STRESS DISTRIBUTION IN THE MATERIAL AND DEVELOPMENT OF LOADS ON THE WALL DURING HOPPER FILLING

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Abstract: Stress distributions developing in granular materials in hoppers during the process of filling is fundamental for an understanding of the phenomena observed in hoppers. Predictions of such stress distributions are therefore essential. In this paper, based on a model which was created to simulate various filling procedures, an (ABAQUS) analysis has been carried out to investigate the development of stress distribution in the material and the loads on the hopper wall when the hopper is filled by the concentric-filling method. Calculations have been carried out either according to a procedure known as switch-on or according to the so-called layer-by-layer procedure. It was found that the maximum stress developed at the end of the filling, not at the bottom, but somewhere in the lower area of the hopper (layer 3). The stresses developed during layer-by-layer filling were greater than those developed during the switch-on filling in the lower area of the hopper, but were smaller in its upper area. Maxima of normal pressure along the wall were not at the outlet, even from the very beginning of filling. Instead, it was located at a position around 2/5 of the length of the wall from the outlet when the filling was finished. Various filling methods would have an effect on the stress distribution within the material and, consequently, affect the type and magnitude of loads on the hopper wall, and particularly at the hopper outlet.

Keywords: hopper, stress distribution, FEM, filling

1. Introduction

When a silo hopper is filled, stresses will develop in the material in the process of piling-up; they will undergo a shift when the outlet is opened. An understanding of such stress distributions is fundamental for phenomena such as arching, velocity discontinuity, stress fluctuations, *etc.* [1–5]. A prediction of stresses developed in granular materials during filling a silo, in particular in the hopper, is essential.

Experimental approaches to the development of stress in granular materials require measurements of the interactions occurring at the single particle level. However, in the past such measurements have been very limited due to the practical difficulties of manufacturing force transducers with spatial resolution sufficient to distinguish individual grains and, at the same time, in numbers sufficient to provide statistically meaningful data [6, 7].

The classic theoretical approach was the Janssen analysis applied to the material in the cylinder section of a silo. It was based on two assumptions. Firstly, that the ratio between the average vertical stress across a horizontal section and the horizontal stress on the wall is constant along the whole filling height (the value of this constant depends on whether the material is in an active or a passive state [8, 9]). Secondly, that wall friction angle is constant along the whole filling height. The Janssen theory was later modified by introducing a distribution factor, D [9–11]. This distribution factor may take values between 0.6 and 1, and the variation could be neglected in an active case. But D could also vary from 1 up to 3 and should not be ignored in a passive state. The main importance of the distribution factor lies in the stress analysis of a conical hopper, where a passive failure normally occurs.

Numerical methods were also introduced in the early 1960s and many advances have been made since then. These methods have been extensively used to find stress fields for both filling and flow in hoppers. For instance, the method of characteristics was first applied to the flow of a material in a hopper by Jenike [12] to validate his radial stress theory. Horne and Nedderman [13] used the same method to calculate limiting stress distributions in two-dimensional vertical-sided bins. The method of characteristics predicts discontinuities in the stress field in many practical situations [14, 15]. The Discrete Element Method (DEM) is also capable of predicting discontinuities [16, 17].

The main stream of numerical methods is the Finite Element Method (FEM). It has been adopted mainly to study macroscopic phenomena such as the flow behaviour of granular solids and the pressure exerted on silo walls [18–22]. Recent examples achievements in stress analysis have been the predictions of shear bands and stress fluctuations [23–25]. In this paper, the Finite Element Method has been adopted to investigate the development of stresses in the material in a hopper and wall loads when the hopper is filled with granular material. In order to simulate the filling process, the zones representing the granular material were partitioned into layers, upon which fine meshes were defined and suspended at the beginning of the analysis. They were then reintroduced either in an operation known as switch-on or layer-by-layer in a designated sequence. By doing so, the effect of these two different procedures of simulating filling the hopper could be investigated.

2. Hopper, granular material and FE formulation

An axi-symmetrical conical hopper made of stainless steel was designed. It was 2400mm in height, with an inlet of 2 400mm and an outlet of 400mm in diameter. The wall of the hopper was 6mm thick. The hopper was filled with the material by the concentric-filling method: the material was falling in the centre, piling up into a cone with a repose angle as shown in Figure 1.

The silo wall was simply modelled as an elastic material, and Young's modulus $E_w = 10^{11}$ Pa and Poisson's ratio $\nu_w = 0.3$ were assumed to be enough to cover its response to loads. The identification of a material model is, however, still a challenge and has not been satisfactorily resolved. Granular materials display varied behaviour, and a mechanical description of such assemblies is an old but still open problem. A general feature observed both in experiments and in simulations is the highly heterogeneous and anisotropic character of the force network arising from intergranular contacts [26–29]. However, in the present investigation a classical and well-established approach has been adopted. The material has been assumed to be a one-phase material, roughly treated on the principles of plasticity theory with Mohr-Coulomb limitation. This treatment is based on an incremental description of stress-strain relations and is practically suitable for numerical implementation. In this model, the main parameters involved are: Young's modulus, E_p, Poisson's ratio, ν_p, bulk density, ρ, the internal friction angle, φ, and the friction angle, ϕ_w, against the hopper wall. These have been assumed basing on the data in references and a numerical convergence requirement, as explained in Section 3.2.

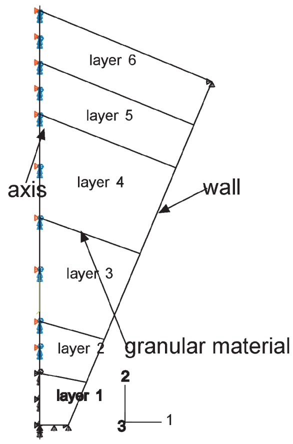

Figure 1. Hopper, granular material and boundary conditions

The interaction between the granular material and the silo wall depends on the material properties of the hopper wall and the granular material; it could be quite complex. For instance, the slip-stick behaviour is quite often observed during wall friction measurements, and the reader may be referred to substantial papers on the subject, as in [30–33] and more recent examples. Modelling such a mechanical interaction can be quite complex, and is still a great challenge. In the current studies, the interaction between the contacting surfaces of the hopper and the granular material was modelled through a simplified constitutive model of Coulomb friction.

Basing on physical models, an ABAQUS input file was created to model the hopper wall, the granular material and the contact interaction between the granular material and the wall. In the model, the region shown in Figure 1 was discretized, an axi-symmetrical shell element was defined for the wall, and a continuum axi-symmetrical element was defined for the granular material. As shown in Figure 1, the hopper was constrained both horizontally and vertically at its top edge and horizontally at its bottom edge. The edge of the granular material at the outlet was fixed vertically. The loading of the granular material was due to its gravity, the hopper was assumed to be weightless. The contact interaction between material and the hopper wall was implemented by a constant friction coefficient.

3. Numerical simulations and results

3.1. Filling procedures

It has been a challenge to simulate a silo filling procedure. In finite element modelling, the edges and the volume of an object are usually required to be defined first. In a real filling process, the volume of the stored material is increasing, and the volume edges are changing. Several attempts have been made in order to get the modelling process closer to the filling process by applying the loads incrementally, such as progressively increasing the density of the material, or incremental "layered filling" with a small preloading in the material, or applying body weight incrementally over the volume of the stored solid [34–37].

In the present study, a different filling procedure has been explored. The region representing the material was partitioned into six layers as shown in Figure 1. All of them, and the properties related to the layers such as meshes, loadings and contacts with the wall were then suspended. They were subsequently reintroduced all together in an operation known as switch-on filling, or in a sequence from layer 1 to layer 6, in another operation called layer-by-layer filling.

3.2. Convergence test and determination of parameters

For these two filling procedures, convergence tests were first carried out after setting the material parameters based on the assumed model. Bulk density is an important parameter since gravity loading is indirectly defined through density and has an influence on the numerical convergence, in combination with the material's Young's modulus, E_p, Poisson's ratio, ν_p, and yield stress. When bulk density was set at $\rho = 1\,000\,\mathrm{kg/m^3}$, convergence was achieved if Young's modulus, E_p, was greater than $2.5 \cdot 10^4\,\mathrm{Pa}$ (the Poisson ratio, ν_p, was 0.3) [38, 39]. The internal friction angle, φ, and the wall friction angle, ϕ_w, were measurable according to the Mohr-Coulomb model and set at $\varphi = 35^\circ$ and $\phi_w = 21^\circ$. They had little influence on convergence.

3.3. Numerical results

3.3.1. Stress development in the material during switch-on filling

It is a common practice in FEM to apply loads as gravity to the region of elements representing the granular material; it is known as switch-on loading. Actually, switch-on filling is a process of consolidation with no initial stresses in the material rather than a filling process. During the process of consolidation, stresses will develop in the material. Examples of such stress distributions are shown in Figure 2 for the shear stresses S12, the vertical stresses S22 and the horizontal stresses S11 along the central lines of each layer.

3.3.2. Stress development in the material during layer-by-layer filling

For layer-by-layer filling, layer 1 was first reintroduced. The elements, the loading and the interaction with the wall were reactivated in this region (they were switched on). The material in layer 1 was then consolidated with no initial stress, and stresses would develop in this region. When the meshes from layer 2 were reintroduced, the material in layer 2 underwent the same process as the material in layer 1 in the first step. The material in layer 1 would be further consolidated under the loads from layer 2, so the stress would be further developed. In the subsequent reactivation

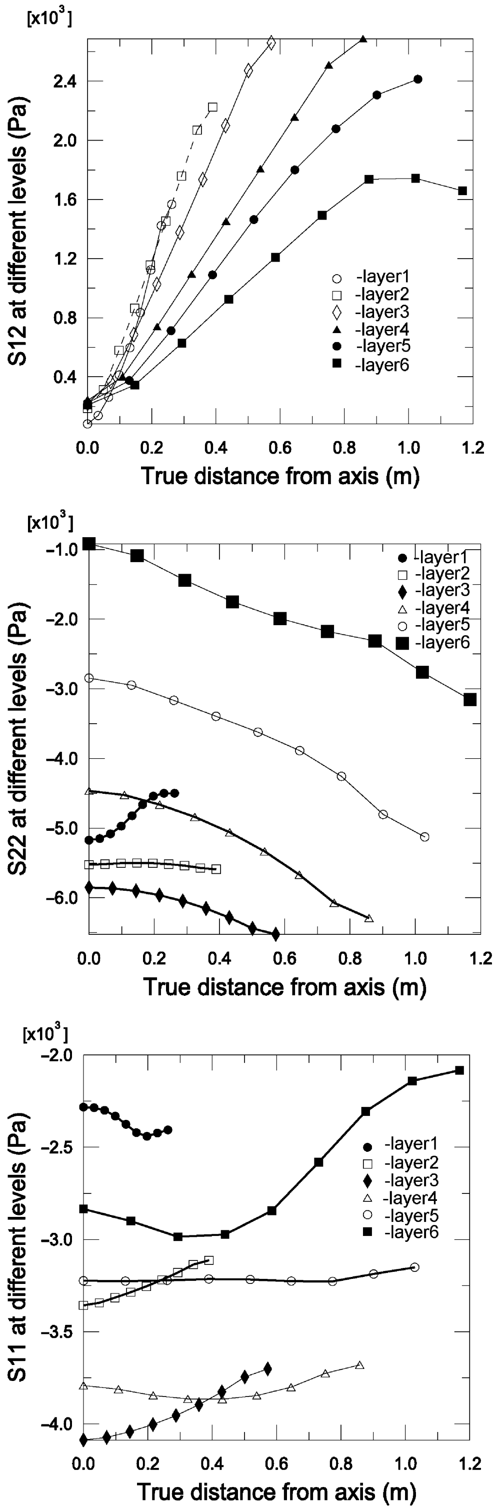

Figure 2. Stress developed along the middle lines of each layer during switch-on filling

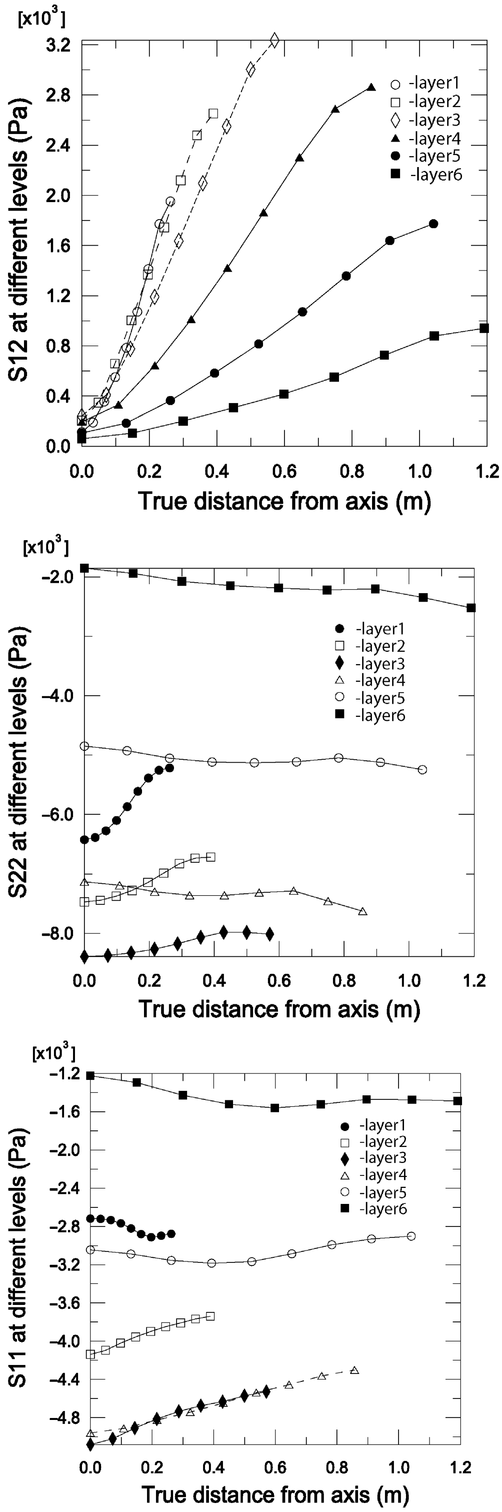

Figure 3. Stress developed along the middle lines of each layer during layer-by-layer filling

of the meshes of the remaining layers, the material in the corresponding layers was consolidated and the material underneath was further consolidated. By doing so, the development of stresses in the material was investigated. They are shown in Figure 3 for the shear stresses S12, the vertical stresses S22 and the horizontal stresses S11 along the same lines as those in Section 3.3.1 at the end of filling.

3.3.3. Loads during switch-on filling

In the present study, the interaction between the material and the wall, as well as the stress loaded at the outlet, were interpreted as loads.

When loading was applied as gravity to the granular material region in the switch-on operation, the contact surfaces of the material and the wall were interacting too. As a result, normal forces as well as shear forces were generated across the interfaces. The results of normal and shear forces between the material surfaces and the wall were the loads on the wall. The normal force was regarded as normal pressure on the wall.

In this investigation the outlet was set as closed. It was mathematically fulfilled by vertically constraining the outlet. When the material was filled into the hopper, it was consolidated due to gravity, instead of passing through the outlet. Stresses developed in the material as shown above. Among the stresses, the vertical stresses S22 at the interpretation points along the outlet were treated as loads at the outlet. Such load development will be shown later for comparison.

3.3.4. Loads during layer-by-layer filling

When the meshes from layer 2 to layer 6 were still suspended, the contacts between them were removed, so that only the contact surfaces of layer 1 and the wall were interacting. As a result, normal pressure and shear stress were at this stage confined to the contacting surfaces of layer 1 and the wall. Subsequently, when the meshes of layer 2 were reactivated in the second stage of filling, the contacts of the surfaces of layer 2 and the surface of the wall were also re-added. The contact interactions were now between the contacting surfaces of layer 1, layer 2 and the wall. This new interaction brought about normal pressures and shear stresses on the surface of a region consisting of layer 1 and layer 2. In further reactivation of layer meshes and re-additions of the corresponding surface contact interactions, normal pressures and friction stresses between them could be calculated. By doing so, the development of the filling loads on the hopper wall was obtained. They are shown in Figure 4, for normal pressure on the wall, and in Figure 5, for shear stress along the wall.

It is evident from Figure 4 that the maximum normal pressures were not at the outlet from the very beginning of filling along the wall. The maximum increased and moved upwards with the development of the filling process, and was located at a position around 2/5ths of the wall length from the outlet when the filling was finished. The normal pressures at the outlet also increased in the process of filling, but tended to approach a constant. Basing on the parameters assumed in the present study, it was about half the maximum normal pressure.

Figure 5 shows the development of shear stress distributions along the surface of the hopper wall. It was similar to that of normal pressure. The position of the maximum shear stress moved upwards with the filling process, and ended up at

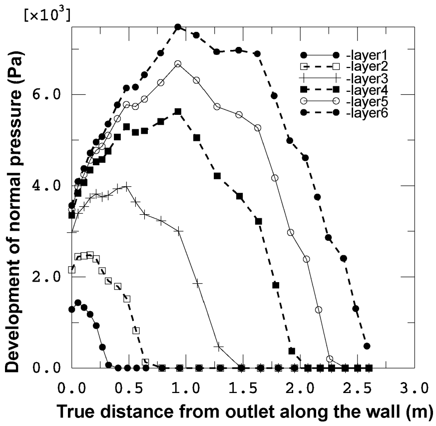

Figure 4. Development of normal pressures along the hopper wall

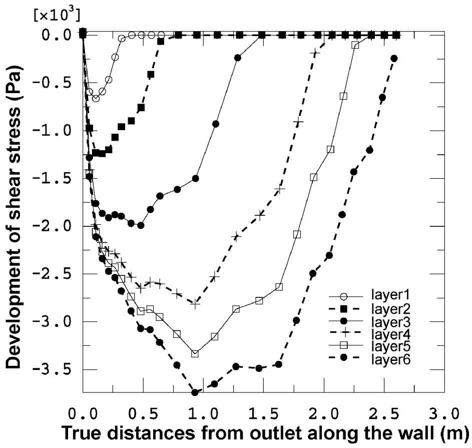

Figure 5. Development of friction stress distributions along the hopper wall

a location around 2/5ths of the wall length from the outlet (please note the zero friction forces at the outlet). It was due to the interaction definition adopted in this analysis. According to this definition, contact surfaces will slip against each other to develop a friction force. At the point of the outlet between the material and the wall, there was no relative movement tendency: shear stress was treated as zero.

The vertical stresses S22 at the interpretation points along the outlet were regarded as loads at the outlet. Accompanying the layer-by-layer filling procedure, the development of this stress is shown in Figure 6.

It is apparent from Figure 6 that loads at the outlet increased when the material was filled up. Interestingly, the loads developed faster in an area close to the centre than in the area close to the hopper wall, and increased quicker at the beginning of the filling than at its later stages. In the partition operation described in Section 4, layers 1, 2, 5 and 6 were designed with the same thickness. The thickness of layers 3 and 4 was doubled. Apparently, the same thickness of layer 2 had a lesser contribution to the load at the outlet than that of layer 1. The same applied to layer 3 and layer 4, and layer 5 and layer 6. One can conclude that the further the material was from the outlet, the less effect it had on the loads at the outlet.

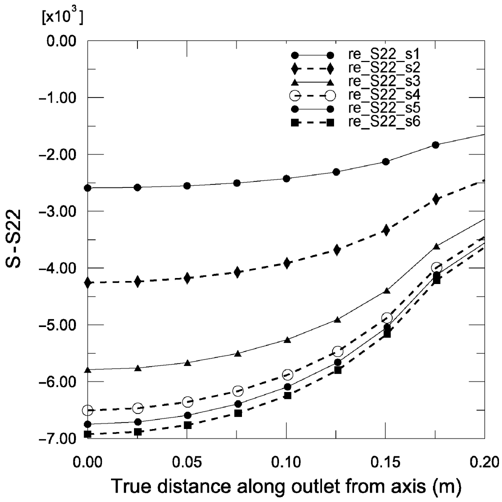

Figure 6. Development of S22 at the outlet

3.3.5. Discussion and comparisons

It is evident from Figures 2 and 3 that the maximum vertical stresses S22 and horizontal stresses S11 developed not at the bottom but somewhere in the region of layer 3, indicating that the material in this region was best consolidated. During switch-on filling, S22 increased in positions along the middle lines of each layer from the axis to the wall between layer 2 and layer 6, but was the greatest at the axis in layer 1. During layer-by-layer filling, S22 also increased in positions from the axis to the wall along the middle lines of layers 4, 5 and 6, and appeared more stable. S22 were the greatest at the axis in the region of layers 1, 2 and 3. S11 exhibited a less obvious tendency, but were greater in the region closer to the axis than to the wall in the middle areas, and appeared to be the opposite in the other regions, both for switch-on and layer-by-layer filling.

Corresponding to the positions where S11 and S22 were known, there existed shear stresses S12, which meant that S22 and S11 were not the principal stresses. Shear stresses along the lines in layers 1, 2 and 3 were more or less the same. For the material in this area, the ratio between shear stress and normal stress (the greatest in layer 3) was the smallest in layer 3. Regarding that the material in layer 3 was best consolidated, one may conclude this is a region where arching is most likely to occur.

With a closer look at the stress distributions shown in Figures 2 and 3, one may also find that during switch-on filling the stresses developed differently to those of layer-by-layer filling, *e.g.* as shown in Figure 7 for S22. Apparently, the stresses developed during layer-by-layer filling were greater than those developed during switch-on filling in the lower area, but were smaller in the upper area. During switch-on filling, the material seemed to be “hung up” and greater stresses could develop in the upper area, compared with the corresponding values developed during layer-by-layer filling. At the same time, during layer-by-layer filling there were less “hanging up” effects and the stresses mainly developed in the lower areas. This was in accordance with the loads developed along the wall and at the outlet, as shown in Figures 8 and 9.

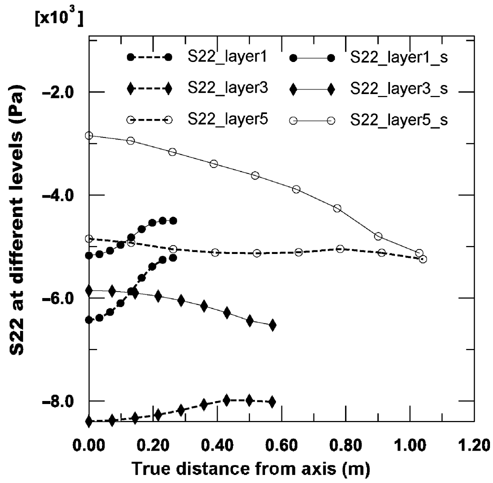

Figure 7. Comparison of stresses developed during various filling procedures

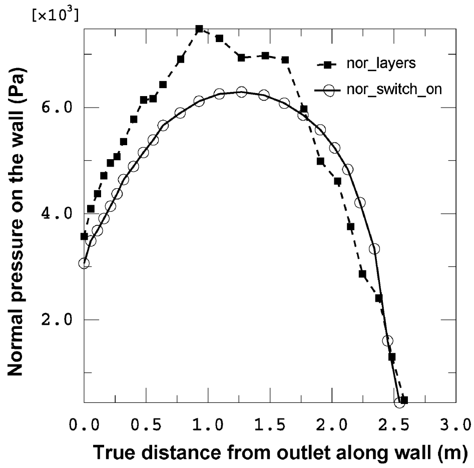

Figure 8. Loads along the wall for the two filling processes

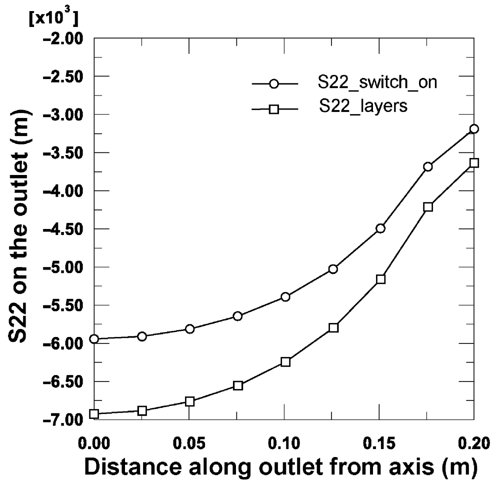

Figure 9. Vertical stresses at the outlet for the two filling processes

Figure 8 shows a comparison between the loads developed during layer-by-layer filling and switch-on filling after filling had been completed. Some differences are noticable. Layer-by-layer filling, as compared with switch-on filling, increases the maximum contact pressure and moves the location of maximum pressure downwards. In the upper area of the hopper wall, it decreases normal pressures, while in the lower area normal pressures are increased.

Figure 9 shows an example of vertical stresses at the outlet developed during the switch-on and the layer-by-layer filling procedures. Apparently, the vertical stresses at the outlet were greater for layer-by-layer filling than for switch-on filling.

4. Conclusions

The present study has explored an approach to the simulation of the filling process in hoppers. In this approach, the meshes, the interaction and gravity loading were suspended in the beginning of the analysis. They were then reactivated in the subsequent analysis. In this paper, the approach of suspension and reactivation of the meshes, interaction and loading, was used to investigate the development of stress in the material and loads exerted on the wall by the granular material during hopper filling.

The maximum stress was found to develop at the end of the filling not at the bottom, but somewhere in the lower area (layer 3) of the hopper. The material in this area was best consolidated, but had the lowest ratio between shear stress and normal stress. The stresses developed differently for the two filling procedures. The stresses developed during layer-by-layer filling were greater than those developed during switch-on filling in the lower area, but were smaller in the upper area.

The maximum of normal pressure along the wall was not at the outlet, even from the very beginning of filling. In consistence with the developments of stress distribution, the maximum pressure increased and moved upwards with the development of filling and was located at a position around 2/5ths of the wall length from the outlet when filling was finished. It has also been shown that different filling methods would affect the kind and magnitude of loads on the hopper wall and, particularly, at the hopper outlet. Layer-by-layer filling, compared with switch-on filling, would produce smaller loads in the upper areas of the hopper, and increased loads in the lower areas and at the outlet.

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