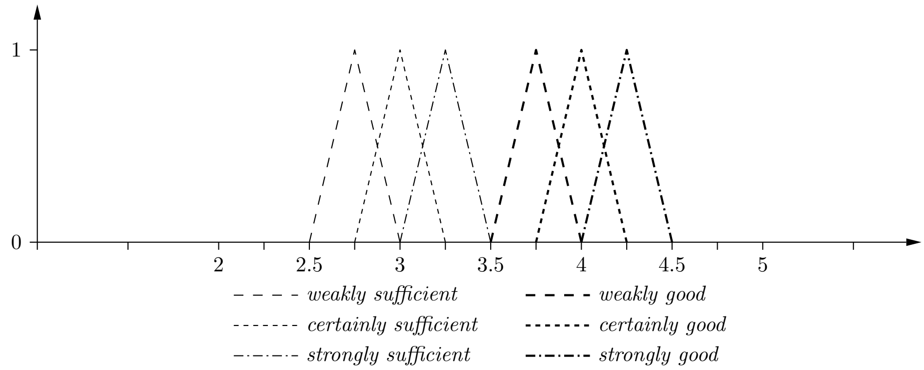

Figure 7. Fuzzy numbers representing the component assessments of Student 1 (Table 1)

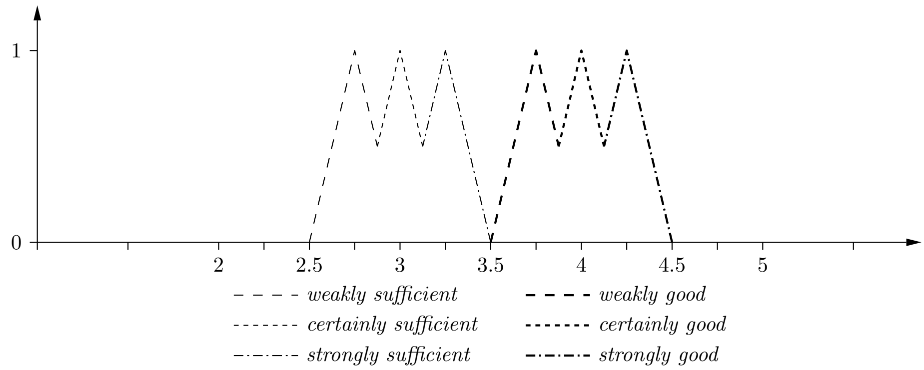

Figure 8. Aggregation of the component fuzzy sets shown in Figure 7

The aggregation of the component fuzzy sets corresponds to the stage of having all the component assessments expressed by words, *i.e. weakly sufficient*, *certainly sufficient*, *strongly sufficient*, *weakly good*, *certainly good* and *strongly good*. The next step is to infer the overall evaluation (EM) by a teacher, based on his perceptions. This stage can be realized by a computer program employing the center-average defuzzification method, mentioned in Section 3.

The center-average defuzzification method allows to obtain a crisp value from the fuzzy set illustrated in Figure 8, using the following formula:

$$EM = \frac{\sum_{i=1}^{n} c_i \mu_{q_i}(c_i)}{\sum_{i=1}^{n} \mu_{q_i}(c_i)}, \tag{1}$$

where μ_{q_i} and c_i, for $i = 1,\ldots,n$, denote membership functions of the fuzzy sets corresponding to the component tasks q_i and centers of these membership functions, respectively.

From Equation (1) and the membership functions shown in Figure 8, we have:

$$EM = \frac{1}{6}(2.75+4.25+3+3.75+4+3.25) = 3.5.$$

Thus, the result determined by this method is the same as the evaluation mark deduced by the teacher, based on his perceptions.

Using the approach applied above, we can easily obtain the evaluation marks (EM) with regard to Student 2 and Student m, presented in Table 1, as well as many others.

As mentioned earlier, the overall evaluation marks (EM) may also be determined by algebraic operations applied to fuzzy numbers. Thus, the arithmetic average value of the fuzzy numbers can be calculated, since the extension principle allows to extend this operation from crisp to fuzzy numbers.

Let Q_i, for $i=1,\dots,n$, denote fuzzy numbers that correspond to the component tasks q_i, respectively. The membership functions, μ_{q_i}, of these fuzzy sets are shown in Figure 7. The average value of these fuzzy sets, which is a fuzzy set, Q, can be obtained as follows:

$$Q=(Q_1\oplus Q_2\oplus\dots\oplus Q_n)\oslash n, \tag{2}$$

where $\oplus$ and $\oslash$ are the extended addition and division operators, *i.e.* the addition and division operators applied to fuzzy numbers (for details see [7, 8, 10]). In this case, the addition operator is employed to n fuzzy sets, and the division operator to one fuzzy set.

It is easy to demonstrate that this approach leads to the same results when the crisp value (corresponding to the overall assessment) is determined as the center of the fuzzy set Q. For the fuzzy numbers depicted in Figure 7, the center of the fuzzy set Q equals 3.5. Let us explain how this result is obtained.

The definition of the extended addition

$$\bar{Q}=Q_1\oplus Q_2\oplus\dots\oplus Q_n$$

has been based on the extension principle, and expressed by membership functions μ_{q_i}, for $i=1,\dots,n$, in the following form [7, 8]:

$$\mu_{\bar{Q}}(y)=\sup_{\substack{x_1,\dots,x_n\\ y=x_1+\dots+x_n}}\min\{\mu_{q_i}(x_1),\dots,\mu_{q_n}(x_n)\},$$

where $\mu_{\bar{Q}}$ is the membership function of the fuzzy set $\bar{Q}$, $x_1\in\mathbf{X}_1$, …, $x_n\in\mathbf{X}_n$, $y\in\mathbf{Y}$, $Q_1\in\mathbf{X}_1$, …, $Q_n\in\mathbf{X}_n$, and $\mathbf{X}_1,\dots\mathbf{X}n$, $\mathbf{Y}\subset\mathbf{R}$.

It is easy to notice that, for the fuzzy numbers shown in Figure 7, the maximal value of $y=x_1+\dots+x_n$, which produces the maximal membership value, $\mu_{\bar{Q}}(y)$, equal to 1, is $y=2.75+3+3.25+3.75+4+4.25=21$. Thus, this value is the center of the fuzzy set $\bar{Q}$. Now, let us apply the extended division, according to Equation (2). This operation results in the fuzzy set of the same shape as $\bar{Q}$ but the support of this fuzzy set, *i.e.* the crisp set (interval) of points that satisfy the inequality $\mu_{\bar{Q}}(y)>0$, is different. The support of the fuzzy set Q, denoted as supp Q, includes the points obtained by dividing the values (points) that constitute the support of the fuzzy set $\bar{Q}$. Hence, $\text{supp}\bar{Q}=(\text{supp}Q)/n$. In our case, when $n=6$, it is obvious that the center of the fuzzy set Q equals $21/6=3.5$. This is the result (EM) presented in Table 1 for Student 1.

In spite of the method described above being more calculation oriented, this approach may also be viewed as perception-based, since our perception is to calculate the average value of the component evaluation marks. Moreover, it is worth noticing that, in the case of exam evaluation, it is not necessary to perform all the calculations to obtain the extended addition of the fuzzy sets. We do not need the exact shape of the fuzzy set that equals to the sum of the component fuzzy sets. The perception is that only the center of the membership function of this fuzzy set should be determined, because it gives the information about the overall assessment. Thus, the computational aspect of the reasoning of this kind can be reduced, and more perception-based techniques may be employed.

One method that allows to lessen the computational effort is the use of α-cuts of fuzzy sets, *i.e.* the crisp subsets of points that satisfy the condition of having values of the membership function equal or greater than α, where $\alpha \in [0,1]$. The definition of α-cuts was introduced by Zadeh [16] (for details, see [6–8, 10]). Applying α-cuts to our example of exam evaluation, instead of using the component fuzzy sets (fuzzy numbers), we can consider their α-cuts, assuming that, for instance, $\alpha = 0.8$. Then, we can employ algebraic operations on crisp intervals (*e.g.* addition of intervals) and determine the overall evaluation (EM) as the center of the interval obtained according to the formula, similar to Equation (2). The well-established arithmetic on crisp intervals are presented *e.g.* in [17, 18].

5. Perception-based neurons

The approach of performing perception-based reasoning by use of extended operators on fuzzy sets, described in Section 4, can be realized when artificial neurons are applied.

Artificial neurons are processing elements of artificial neural networks (called neural networks, for short, see *e.g.* [6, 8, 19]). Artificial neural networks were inspired by the modeling of networks of natural (biological) neurons in the human brain. Neural networks are connectionist architectures composed of simple processing elements (neurons). The mathematical model of an artificial neuron (or simply neuron) is based on the biological neuron. The neuron receives input values from other neurons or from an external input stimulus. These inputs are multiplied by corresponding connection weights and all these signals are added together. The output of the linear part of the neuron thus equals the weighted sum of the inputs. This output value is then usually transformed by a nonlinear activation function (or transfer function), which in most cases is the sigmoidal function. In its simplest form, the linear neuron can be presented as per Figure 9. This model may also include the bias input [19].

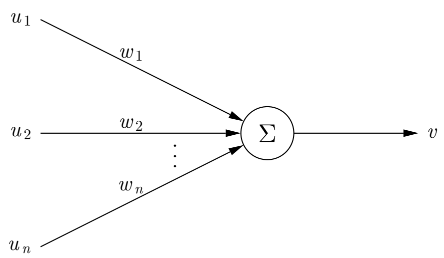

Figure 9. The simplest linear neuron

In the model of the neuron shown in Figure 9, values of the inputs and the output, $u_1,\ldots,u_n$ and ν, as well as the weights, $w_1,\ldots,w_n$, are crisp values, and the addition (denoted by $\sum$) and multiplication operators (that multiply the inputs by the weights) are classical addition and multiplication operators on crisp numbers.

Apart from the classical neural networks, fuzzy neurons and fuzzy neural networks have also been introduced (see *e.g.* [6, 20, 21]). The name "fuzzy neural network" suggests that it refers to neural networks that are fuzzy. This means that some kind of fuzziness has been introduced to standard neural networks, resulting in networks with fuzzy signals and fuzzy weights.

Fuzzy neurons were first proposed by Lee and Lee [22]. Their fuzzy neurons were understood as a fuzzy generalization of the classical McCulloch-Pitts' neuron model [23], which was, historically, the first introduced model, very similar to that presented in Figure 9. Then, fuzzy neural networks were developed by incorporating fuzzy sets into neural networks [24].

Let us notice that the approach of performing perception-based reasoning by use of the extended operators on fuzzy sets, described in Section 4, can be realized by means of the fuzzy neuron. The model of this fuzzy neuron is the same as shown in Figure 9, but input values are fuzzy sets, and the addition and multiplication operators are the extended operators [7]. The input fuzzy sets are the component fuzzy sets, Q_i, for $i=1,\ldots.n$, labeled by words, such as *weakly sufficient*, *strongly good*, *etc.* In order to realize Equation (2) by the fuzzy neuron, the extended division operation must be replaced by the extended multiplication operation, so that this equation has the following form:

$$Q=\frac{1}{n}\otimes(Q_1\oplus Q_2\oplus\cdots\oplus Q_n), \tag{3}$$

where $\otimes$ and $\oplus$ are the extended multiplication and addition operators, respectively.

It is easy to notice that the fuzzy neuron that performs Equation (3) is the neuron illustrated in Figure 9, where the inputs are fuzzy numbers, $Q_1,\ldots,Q_n$, the output is the fuzzy set Q, values of the weights $w_1=\cdots w_n=1/n$, and the multiplication and addition operators are the extended operators. This fuzzy neuron is portrayed in Figure 10, where the input, output, and weight values correspond to the example of Student 1 in Table 1. This neuron reflects Equation (3), because fuzzy sets, Q_i, for $i=1,\ldots,n$, are positive fuzzy numbers, which means that $\mu_{q_i}(x_i)=0$, $\forall\ x_i<0$. Therefore, Equation (3) can be rewritten as follows [7, 10]:

$$Q=\left(\frac{1}{n}\otimes Q_1\right)\oplus\left(\frac{1}{n}\otimes Q_2\right)\oplus\cdots\oplus\left(\frac{1}{n}\otimes Q_n\right).$$

The operator $\sum$ in Figure 9 has been replaced by $\oplus$ in Figure 10, and $n=6$ in this case. The output is the fuzzy set Q, labeled as "about 3.5". A defuzzifier can be added to the neuron in order to obtain the crisp value 3.5, determined as the center of the fuzzy set Q.

It has been shown in this section that perception-based reasoning, which is performed using fuzzy sets representing perceptions expressed by words, can be realized by fuzzy neurons. As mentioned above, artificial neurons imitate biological neurons, which are elements of neural networks in the human brain. In Figure 10, we can see that fuzzy neurons operate on fuzzy sets representing perceptions, in a way similar to that of the human brain. Thus, both fuzzy sets and neural networks would

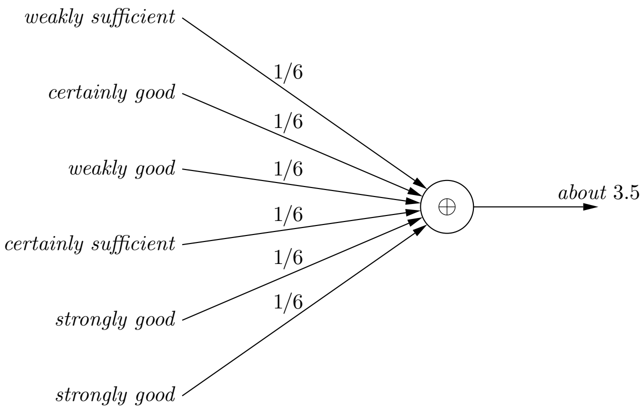

Figure 10. A fuzzy neuron performing perception-based reasoning

seem obviously suitable for modeling artificial intelligence and creating intelligent systems to imitate human intelligence.

A special type of the neuron illustrated in Figure 10 may be obtained when methods employing α-cuts, and algebraic operators on crisp intervals are used (see Section 4).

For problems more complicated than the exam evaluation example, not only single neurons but whole fuzzy neural networks can be applied to operate on perceptions and infer the proper solution. There are many situations similar to exam assessment, but requiring introduction of weights with regard to the component evaluation marks. Even when an exam is considered, one task may be more important than others, so weight values in the fuzzy neuron may be different, unlike in Figure 10. Such an evaluation system is very helpful, because it is much more difficult for a human (teacher) to assess students if various weights are associated with the particular tasks of the exam.

6. Conclusions

The example of the evaluation system considered in this paper is very simple, but definitely helpful from the practical point of view. It is convenient to have a system of this kind when there are many tasks to assess as parts of an exam or a knowledge test. It is much easier to evaluate particular tasks using perceptions (labeled by words) and enter the component assessments into a computer, than evaluate every student's job in a traditional way. If a computer program performs perception-based reasoning to infer the overall evaluation mark, the assessment is much more equitable. A human (teacher) sometimes makes mistakes, especially when he deduces the overall mark from the component evaluation marks, because he may be tired. A computer always infers correctly, and performs the job faster.

The evaluation of exam tasks is only one example of applications of perception-based evaluation systems. Let us imagine that some products, like electronic equipment or cars, must be evaluated for their functionality, quality, and so on. This kind of evaluation may be composed of many assessments of their elements, such as car engines, safety equipment, *etc.* (with regard to cars). Other features are taken into account when electronic devices are evaluated. Perception-based evaluation systems, in the form of a computer program, are certainly very useful in such cases and in many others.

If a perception-based evaluation system was applied in industry or advertising, the reasoning process might be more sophisticated, because component assessments used in those applications could not be as simple as in the example presented in this paper. However, the idea of inference can be incorporated in more complex situations. In addition, some other techniques of manipulation of fuzzy sets may be employed, *e.g.* ordering of fuzzy sets and fuzzy IF-THEN rules [6–8, 10].

It is worth mentioning that the process of thinking (and reasoning), which is perception-based in the human brain, employs rules of the form IF-THEN, and these rules are fuzzy rather than crisp logical rules. It is obvious that people think with fuzzy concepts of perceptions expressed by words, such as "small", "big", "tall", *etc.* The thinking of this kind is called logical thinking and referred to as serial processing [25]. At the same time, the brain is composed of biological neural networks which perform parallel processing (parallel thinking). Thus, both fuzzy IF-THEN rules and artificial neural networks should be applied in order to simulate the natural process of reasoning.

Perception-based intelligent systems have started a new direction in Artificial Intelligence [1, 2]. It seems that the concepts of computing with words and computational theory of perceptions, as developed by Zadeh and mentioned in Section 1, ought to be applied in AI, since they try to imitate the human way of thinking and reasoning. This paper shows that this direction, based on soft computing methods, especially fuzzy the set theory and fuzzy logic, but also neural networks, is promising in the construction of intelligent systems. The systems of this kind are intended to solve many problems using their "intelligence" in a way similar to that of humans. The main point is that these systems should be perception-based systems.

As mentioned in Section 2, the fuzzy sets that represent the perceptions of the evaluation marks can be considered as fuzzy granules. The words associated with the evaluation marks may be viewed as labels of the fuzzy granules. In fuzzy logic, granules are clumps of objects drawn together by similarity, and the objects are grouped into fuzzy granules. The foundation of the theory of fuzzy information granulation, introduced by Zadeh [26, 27] comes from his concept of linguistic variables and fuzzy IF-THEN rules [28, 29]. The theory of fuzzy granulation provides a basis for computing with words (see *e.g.* [6, 30]). It is worth emphasizing that CW is closely related to CTP, as explained in Section 1, and fuzzy granulation plays a significant role in this approach. The example of evaluation systems, presented in this paper, shows the idea of fuzzy granulation which can be incorporated into perception-based systems. Granular computing with regard to neural networks is also considered in [31].

References

[1] Zadeh L A 1999 *IEEE Trans. Circ. Syst. – I: Fundamental Theory and Application* **45** (1) 105

[2] Zadeh L A 2001 *AI Magazine* **22** (1) 73

[3] Zadeh L A 1996 *IEEE Trans. Fuzzy Syst.* **4** 103

[4] Zadeh L A 1974 *Information Processing* **74** 591

[5] Zadeh L A 1965 *Information and Control* **8** (3) 338

[6] Rutkowska D 2002 *Neuro-Fuzzy Architectures and Hybrid Learning*, Springer-Verlag, Heidelberg, New York

[7] Dubois D and Prade H 1980 *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, London

[8] Rutkowska D, Piliński M and Rutkowski L 1997 *Neural Networks, Genetic Algorithms, and Fuzzy Systems*, PWN, Warsaw, Poland (in Polish)

[9] Klir G J 2000 *Fuzzy Sets: An Overview of Fundamentals, Applications, and Personal Views*, Beijing Normal Univ. Press, Beijing

[10] Zimmermann H-J 1991 *Fuzzy Set Theory and its Applications*, (second edition) Kluwer, Boston, Dordrecht, London

[11] Calvo T, Gaspar M and Mesiar R 2002 *Aggregation Operators: New Trends and Applications*, Springer-Verlag, Heidelberg, New York

[12] Driankov D, Hellendoorn H and Reinfrank M 1993 *An Introduction to Fuzzy Control*, Springer-Verlag, Heidelberg, New York

[13] Klir G J and Yuan B 1988 *Fuzzy Sets, Uncertainty, and Information*, Prentice Hall, Englewood Cliffs, New Jersey

[14] Kaufmann A and Gupta M M 1985 *Introduction to Fuzzy-Arithmetic: Theory and Applications*, Van Nostrand, New York

[15] Pedrycz W 1993 *Fuzzy Control and Fuzzy Systems*, Wiley, New York

[16] Zadeh L A 1971 *Information Science* **3** 177

[17] Moore R E 1966 *Interval Analysis*, Prentice Hall, Englewood Cliffs, NJ

[18] Alefeld G and Herzberger J 1983 *Introduction to Interval Computation*, Academic Press, New York

[19] Żurada J M 1992 *Introduction to Artificial Neural Systems*, West Publishing Company

[20] Rutkowska D and Hayashi Y 1999 *J. Adv. Comp. Intelligence* 3 (3) 177

[21] Buckley J J and Feuring T 1999 *Fuzzy and Neural: Interactions and Applications*, Springer-Verlag, Heidelberg, New York

[22] Lee S C and Lee E T 1970 *Proc. 4^{th} Princeton Conf. Information Science Systems*, pp. 381–385

[23] McCulloch W S and Pitts W 1943 *Bull. Math. Biophysics* **5** 115

[24] Lee S C and Lee E T 1974 *J. Cybernetics* **4** 83

[25] Grmela A 1997 *Neural Computing and Neural Science: A Theory of Information Processing in the Human Brain*, AGCES, Praha, Czech Republic

[26] Zadeh L A 1979 *Advances in Fuzzy Set Theory and Applications* (Gupta M, Ragade R and Yager R, Eds.), North Holland, Amsterdam, pp. 3–18

[27] Zadeh L A 1997 *Fuzzy Sets and Systems* **90** 111

[28] Zadeh L A 1975 *Inform. Sci.* **8** 199; *ibid.* 301

[29] Zadeh L A 1975 *Inform. Sci.* **9** 43

[30] Rutkowska D and Hayashi Y 2002 *TASK Quart.* **6** (4) 1 (in this volume)

[31] Pedrycz W 2000 *Int. J. Appl. Math. Comp. Sci.* **10** (4) 723