

## WHY EVERYTHING GETS SLOWER?

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**Abstract:** A social system is represented by the Barabási-Albert model. At each node of the graph, an Ising spin is placed,  $S = \pm 1$ , with antiferromagnetic interaction between connected nodes. The time to reach equilibrium via Glauber kinetics does not depend on system size. The average energy associated with the rare spin flips in equilibrium oscillates with the number  $m$  of edges of new nodes. The conclusions are illustrated with events from recent European history, where after some strong change a rather immobile society evolved.

**Keywords:** sociophysics, stability, Barabási-Albert, Ising antiferromagnet

### 1. Introduction

Some of us wonder sometimes why it is so difficult to reorganize a social system [1, 2]. There are many examples in history when a society prevents changes just by means of inertia. A commonly accepted example is provided by the history of Soviet Union in the 60's, when reforms of Khrushchev failed despite an apparent lack of objections. In fact, the famous 'perestrojka' in the 80's could happen only because members of the Politburo in Moscow were pretty sure that Gorbachev does not talk seriously [3].

Here, we are interested in the time dependence of the flexibility of a social system, described in the simplest possible way. We profit from recent works of Barabási and Albert [4–6], where social connections are modeled as edges of graphs. A special algorithm was introduced in [4, 5] to construct the so-called scale-free networks of nodes connected by edges. The resulting systems were proven to show qualitative similarities with networks known from different branches of knowledge, such as biology, ecology or linguistics. For a review of this new subject, see [6].

A social system is represented by a scale-free Barabási-Albert network, with the number  $m$  of edges of successively added nodes as the only parameter. Ising spins  $S = \pm 1$  are assigned to all nodes. The interaction is chosen to be antiferromagnetic and it is limited to the nodes directly connected by edges. This choice may be motivated

heuristically as follows. There is a natural tendency of social units (people, enterprises or political fractions) to differ and to compete. As a result of the competition, one unit of each two becomes more self-concentrated, egoistic and powerful than the other – and it wins. In this sense, a stable state of each pair of units is: one up (positive, subordinated) and one down (negative, prevailing). At the initial state of a newly created system, however, all units are idealistic and claim to serve the community. A stable state of the whole system is attained by a gradual evolution of the units, which consists in local competition between members of interacting pairs. A similar splitting of an evolving society into performers of two opposite strategies was described by Mitchell *et al.* in terms of rules of cellular automata [7].

Our time evolution accords with the Monte-Carlo Glauber dynamics [8]. However, we believe that in our problem the character of the evolution rule is of secondary importance. The relevant feature is the existence of the working function – energy in the Ising model – which is minimized apart from thermal fluctuations. As it was proven by Derrida [9], in this case, a local equilibrium state is well defined, and the length of the limit cycle is at most 2. In fact, we need only the existence of an absorbing state, which is stable against existing disturbances. This condition seems to be fulfilled also in the world of social phenomena, at least until a Great Disturbance appears.

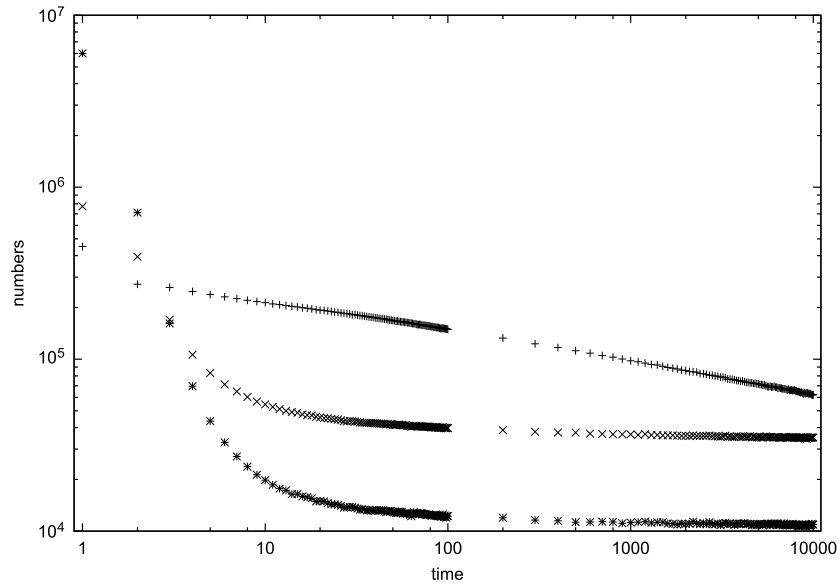
In contrast to spin glass simulations since a quarter of a century, we do not discuss here phase transitions, ordering below the phase transition temperatures, or ground states. Instead, we look at the initial nonequilibrium dynamics as a model for the development of social rigidity.

## 2. Results

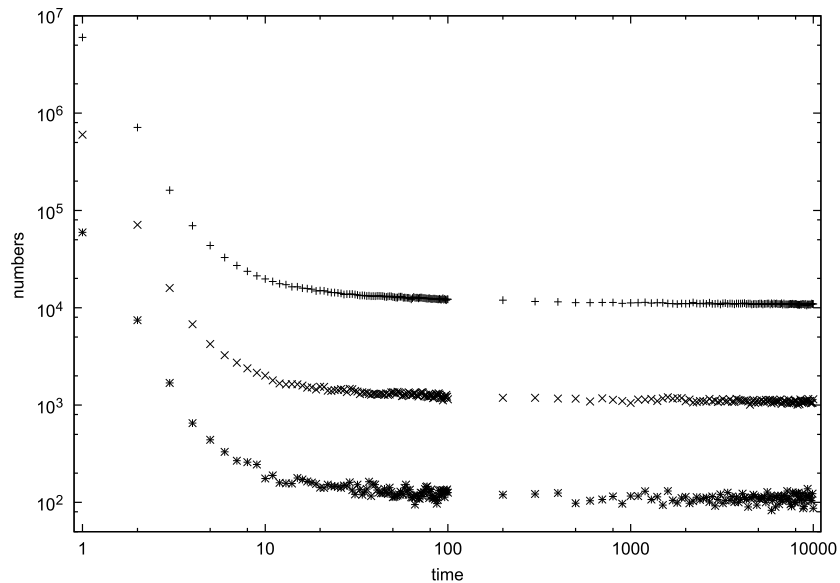
We used the program employed already by Aleksiejuk *et al.* [10] and replaced ferromagnetic couplings by antiferromagnetic ones. The Barabási-Albert network started with  $m$  mutually connected sites and was then grown to  $N + m$  sites according to the standard rule that a newly added site selects  $m$  neighbours from the already existing network sites, with a selection probability proportional to the number  $k$  of neighbours the selected site had before. The Ising spins, initially all up, were flipped with heat bath (Glauber) dynamics, *i.e.* up with probability  $x/(1+x)$  and down with probability  $1/(1+x)$ , where  $x = \exp(-nJ/k_B T)$  is the Boltzmann thermal probability and  $n = \sum_j S_j$  is the sum over all  $k(i)$  neighbours of spin  $i$ , with  $S_j = \pm 1$  and a positive exchange energy  $J$ . One iteration  $t \rightarrow t+1$  is one Monte Carlo step per spin. During each iteration, we counted the total number of spins which are flipped, the total energy change ( $|n|$  summed over all spin flips), and the magnetization  $M = \sum_i S_i$ .

Figure 1, for  $m = 5$ , shows that the magnetization decays weakly with time, the total energy changes strongly, while the number of flipped spins is mostly in between. Changing the network size  $N$  from 2 million to 20 000 gives similar results, see Figure 2. Figures 3 and 4 show a slightly faster equilibration, with  $m = 1$  or 2 instead of 5, and Figure 5 summarizes this trend for  $m = 2, 4, 6, 8, 10$ .

In Figure 2, we see that for  $m = 5$  the energy change stabilizes in about 100 timesteps, and this characteristic time does not depend on the system size. (It also remains about the same if we use lower temperatures.) The stable value is a small



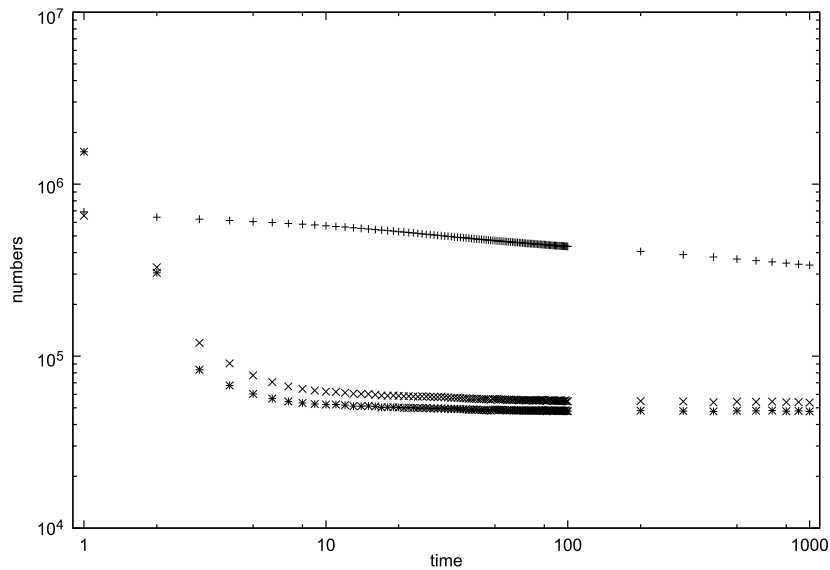
**Figure 1.** Relaxation for  $m = 5$  at  $k_B T/J = 0.5$  for two million nodes. Numbers (from top): magnetization, flipped spins, changed energies



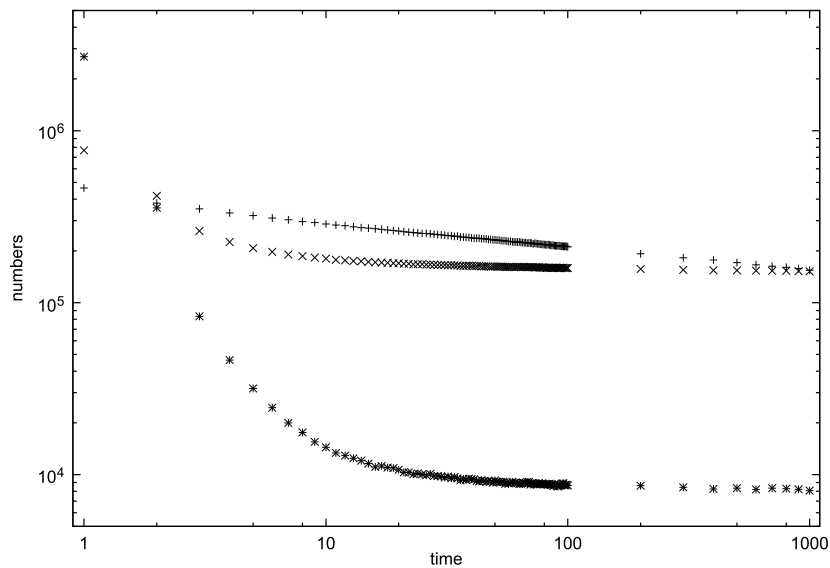
**Figure 2.** Sum of energy changes for small ( $N = 20\,000$ ), medium ( $N = 200\,000$ ) and large ( $N = 2\,000\,000$ ) systems; again  $m = 5$  at  $k_B T/J = 0.5$

fraction of the number of nodes, *viz.* about  $0.005N$ . Similarly, the number of spin flips stabilizes near  $0.015N$ . The magnetization changes reveal two stages: after a short transient, magnetization decreases with time  $t$  proportionally to  $t^{-\beta}$ , with  $\beta$  less than 0.1.

The  $m$  dependence of the results, Figure 6, reveals a kind of oscillations with amplitude decreasing with  $m$ . This effect can be assigned to the difference



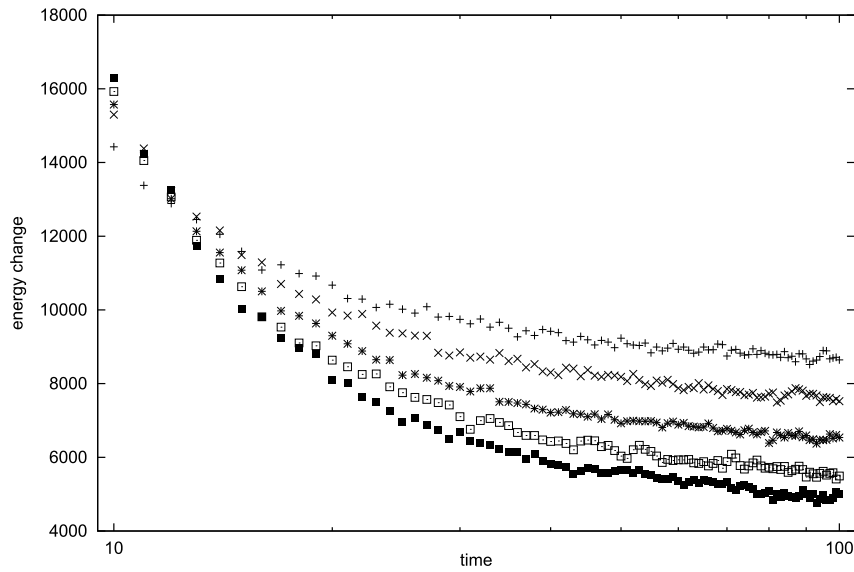
**Figure 3.** As Figure 1 but with  $m=1$  instead of  $m=5$



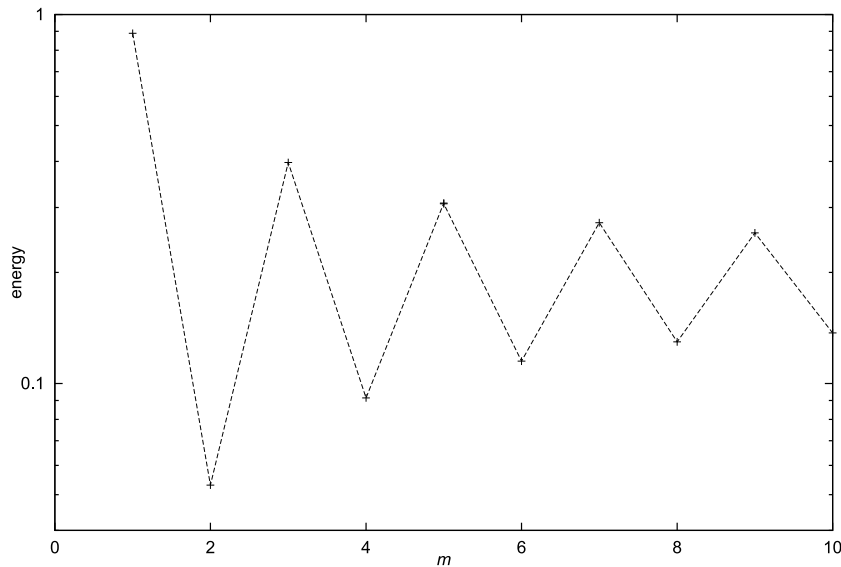
**Figure 4.** As Figure 1 but with  $m=2$  instead of  $m=5$

between  $m$  even and odd, which is expected to alter the density of frustration in the system: a cycle with an even number of antiferromagnetic edges allows all spins to be antiparallel to their neighbours on this cycle; for an odd number of edges this is impossible, which leads to frustrated edges connecting parallel spins even at zero temperature.

In the above results, the flexibility of the modeled system is measured by the number of spins flipped at a given iteration step, and by the energy change during these flips. Continuing our allegorical interpretation, we assign the number of personnel changes to the number of spin flips. This is justified by our every-day



**Figure 5.** Central part of energy curves from Figure 4 combined with other  $m$  values, showing  $m = 2, 4, 6, 8, 10$  from top to bottom, and a relaxation time increasing with  $m$ . Using odd  $m$  only the plot (not shown) is similar except that the curve for  $m = 1$  is more separated from the others. Summed energy changes for  $m = 2, 4, 6, 8, 10$  from top to bottom,  $N = 2\,000\,000$ ,  $J/kT = 2$



**Figure 6.** Semilogarithmic plot of energy change per spin flip during the last of 1000 iterations (Monte Carlo steps per spin);  $N = 2\,000\,000$ ,  $k_B T/J = 0.5$ . Every successful spin flip attempt is taken into account

experience that it is easier to change a person than his/her attitude. Even more important is the change of energy, which can be seen as a measure of the social cost of changes. An exchange of a politician without a resulting change in the problems of the society corresponds to a spin flip with zero energy change,  $n = 0$ . These flips dominate at long times in our simulations.

### 3. Conclusions

For social systems, two conclusions can be drawn from our results. First, after a big (external) disturbance, the ability of a system to change decreases with time and soon it becomes very small. Actually, this result follows as a simple consequence of our model of a social system as an interacting medium, with the energy as a work function which is minimized during the time evolution. With this assumption, the proof of Derrida [9] holds. Our second conclusion is more subtle. If the above interpretation is valid, personnel changes or even changes of attitude of the administration are not equivalent to changes of the social structure, which can remain unharmed. A real change demands a new initial state far from equilibrium, when most units agree.

These conclusions can be easily illustrated by examples from recent history of Europe. In years 1989–91 the reforms of the democratic Polish government transformed the life in the country much more than the changes in the ten following years. The characteristic time can then be evaluated as one or two years. The moral is: once you can change something, do it quickly. On the other hand, there is continuity of the so-called 'national interests' of many countries, and they are present in political strategies through centuries. At the reverse of the same effect, we find that the amounts of unemployed people in several countries (including ours) seem to be very resistant with respect to changes of governments and parliaments, not to mention "New Policies" frequently claimed during elections.

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