NON-LINEAR OPTIMIZATION METHODS FOR SMALL EARTHQUAKE LOCATIONS

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(Received 6 June 2004; revised manuscript received 18 August 2004)

Abstract: The problem of locating mine tremors using P-wave arrival times is revisited in the paper. A multidimensional, global, non-linear, constrained optimization method is used as a minimization algorithm for tremor location.In order to see the general properties of the minimized function a few images showing its basins of attractions have been constructed. These pictures enable us to choose efficient algorithms needed to solve location problems. The classical genetic algorithm, pure random search and the most efficient multistart algorithm have been tested. Local minimization methods should be introduced to the location procedure to increase the efficiency of tremor location.

Keywords: global optimization, basin of attraction, earthquake location

1. Introduction: an overview of the P-wave location method

Tremor location in mines is an important and difficult task, especially when the coordinates of the seismic source need to be determined with high accuracy. The well-known and most common method of tremor location used in Polish mines is the P-wave method, which uses P-wave first arrival times [1, 2]. The main problem of the location procedure consists in the resolution of the algorithms. As a non-linear optimization problem, it should be optimized with an appropriate algorithm.

After an earthquake or mine tremor, a lot of different waves are recorded by geophone networks. It is most convenient to use a P-wave for source location, as it is easy to find its exact first arrival time at the seismogram. As the fastest wave in the rock mass, it reaches the geophones first. Therefore, the first part of the signal is not disturbed by other waves and easy to extract.

The travel time of a P-wave depends on the location of its source (hypocenter), the coordinates of the geophone and the P-wave velocity distribution in the rock mass.

In the simplest case of a homogeneous and isotropic medium one can compute this time using the following relation [1]:

$$T_i = \frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{v}, \tag{1}$$

where the symbols denote: T_i – the P-wave travel time from the hypocenter (x,y,z) to the i^{th} geophone; (x,y,z) – the coordinates of the hypocenter (source); (x_i,y_i,z_i) – the i^{th} geophone's coordinates; v – velocity of the P-wave in the homogeneous rock mass.

In this paper, a simple, deterministic, homogeneous and isotropic rock mass is assumed. This is the most common rock mass model used in Polish mines to locate tremors, especially when distances between geophones are not very large. Its simplicity also allows us to increase the number of evaluations of the target function.

Let us assume that the hypocenter has approximate coordinates $(x,y,z;t)$ (Figure 1). We can create an error function that tells us how much these coordinates differ from the true and unknown coordinates $(x_0,y_0,z_0;t_0)$. The non-linear function is given by the following equation:

$$f(x,y,z;t) = \sum_{i=1}^{n} [t_i - t - T_i]^2, \tag{2}$$

where T_i denotes the evaluated travel time of the P-wave from point (x,y,z) to the i^{th} geophone and t_i – the recorded P-wave arrival time detected at the i^{th} geophone. Therefore, in homogeneous medium, we obtain the following from Equations (1) and (2):

$$f(x,y,z;t) = \sum_{i=1}^{n} \left[t_i - t - \frac{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}{v} \right]^2. \tag{3}$$

Function (3) has the following properties:

- the global minimum of f is equal to zero if there are no measurement errors of P-wave arrival times, t_i;

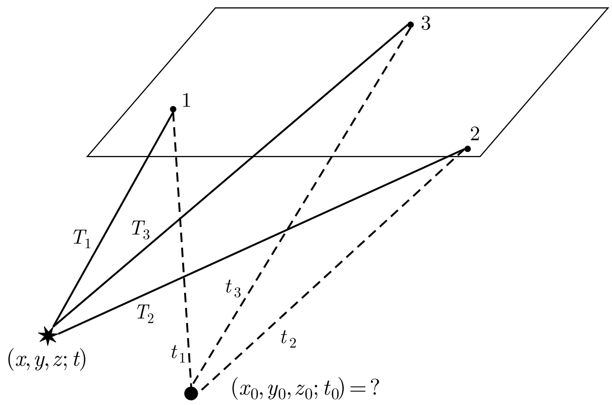

Figure 1. Location of the hypocenter using the P-wave arrival time: 1, 2, 3 – positions of the geophones; t_1, t_2, t_3 – measured first P-wave arrival times at seismometers 1, 2, 3; T_1, T_2, T_3 – evaluated first P-wave arrival times when the location of the hypocenter is at the (x,y,z,t) point; x_0, y_0, z_0, t_0 – real (unknown) position of the hypocenter

- there is a unique global minimum if the number of geophones is greater than four;
- the global minimum indicates the real hypocenter's coordinates $(x_0, y_0, z_0; t_0)$.

The problem of finding the global minimum of f is usually solved by linearization of function (3) [3]. As numerical evaluations have shown, f contains a number of local minima and thus linearization is often incorrect.

2. Methodology

The following procedure is proposed in order to choose the best location algorithm:

- example images are constructed demonstrating basins of attraction of the minimized function f;
- a few algorithms are selected for testing purposes regarding the function's values in minima, the geometry of the basins of attraction and the number of minima;
- each algorithm is tested by modeling many earthquakes and inverting their location;
- the algorithm which has generated the smallest location error $\Delta x, \Delta y, \Delta z$ is selected as the best location method.

Target function (3) is multidimensional. It depends on origin coordinates of the hypocenter, x, y, z, origin time of the tremor, t, P-wave velocity, v, and $3n$ coordinates of the geophones in the network. Thus, the multivariate target function is very complicated and difficult to analyze comprehensively, but the algorithm design scheme presented above allows us to deal with this complicated formula.

First we tried to recognize the type of the target function. Was it smooth enough to use local methods repeatedly? What was the range of the target function's values? Did it contain large peaks and deep valleys? That could be seen in the example images, which presented only a particular case of the target function.

Then the chosen algorithms were tested. Many different geophone networks, tremors and measurement errors were modeled. Tremors were inverted (located) after each modeling.

The algorithm which generated the least location error was considered the best.

An example of parameters of the target function is presented in Table 1. Eight geophones were chosen at random. Then, a tremor was modeled as presented in Table 1.

First the figure of the target function was analysed. Time and the vertical coordinate, z, were fixed as $t = -500\,\mathrm{ms}$, $z = -1000\,\mathrm{m}$, while the x, y variables could change. Zero for the time variable denotes the least time recorded by geophones and is equal to 01:00:00, 150.456ms. A contour map of the target function's surface is presented in Figure 2. The target function's values are very large, which is typical for a least square error function far from an exact solution.

In order to construct the map of basins of attraction, a local minimization procedure was launched at various points. Powell's conjugate directions algorithm [4] was used as the local minimization method. The grid of starting points is given in Table 1.

Table 1. The parameters of the plotted example of the target function and the definition of the grid of points while evaluating the basins of attraction; time $\tau(**)$ is equal to the least recorded value of the P-wave arrival time (01:00:00, 150.456ms)

P-wave velocity [m/s]	$v=1000$
variables' boundaries [m]	$0\leq x\leq 2000$ $0\leq y\leq 2000$ $-1000\leq z\leq 0$
hypocenter coordinates of the modeled tremor [m]	$x=1000$ $y=1000$ $z=-500$ $t=$00:01:00 000ms

The geophone network:				
number	coordinates			P-wave arrival time and error
i	x [m]	y [m]	z [m]	$t+N(0,1)*\sigma$
1	936.70	1071.16	−389.10	01:00:00, 150.456ms
2	508.57	1096.08	−474.68	01:00:00, 488.783ms
3	497.53	1248.34	−200.05	01:00:00, 642.642ms
4	1997.54	1953.66	−656.24	01:00:01, 391.757ms
5	1946.61	1965.80	−477.71	01:00:01, 337.296ms
6	382.03	282.08	−302.09	01:00:00, 952.877ms
7	416.47	1317.21	−819.08	01:00:00, 720.721ms
8	526.88	1959.66	−228.66	01:00:01, 119.099ms
σ of the modeled Gaussian error				3ms

Grid of points for the local minimization method:		
variable	bounds	step
x [m]	$0\leq x<2000$	100
y [m]	$0\leq y<2000$	100
z [m]	$-1000\leq z<0$	100
t [ms]	$-500\leq t<\tau(**)$	10

3 407 minima were evaluated. A particular minimum was separated if it differed at least 20m in its x, y or z coordinates from other minima. The standard way of presentation of basins of attraction assigns different color to each basin. Therefore, an appropriate color is assigned to each starting point $(x,y,z;t)$. Unfortunately, such a large number of minima makes this approach inconvenient.

Instead of using colors, the value of the target function at the local minimum was used. This minimum was returned by the local minimization method launched at the starting point. It is represented by:

$$P \longrightarrow f(\mathrm{localMethod}(P)), \tag{4}$$

where the symbols denote: P – the point that belongs to the target function's domain; f – the target function; localMethod – Powell's local minimization method.

For example, for the point at $x=100$m, $y=0$m, $z=-1000$m, $t=-500$ms, Powell's local method returns a minimum of $x=1002.04$m, $y=985.40$m, $z=-519.31$m, $t=-163.58$ms with a target function value of $f=415.53$. Relation (4) assigns a target function value at the minimum ($f=415.53$ in the example) to each

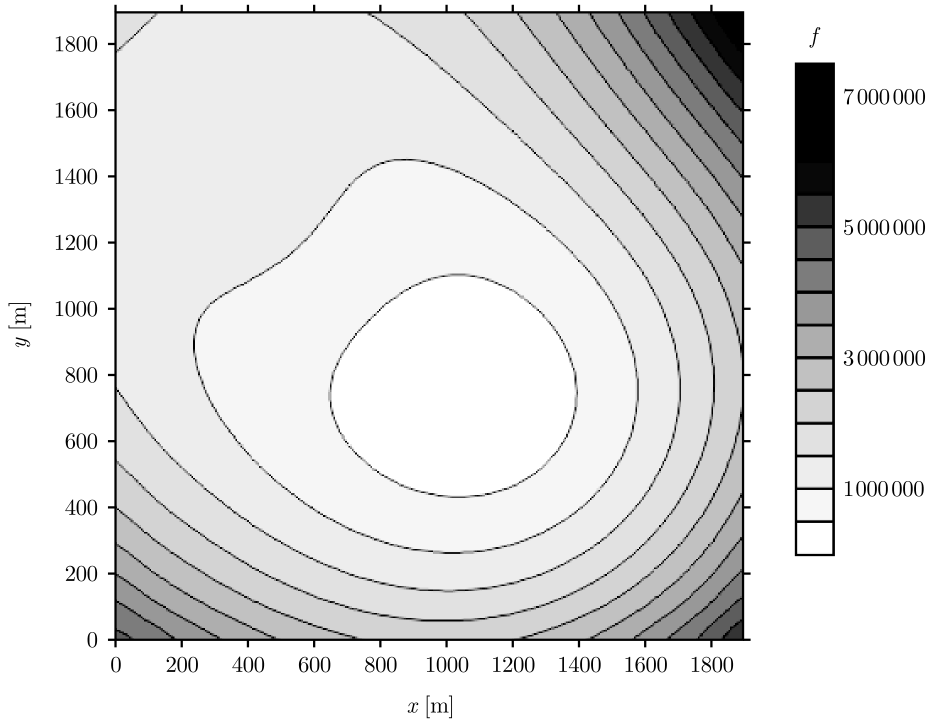

Figure 2. A contour plot of the target function; fixed variables: $t = -500$ms, $z = -1000$m

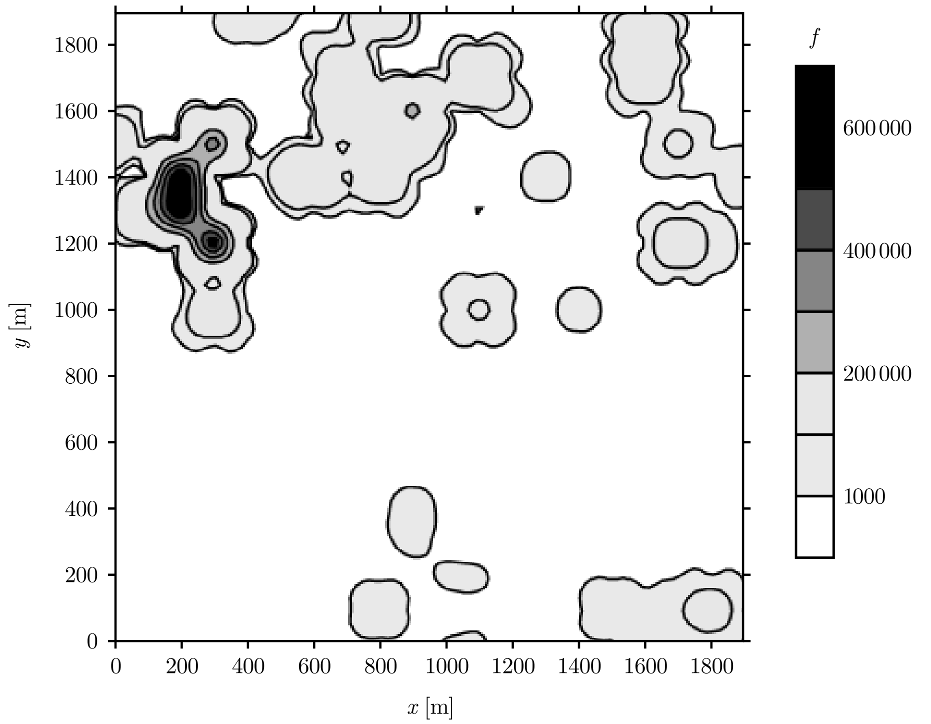

Figure 3. Values of the target function at minima obtained by Powell's local method; fixed variables: $t = -1000$ms, $z = -500$m; x and y vary in the $(0, 2000\text{m})$ region

starting point such as ($x=100$m, $y=0$m) (t, z were fixed). The results are presented in Figure 3.

The most important result presented in Figure 3 is the area of the function's values below 1000. It represents the areas belonging to the basin of attraction of the global minimum. By launching the local method from a point within this area, the algorithm would find the proper global minimum. But the areas of other, local minima cannot be neglected. Therefore, it is obvious that the local optimization method is not sufficient.

3. Selected algorithms

Numerical experiments have shown that the target function contains a large number of local minima. The target function's values at these minima differ significantly (Figure 3). Therefore the pure random search, multistart and classical genetic algorithms were chosen for testing.

Pure random search (PRS) is the simplest Monte-Carlo algorithm. The domain of possible solution is surveyed at random with uniform distribution. The point where the function's value is minimal is returned as a solution. After a given number of samplings, the local method is launched at the solution point.

Multistart is similar to PRS, but the local method is launched after each sampling.

The classical genetic algorithm [5] allows us to explore the domain in a different way, imitating the biological process of evolution. A number of points (the population) is processed. Each point is represented by a bit-string coding the x, y, z set of variables. Two strings chosen at random can be divided into parts (each string into two parts); when exchanged, they form two new points (offspring). One bit of the newly created string can be changed at random. This is a very important process (*viz.* mutation) that allows the algorithm to explore the function's domain. Let the points reproduce with probability proportional to the goodness of the solution that each string represents. The point which returns the smaller target function is considered better, in terms of goodness. A new population of points is formed (new generation) after many reproduction processes. After many generations, the population splits into groups which enclose deep minima. In the tested algorithm, the best point was chosen from the population after a given number of evolutionary generations. Then the local minimization method was launched.

4. Testing the algorithms

A scheme of the test is presented in Figure 4. The tremor phenomenon was modeled many times for different networks of geophones. An error of Gaussian distribution was added in order to imitate real measurement. After the modeling, the coordinates of the earthquake hypocenter were inverted. The total error of location was then evaluated. All the parameters of the test procedure and the tested algorithms are shown in Table 2.

The question is whether it is possible to compare different global optimization algorithms. The tested algorithms differ in the spirit of their optimization strategy, depend on a number of different parameters, *etc.* The designed test first checked

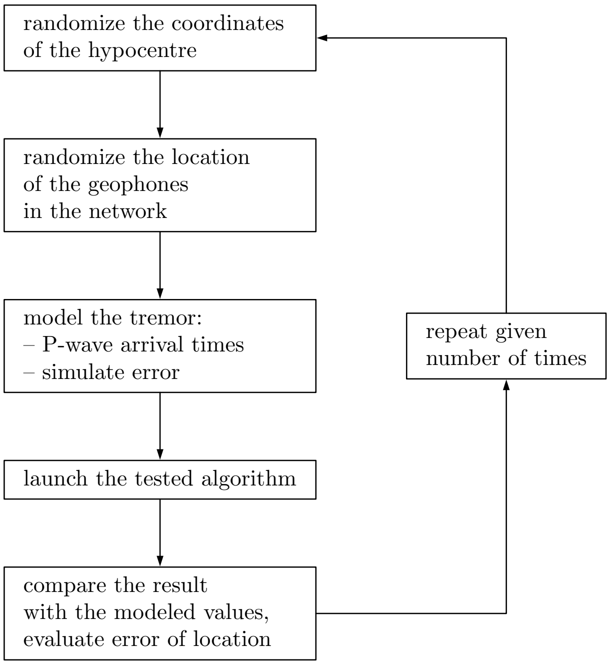

Figure 4. A scheme of the algorithms' test

Table 2. The parameters of the test and of the tested algorithms

Parameters of the test

geological environment	bounds of x [m] bounds of y [m] bounds of z [m] velocity [m/s]	$0 \le x \le 2000$ $0 \le y \le 2000$ $-1000 \le z \le 0$ 1000
geophone network	number of geophones: geophone coordinates:	8 chosen at random at each test's iteration
tremor modeling	hypocenter location: modeled value:	randomized at each test's iteration first P-wave arrival times
modeled error	P-wave arrival times	Gaussian error, $\sigma = 3$ms
test	number of iterations	100

Parameters of the algorithms

Pure random search (PRS)	number of iterations	1000000
Multistart	number of iterations	100
Classical genetic algorithm	population size parent population size number of bits/variable probability of mutation number of generations	100 100 10 5% 1000

whether the algorithms are able to locate tremors correctly. Secondly, the number of iterations was experimentally adjusted for PRS and multistart so that the total number of the target function evaluations were the same. The results of the test are presented in Table 3.

Table 3. The efficiency of the tested algorithms; the most efficient is that with the least location error Δx, Δy, Δz

algorithm	total [m]	Δx [m]	Δy [m]	Δz [m]
Classical genetic algorithm	206	134	96	123
Pure random search	43	17	21	32
Multistart	13	7	7	7

5. Summary

The numerical experiments have shown multistart to be the best algorithm for tremor location, as far as the location error is used as a measure of quality. It has an acceptable location error. A linearized version of location is used in Polish mines. If location is unsuccessful, the tremor is not processed. Incorrect location can be a consequence of inappropriate global minimum recognition. A linearized version may work fine, due to the target function's property seen in Figures 2 and 3. The basin of attraction of the global minimum is quite wide, and thus a linearized version of the algorithm is a suitable solution. The global multistart method should be used in order to improve efficiency.

The failure of the classical genetic algorithm in the test demonstrates that the classical version of the genetic algorithm is insufficient. The weak location resolution of the genetic algorithm may be due to the target function's deep minima.

In the authors' opinion, the multistart algorithm is capable of increasing the number of successful tremor locations in mines and probably decreasing the total error of location, usually equal to 30m.

Acknowledgements

The work has been financially supported by Faculty of Geology, Geophysics and Environmental Protection, AGH University of Science and Technology, grant no. 10.10.140.142.

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