COMMUNICATION AMONG AGENTS: A SET THEORETIC APPROACH

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Abstract: This paper uses the notion of relative sets in relation to fuzzy set theory to provide a mathematical framework to analyze communication among agents. Each relative set partitions all objects into four distinct regions corresponding to four truth-values of Belnap's logic. Two orderings on relative sets are considered; one is an extension of the classical set inclusion ordering while the other is a new ordering of knowledge or information. According to these orderings, we can divide set theoretic problems into two major categories: *reasoning problems* and *communicating problems*. In the first category, an agent tries to extract a sound decision through granular reasoning. In this case, a granule represents a concept or a word. In the second category, each granule relates to an agent, and the problem is to compare agents' knowledge about concepts by their related granules, *e.g.* a knowledge reduction problem. Then, we concentrate on the second category of problems and try to investigate this kind of problems in the context of fuzzy set theory. In this way, we could provide a basis for modeling and analyzing the relations among machines, which could communicate with each other using words and granules.

Keywords: computing with words, granular computing, fuzzy sets, rough sets, multi-valued logic

1. Introduction

In 1977 Belnap introduced a four-valued logic [1, 2] to deal with incomplete and inconsistent information. Two truth-values of these four values are the classical *True* and *False* values and the others are new ones, called *None* and *Both*. By doing this, he actually provided a new ordering of knowledge such that a truth-value, regardless of its measure of truth, receives a state of determination or a degree of knowledge. According to truth ordering, *True* has the maximum value, *False* has the minimum value, and *None* and *Both* are two intermediate values, while according to knowledge ordering *Both* has the maximum value, *None* has the minimum value, and *True* and *False* are two intermediate values.

A theory appropriate for dealing with several information sources should have the capability of dealing with the incompleteness of and discrepancies in the available information. In [3–6], we have introduced relative sets as a counterpart of Belnap's four-valued logic, to deal with several sources of information. Each relative set partitions all objects into four distinct regions, according to the four values of Belnap's logic. Like Belnap's four truth-values, relative sets have two orderings: one is an order of inclusion, which is an extension of the classical set inclusion ordering, and the other is an ordering of knowledge or information, which is a new ordering.

According to these orderings, we can divide set theoretic problems into two main categories; *reasoning problems*, corresponding to the order of inclusion, and *communicating problems*, corresponding to the order of knowledge. We claim that in a reasoning problem, the main concern is to derive a sound decision from some information, while in a communicating problem, the aim is to investigate the relations and dependencies among several information sources. We also investigate this distinction in relation to fuzzy set theory to provide a more general framework to analyze relations among agents.

In Section 2, we briefly review Belnap's four-valued logic and the notion of relative sets. In Section 3, an explanatory description of relative sets is proposed. Using this description, we present our division of set theoretic problems into two major categories, named *reasoning problems* and *communicating problems*. A communicating problem investigates the relations of several information sources, each of them called an agent. Section 4 introduces two approaches to develop the notion of relative set and communicating problems to the theory of fuzzy sets. One is based on Klir's views on fuzzy sets [7], while the other has been obtained by defining the notion of *a relative fuzzy set*. In Section 5, we briefly refer to the notion of rough communication as an integrated approach to relative sets and rough sets, which is discussed in depth in [6, 8].

2. Relative sets

The origin of relative sets is in Belnap's four-valued logic, introduced in 1977 [1, 2]. It suggests four truth-values $V = \{\textit{True,False,Both,None}\}$ for each sentence p. The intuitive meanings of these values are:

1. p is stated to be true only (*True*),
2. p is stated to be false only (*False*),
3. p is stated to be both true and false (*Both*),
4. p's state is unknown, *i.e.* neither true nor false (*None*).

The truth-values of Belnap's logic mentioned above have two natural orderings: one ordering, $\leq_t$, records degree of truth. According to this order, *False* is the minimal element, *True* is the maximal one, and *Both*, *None* are two intermediate values that are incomparable. $(V, \leq_t)$ is a lattice with an order reversing involution $\neg$, for which $\neg$*Both*=*Both* and $\neg$*None*=*None*. The lattice's meet and join operators are denoted by $\wedge$ and $\vee$, respectively. The other ordering, $\leq_k$, reflects degree of information or knowledge, in which *Both* has the maximum value, *None* has the minimum value, and *True* and *False* are two intermediate values. So, we have another lattice with the same set V and the ordering of $\leq_k$. We shall denote the meet and join of the $\leq_k$ by $\otimes$

and $\oplus$, called *consensus* and *gullibility*, respectively. Note that the negation operator $\neg$ is an order preserving operator with respect to $\leq_k$. A diagram of these two orders is depicted in Figure 1. The structure which consists of these four elements and the five basic operators ($\wedge$, $\vee$, $\neg$, $\otimes$, $\oplus$) is usually called "FOUR" [9].

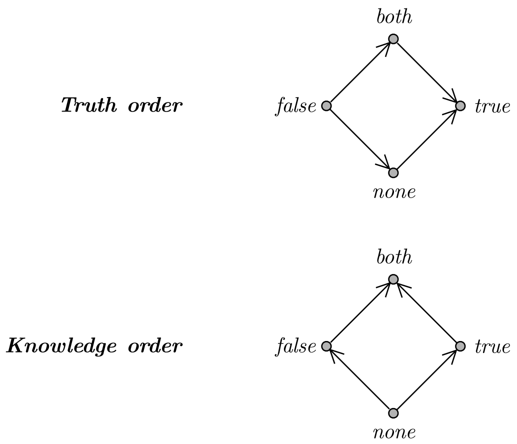

Figure 1. Schematic diagram of two orders. Two comparable values are connected with a directed line. The direction of the line is from a lower value to a higher value

Using Belnap's four-valued logic in [3–6], an extension of classical sets were introduced. The main idea is that constructing a set for a concept is completely dependent on the knowledge or opinions of the agent who constructs the set. Therefore, each set is defined relative to an agent and called a *relative set*. Consequently, we suggest using two sets rather than one to represent a concept; one is the positive region which consists of all elements for which the agent has evidence of belonging to the concept, and the other is the set of all elements for which the agent has evidence that they do not belong to the concept. This can be achieved by using a pair of classical sets (A^+, A^-) to represent a concept which is called a relative set. A^+ is the set of all objects for which there is some evidence that they belong to the concept and is called the positive region of the relative set. A^- consists of all objects for which there is evidence against their belonging to the concept and is called the negative region of the relative set. We will say that:

1. a belongs to (A^+, A^-) if $a \in A^+$ and $a \notin A^-$,
2. a does not belong to (A^+, A^-) if $a \notin A^+$ and $a \in A^-$,
3. a has contradictory behavior in (A^+, A^-) if $a \in A^+$ and $a \in A^-$,
4. belonging of a to (A^+, A^-) is unknown if $a \notin A^+$ and $a \notin A^-$.

According to these definitions, a relative set partitions all objects into four distinct regions:

1. The region of all objects that belong to A^+ and do not belong to A^-. This region corresponds to the *True* value of Belnap's logic.
2. The region of all objects that belong to A^- and do not belong to A^+. This region corresponds to the *False* value of Belnap's logic.
3. The region of all objects that belong both to A^+ and A^-, which corresponds to the *Both* value of Belnap's logic.

4. The region of all objects that belong neither to A^+ nor A^-, which corresponds to the *None* value of Belnap's logic.

The five defined operations (intersection, union, complement, consensus and gullibility) are defined on relative sets as follows:

1. Relative sets' intersection:

$$(A^+,A^-)\cap_R(B^+,B^-)=(A^+\cap B^+,B^-\cup A^-)$$

2. Relative sets' union:

$$(A^+,A^-)\cup_R(B^+,B^-)=(A^+\cup B^+,B^-\cap A^-)$$

3. Relative sets' complement:

$$\sim\left(A^+,A^-\right)=\left(A^-,A^+\right)$$

4. Relative sets' consensus:

$$\left(A^+,A^-\right)\otimes\left(B^+,B^-\right)=\left(A^+\cap B^+,A^-\cap B^-\right)$$

5. Relative sets' gullibility:

$$\left(A^+,A^-\right)\oplus\left(B^+,B^-\right)=\left(A^+\cup B^+,A^-\cup B^-\right)$$

Consensus of two relative sets can be understood as an intersection of two agent's knowledge about the same concept, and gullibility of two relative sets can be understood as a union of two agents' knowledge about the same concept. Furthermore, like Belnap's four truth-values, we can distinguish two distinct orders for relative sets:

1. Relative sets' inclusion ordering:

$$\left(A^+,A^-\right)\subseteq_I\left(B^+,B^-\right)\Leftrightarrow A^+\subseteq B^+,B^-\subseteq A^-$$

2. Relative sets' knowledge ordering:

$$\left(A^+,A^-\right)\subseteq_K\left(B^+,B^-\right)\Leftrightarrow A^+\subseteq B^+,A^-\subseteq B^-$$

The first ordering, $\subseteq_I$, is an extension of the classical set inclusion, which corresponds to $\leq_t$ ordering of FOUR. $(2^U\times 2^U,\subseteq_I)$ is a lattice whose meet and join operators are relative set intersection, $\cap_R$, and union, $\cup_R$, respectively, and the relative set complement is an order reversing involution of this lattice. The second ordering, $\subseteq_K$, is new and can be understood as an order of information exhibited by each relative set. It corresponds to $\leq_k$ ordering of FOUR, and $(2^U\times 2^U,\subseteq_K)$ will be a lattice in which the relative set consensus $\otimes$ and the relative set gullibility $\oplus$ are its meet and join operators, respectively. The relative set complement $\sim$ is an order-preserving operator of this lattice.

Example 1:

Table 1. Results of three referees' judgments

Article	1	2	3	4	5	6	7	8	9	10
Referee 1	G	G	W	G	M	M	G	W	W	—
Referee 2	G	—	W	G	—	M	G	W	W	—
Referee 3	G	M	W	G	M	G	—	—	M	W

Suppose there are three referees who are asked to classify ten numbered articles as Good (G), Medium (M) and Weak (W). Table 1 shows their judgements. The symbol "—" in this table shows that the specific referee could not judge about the related article. If somebody asks us to determine the set of good articles, we can base our judgment on each of the referees opinion. Thus, we can propose three sets as follows:

- According to referee 1: The set of good articles $=\{1,2,4,7\}$
- According to referee 2: The set of good articles $=\{1,4,7\}$
- According to referee 3: The set of good articles $=\{1,4,6\}$

In other words, for the concept of "good" among articles, there are three sets such that each of them could be a sound choice. But, we must consider another point, as well. For every referee, there are some articles that he or she can not judge. If we omit these articles from the universe set of each referee, then we will have different universe sets for different referees:

- According to referee 1: Universe set $=\{1,2,3,4,5,6,7,8,9\}$
- According to referee 2: Universe set $=\{1,3,4,6,7,8,9\}$
- According to referee 3: Universe set $=\{1,2,3,4,5,6,9,10\}$

Hence, the set complement of the set "good" defined by:
Set complement of the concept good = Universe set − Set of the concept good is calculated in a different completely way for each referee:

- According to referee 1: The set of non-good articles $=\{1,2,3,4,5,6,7,8,9\}-\{1,2,4,7\}=\{3,5,6,8,9\}$
- According to referee 2: The set of non-good articles $=\{1,3,4,6,7,8,9\}-\{1,4,7\}=\{3,6,8,9\}$
- According to referee 3: The set of non-good articles $=\{1,2,3,4,5,6,9,10\}-\{1,4,6\}=\{2,3,5,9,10\}$

In other words, in each case we have a relative set. Furthermore, someone may ask us to combine this information to get a better judgment. Two reasonable ways are as follows:

1. Accepting only those parts of information, which the three referees agree on (consensus of two relative sets). So, the set of good articles will be the intersection of the three proposed sets of good articles:

   $$\text{The set of good articles} = \{1,4\}.$$

   If we consider the information obtained in this way as a consensus referee, then we have Table 2 for our information. In this way, we have obtained the information with a high degree of confidence, however, we miss some information.
2. Accepting all the information (gullibility of two relative sets). In this case, the set of good articles will be the union of the three proposed sets of good articles:

   $$\text{The set of good articles} = \{1,2,4,6,7\}.$$

   If we consider this information as a gullibility referee, then we have Table 3 for our information. In this way, we do not miss any information, but the obtained information is not reliable and, also, we have contradictory information.

Table 2. Consensus referee

Article	1	2	3	4	5	6	7	8	9	10
Consensus referee	G	—	W	G	—	—	—	—	—	—

Table 3. Gullibility referee

Article	1	2	3	4	5	6	7	8	9	10
Gullibility referee	G	M,G	W	G	M	M,G	G	W	W,M	W

Another point is that, if we compare the classification of referee 1 with that of referee 2 with regard to good articles, we see that their judgments are the same except for article 2, where referee 2 has no idea. In other words, comparing the results of classification of good articles according to referees 1 and 2, we find out that the judgement of good articles of referee 1 has more knowledge than that of referee 2.

3. Communication among agents

In the classical set theory, each concept or label (which is a word like red, tall,*etc.*) determines a crisp granule or set of objects on the universe set. So, this theory enables us to deal with labels instead of dealing with objects and reduces our computation in decision making [10]. In relative sets, however, an agent (a person or a machine) and a concept together determine a relative granule or set in the universe. That is, a granule for a concept is completely dependent on the related agent's knowledge. For example, the set of good articles in Example 1 varied from one referee to the other. Classical sets could be considered as special cases of relative sets where only one agent is present. Figure 2 shows three collections: collection of agents $A=\{a_1,a_2,\ldots,a_m\}$, collection of concepts $C=\{c_1,c_2,\ldots,c_n\}$ and a collection of objects $U=\{x_1,x_2,\ldots,x_n\}$. As far as classical sets are concerned, we do not deal with agents and there is just mapping between concepts and granules of objects. In other words, we have the following mapping:

$$L_c : C \to 2^U .$$

Relative sets, however, deal with agents as well as concepts. Hence, for relative sets, we have this mapping:

$$L_r : A \times C \to 2^U \times 2^U .$$

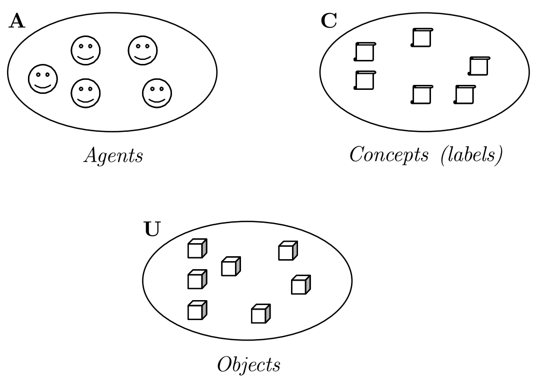

Figure 2. Three collections

$$
\begin{array}{c|cccc}
a_m & R_{m1} & R_{m2} & \cdots & R_{mn} \\
\vdots & \vdots & \vdots & & \vdots \\
a_2 & R_{21} & R_{22} & \cdots & R_{2n} \\
a_1 & R_{11} & R_{12} & \cdots & R_{1n} \\
\hline
 & c_1 & c_2 & \cdots & c_n
\end{array}
$$

Knowledge order

Inclusion order

Figure 3. Entry R_{ij} is equal to $L_r(a_i,c_j)$; each row consists of equi-knowledge relative sets and each column consists of equi-concept relative sets

This mapping relates a pair of sets (a relative set) to a pair of an agent and a concept. So, the considered relative set may vary from one concept to another and also from one agent to another. Figure 3 is an illustrative diagram of this mapping, in which a_1, a_2,..., a_m are agents, c_1, c_2,..., c_n are concepts and entry R_{ij} is the relative set $L_r(a_i,c_j)$ of concept c_j due to agent a_i. In this diagram, each row consists of relative sets for different concepts due to one agent, called *equi-knowledge* relative sets, and each column consists of relative sets for one concept with respect to different agents, called *equi-concept* sets. With this definition, all classical sets on a universe set may be considered as equi-knowledge sets. Now, we could divide our problems into two major categories: *reasoning problems* and *communicating problems.* A reasoning or decision making problem deals with equi-knowledge sets. In other words, we have only one agent or knowledge base and the main point is extracting a sound conclusion to make a right decision. In problems of this kind, we investigate the relations among concepts using intersection, union and set inclusion order as its basic operations and relations. A communicating problem deals with equi-concept sets. That is, we deal with agents and, having only one concept, we want to investigate the relations among agents using consensus, gullibility and knowledge order as the basic definitions. In Table 4, each row exhibits the definitions which have the same role in each kind of problems. In this paper, we would like to concentrate on communicating problems to provide a mathematical basis for analyzing machines which could communicate with words or granules.

Table 4. Corresponding definitions of two kinds of problems

Reasoning problem	Communicating problem
Concept	Agent
Intersection	Consensus
Union	Gullibility
Inclusion order	Knowledge order
Decision making	Knowledge reduction

Example 2 (A reasoning problem):

Suppose that referee 1 who classified articles as good, medium and weak as in Table 1, classifies articles once again as either Practical (P) or Non-practical (N), as in Table 5. Comparing these two classifications, one can see that the set of good articles

Table 5. Another classification of referee 1

Article	1	2	3	4	5	6	7	8	9	10
Referee 1	P	P	N	—	N	N	P	N	N	—

includes the set of practical articles and one can deduce the following for referee 1: *"If an article is practical, then it is good."* Referee 1 can also make sets for concepts such as: *"good and non-practical"*, *"weak or practical"*, *etc.*, using intersection and union operators.

Example 3 (A communicating problem):

Consider the classifications of Table 1. Comparing the classifications of agents 1 and 2 for good articles, one can come to this conclusion: *"If referee 1 judges about an article being good or not being good, then we do not need the judgement of referee 2."* In other words, when articles are considered to be good or not good, referee 1 has more knowledge about articles than referee 2. It is also possible to have some sets of good articles with respect to: *"consensus of referees 1 and 2"*, *"gullibility of referees 1 and 3"*, *etc.*, using consensus and gullibility operators.

4. Fuzzy communication

To extend the notion of relative sets to fuzzy sets, we could adopt two approaches: one is based on Klir's views on fuzzy sets [7], which leads us to defining a *fuzzy knowledge function* for each agent, and the other consists in defining a pair of fuzzy sets as a *relative fuzzy set*. These extensions are considered in the following two sections.

4.1. Approach 1: fuzzy knowledge

The first approach is based on Klir's views on fuzzy sets. He has claimed that fuzziness arises when different agents assign different crisp sets to a concept. So, more assignments of an object to a set result in a higher value of membership of that object in the set. In other words, if we average different agents crisp sets for a concept, the fuzzy set of that concept is obtained. By changing the role of agents and concepts in this description of fuzzy sets, we arrive to an extension of fuzzy sets, interpreted thus: if an agent knows more about belonging or non-belonging of an object to different concepts, he will know more about that object. In this interpretation, instead of *a membership function*, there is a function called *knowledge function*. In other words, membership function is a result of averaging on equi-concept sets, while a knowledge function is an averaging of equi-knowledge sets.

To clarify these notions, let us suppose that there are m agents and n concepts like in Figure 2. According to our discussion, there is a fuzzy set corresponding to each concept, or a row in Figure 3. If the membership of concept c_i is denoted by $\mu(c_i,x)$, then we have:

$$\mu(.,.): C\times U \to [0,1].$$

Hence, we have n membership function for n concepts. There is also m knowledge function corresponding to m agents, or a column in Figure 3. If the knowledge function of agent a_j is denoted by $K(a_j,x)$, we have:

$$K(.,.): A\times U \to [0,1].$$

As the operations $\cap$ and $\cup$ are related to concepts, these operations are extended to fuzzy membership function and, similarly, as the operations $\otimes$ and $\oplus$ are related to agents, they have to be extended to the knowledge function. The mathematical definitions of $\otimes$ and $\oplus$ could be the same as $\cap$ and $\cup$.

Example 4:

Table 6. Classifications of person 1

Animal	1	2	3	4	5
Kind	Lion	Lion	Non-lion	Non-lion	—
Color	Gray	Gray	Gray	Non-gray	—
Size	Big	Big	Non-big	—	—

Table 7. Classifications of person 2

Animal	1	2	3	4	5
Kind	—	Lion	Non-lion	Non-lion	—
Color	Gray	Non-gray	Gray	—	—
Size	—	Big	Non-big	Big	Non-big

Table 8. Classifications of person 3

Animal	1	2	3	4	5
Kind	Lion	Lion	—	Non-lion	Non-lion
Color	—	Gray	Gray	Non-gray	Gray
Size	Big	Non-big	Non-big	Non-big	Non-big

Consider three persons who are classifying the attributes of five given animals as in Tables 6–8 with respect to their kind, color and size. Firstly, we average the three persons' classification to obtain the membership functions of the concepts such as lion, gray and big (Figures 4–6, p. 62). For example, two persons (1 and 3) say that animal 1 is a lion and all of them say that animal 2 is a lion. So, the value of membership function of animal 1 to the concept of lion will be 2/3, where the value of membership function of animal 2 to the concept of lion will be 1. Secondly, we average the concepts to obtain the knowledge functions of persons (Figures 7–9, p. 62). According to the tables, we find that person 1 has no knowledge about animal 5 and also has more knowledge about animal 4 compared to animal 3. Hence, the value of knowledge function of person 1 for animal 5 will be zero, and its value for animal 3 will be higher than that for animal 4.

4.2. Approach 2: relative fuzzy sets

In this section, we assume that the concepts are inherently fuzzy with respect to all agents. In other words, each agent assigns a fuzzy granule to each concept. So, in this case, communication among agents will be through fuzzy granules. To extend the notion of relative sets to provide a basis for communication with fuzzy granules, the notion of a relative fuzzy set is defined. As a relative set is a pair of classical sets, a relative fuzzy set is defined as a pair of membership function. One of them represents

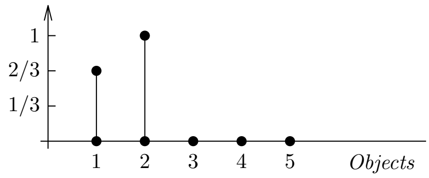

Figure 4. Membership function of the concept "lion"

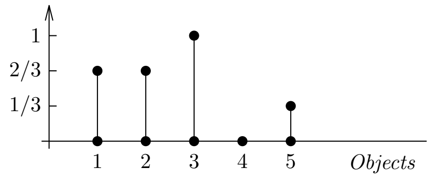

Figure 5. Membership function of the concept "gray"

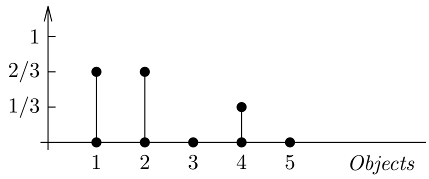

Figure 6. Membership function of the concept "big"

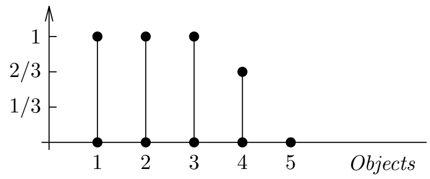

Figure 7. Knowledge function of person 1

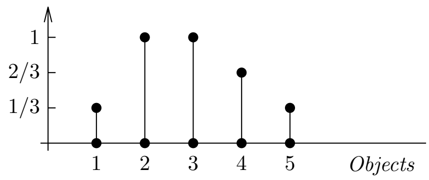

Figure 8. Knowledge function of person 2

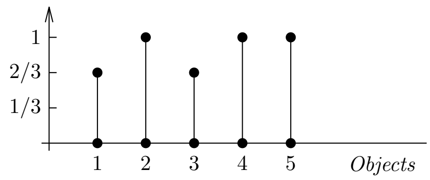

Figure 9. Knowledge function of person 3

the degree of belonging of objects to the concept and the other is the degree of not belonging of objects to the concept. So, a relative fuzzy set is a pair of membership functions as follows:

relative fuzzy set: $(\mu_A^+(x), \mu_A^-(x))$,
positive membership function: $\mu_A^+(\,.\,): U \to [0,1]$,
negative membership function: $\mu_A^-(\,.\,): U \to [0,1]$.

So, for an object x:

1. $\mu_A^+(x) + \mu_A^-(x) < 1$ shows lack of knowledge about object x,
2. $\mu_A^+(x) + \mu_A^-(x) > 1$ shows contradictory information about object x.

Now, we can extend the operations and relations of relative sets to fuzzy sets:

1. Relative fuzzy sets' intersection:

$$(\mu_A^+(x), \mu_A^-(x)) \cap_{RF} (\mu_B^+(x), \mu_B^-(x)) = (\mu_A^+(x) \cap_F \mu_B^+(x), \mu_A^-(x) \cup_F \mu_B^-(x));$$

2. Relative fuzzy sets' union:

$$(\mu_A^+(x), \mu_A^-(x)) \cup_{RF} (\mu_B^+(x), \mu_B^-(x)) = (\mu_A^+(x) \cup_F \mu_B^+(x), \mu_A^-(x) \cap_F \mu_B^-(x));$$

3. Relative fuzzy sets' consensus:

$$(\mu_A^+(x), \mu_A^-(x)) \otimes_{RF} (\mu_B^+(x), \mu_B^-(x)) = (\mu_A^+(x) \cap_F \mu_B^+(x), \mu_A^-(x) \cap_F \mu_B^-(x));$$

4. Relative fuzzy sets' gullibility:

$$(\mu_A^+(x),\mu_A^-(x)) \oplus_{RF} (\mu_B^+(x),\mu_B^-(x)) = (\mu_A^+(x) \cup_F \mu_B^+(x), \mu_A^-(x) \cup_F \mu_B^-(x));$$

5. Relative fuzzy sets' complement:

$$\sim (\mu_A^+(x),\mu_A^-(x)) = (\mu_A^-(x),\mu_A^+(x)),$$

where $\cap_F$ and $\cup_F$ are fuzzy intersection and union, respectively. Furthermore, two orders of inclusion and knowledge may be distinguished on relative fuzzy sets, using fuzzy set inclusion order instead of the classical set inclusion order in the definitions of two orders of relative sets.

4.3. Intuitionistic fuzzy sets

In [11, 12] Atanassov has proposed a generalization of fuzzy sets in which he used two degrees of membership and non-membership to describe the vinculation of an element to a set, so that the sum of these degrees is always less or equal to 1. Formally, an intuitionistic fuzzy set (IFS) on a non-empty set X is an expression A given by:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\},$$

where

$$\mu_A : X \to [0,1], \nu_A : X \to [0,1],$$

with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all x belonging to X. μ_A is called the degree of membership and ν_A is called the degree of non-membership of x to A. The amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the hesitation part.

Definitions:

If A and B are two IFSs of the set X, then:
$A \subset B$ iff $\forall x \in X, [\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)]$,
$A = B$ iff $\forall x \in X, [\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)]$,
$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle | x \in X\}$,
$A \cap B = \{\langle x, \min(\mu_A(x),\mu_B(x)), \max(\nu_A(x),\nu_B(x))\rangle | x \in X\}$,
$A \cup B = \{\langle < x, \max(\mu_A(x),\mu_B(x)), \min(\nu_A(x),\nu_B(x))\rangle | x \in X\}$.

In this context, our second approach will not be a new notion any more, but we still insist on calling this extension a relative fuzzy set, because it suggests two new operators on sets, *viz.* consensus and gullibility, and intuitive meanings can be found for them if one thinks about IFSs as relative sets. Besides, the second approach is capable of modeling cotradictory information, which may be due to having several information sources. As our main interest in this paper is to provide a mathematical basis for communication among agents, and the second approach makes it possible to have bridge intuitionistic fuzzy sets and relative sets, it would be interesting to make some extensions on IFSs to make them a proper tool for analysis of relations among agents.

5. Rough communication

Rough set theory is a generalization of the classical set theory for modeling uncertainty or incomplete information [13, 14]. In other words, it provides a formal tool of dealing with a source of information which may be incomplete or imprecise. Besides, this theory has the ability of knowledge reduction in which we probe knowledge for dependencies. In [8], we have introduced the distinction presented in Section 3, as two facets of rough sets. The first facet of a rough set refers to solving reasoning problems and the second concerns communicating problems. These two facets also propose two sources which cause the roughness of a set; the first facet considers the roughness as a result of incompleteness of available information, while the second proposes it to be a result of granular information transformation between two agents, who are not using exactly the same languages.

6. Discussion

In this paper, with regard to a new ordering on sets, we claim that set theoretic problems could be divided into two major categories: reasoning and communicating problems. We have tried to investigate the communicating problems by taking into account fuzzy set theory. We have also tried to provide a mathematical framework to deal with communicating problems, using an extension of the fuzzy set theory. The interesting point is that the relations used for communicating problems are completely similar to those of reasoning problems, and this similarity enables us to use methods used in reasoning problems to solve communicating problems.

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