PARALLEL AND DISTRIBUTED SEISMIC WAVE-FIELD MODELING

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Abstract: Elastic or acoustic wave-field modeling is an important part of seismic exploration. It can be used during the planning, processing and interpretation stages of seismic investigation. First attempts of using wave-field modeling were undertaken in the seventies by Alford, Kelly and others [1, 2]. These attempts were restricted by the limitations of computers at that time. Even now, computation for models of the standard exploration scale could last many hours, and many days in case of longer recording times. One of the best methods to overcome this disadvantage is parallelization of computations [3, 4]. This paper presents the results of distributed parallelization of elastic and acoustic wave-field modeling based on a Parallel Virtual Machine.

Keywords: wave field modeling, PC clusters, finite difference method

1. Introduction

The advantages of using clusters of standard PC's in mass calculation can hardly be overestimated. Modern PC clusters are strong rivals of very expensive multiprocessor supercomputers and can be an economical and reliable alternative for time-consuming scientific calculations (incl. wave-field modeling), even if there are bandwidth limitations of the computer network [5, 6]. Wave-field modeling is a problem easy to parallelize. Its parallelization can be performed in a heterogenous or a homogenous way [7]. Both of them can be easily coded in programs which use a Parallel Virtual Machine as a background of parallelization.

A PVM package has been created at the Oak Ridge National Laboratory. It can be used in heterogenous networks of machines form 386 to Cray, with various operating systems. The PVM system has two parts. The first is a daemon which must be started on all machines, while the second is a library of message passing and task control routines in C and Fortran.

2. Theoretical backgrounds of acoustic and elastic wave-field modeling

An Acoustic wave equation for a two-dimensional isotropic medium can be written as follows:

$$\frac{\partial^2 p}{\partial t^2} - c^2\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right) = f(x,z,t), \tag{1}$$

where $p(x,z)$ is pressure, $c(x,z)$ is velocity of the acoustic wave, t is time, and f denotes a function which describes pressure change at the source.

Adopting the finite difference to approximate the above equation without source term we can obtain pressure results for each point i, j of the computation grid in time $k+1$ through neighbouring points in time k and $k-1$ as follows [1]:

$$\begin{aligned} p(i,j,k+1) = {} & 2(1-2\gamma^2)p(i,j,k) - p(i,j,k-1) \\ & + \gamma^2(p(i+1,j,k) + p(i-1,j,k) + p(i,j+1,k) + p(i,j-1,k)), \end{aligned} \tag{2}$$

where $\gamma = c\Delta t/\delta h$, Δt is the time sampling interval, Δh is the distance between grid points in the x and z directions. The stability criterion for the above scheme is: $\gamma \leq 1/\sqrt{2}$.

To obtain final results, we also have to define proper schemes for border conditions. To avoid reflections from model borders, we have decided to use absorbant boundaries described by Reynolds [8]:

- the left border :

$$\begin{aligned} p(1,j,k+1) = {} & p(1,j,k) + p(2,j,k) - p(2,j,k-1) + c(1,j)\frac{\Delta t}{\Delta x} \\ & \big(p(2,j,k) - p(1,j,k) - (p(3,j,k-1) - p(2,j,k-1))\big), \end{aligned} \tag{3}$$

- the right border:

$$\begin{aligned} p(n+1,j,k+1) = {} & p(n+1,j,k) + p(n,j,k) - p(n,j,k-1) + c(n+1,j)\frac{\Delta t}{\Delta x} \\ & \big(p(n,j,k) - p(n+1,j,k) - (p(n-1,j,k-1) - p(n,j,k-1))\big), \end{aligned} \tag{4}$$

- the top border:

$$\begin{aligned} p(i,1,k+1) = {} & p(i,1,k) + p(i,2,k) - p(i,2,k-1) + c(i,1)\frac{\Delta t}{\Delta x} \\ & \big(p(i,2,k) - p(i,1,k) - (p(i,3,k-1) - p(i,2,k-1))\big), \end{aligned} \tag{5}$$

- the bottom border:

$$\begin{aligned} p(i,m+1,k+1) = {} & p(i,m+1,k) + p(i,m,k) - p(i,m,k-1) + c(i,m+1)\frac{\Delta t}{\Delta x} \\ & \big(p(i,m,k) - p(i,m+1,k) - (p(i,m-1,k-1) - p(i,m,k-1))\big). \end{aligned} \tag{6}$$

While an acoustic wave equation describes pressure changes and cannot offer a solution for shear waves, an elastic wave equation describes displacements of medium particles and, therefore, can deal with shape changes. The elastic wave equation in two dimensions is slightly more complicated then the acoustic one:

$$\begin{aligned}\rho\frac{\partial^2 u}{\partial t^2} &= \frac{\partial}{\partial x}\left[\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+2\mu\left(\frac{\partial u}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right],\\ \rho\frac{\partial^2 w}{\partial t^2} &= \frac{\partial}{\partial z}\left[\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+2\mu\left(\frac{\partial w}{\partial z}\right)\right]+\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right],\end{aligned} \tag{7}$$

where u and w are the horizontal and vertical components of displacement, ρ is density, while λ and μ stand for Lame's constants.

The finite difference scheme for the above equation can be written for a homogenous medium as [2]:

$$\begin{aligned}u(i,j,k+1) &= 2u(i,j,k)-u(i,j,k-1)+F^2[u(i+1,j,k)-2u(i,j,k)+u(i-1,j,k)]\\ &\quad +F^2(1-\gamma^2)[w(i+1,j+1,k)-w(i+1,j-1,k)-w(i-1,j+1,k)\\ &\quad +w(i-1,j-1,k)]/4+F^2\gamma^2[u(i,j+1,k)-2u(i,j,k)+u(i,j-1,k)],\\ w(i,j,k+1) &= 2w(i,j,k)-w(i,j,k-1)+F^2[w(i+1,j,k)-2w(i,j,k)+w(i,j-1,k)]\\ &\quad +F^2(1-\gamma^2)[w(i+1,j+1,k)-u(i+1,j-1,k)-u(i-1,j+1,k)\\ &\quad +u(i-1,j-1,k)]/4+F^2\gamma^2[w(i+1,j,k)-2w(i,j,k)+w(i-1,j,k)],\end{aligned} \tag{8}$$

where $\gamma=\alpha/\beta$, $F=\alpha\Delta t/h$, α and β are velocities of compressional (P) and shear (S) waves, h is distance between grid points.

The stability criterion for this scheme is as follows:

$$\Delta t \le \frac{h}{\sqrt{\alpha^2+\beta^2}}. \tag{9}$$

In an inhomogenous medium (*i.e.* on borders between geological layers) the finite difference schemes for elastic wave Equation (7) are even more complicated:

$$\begin{aligned}&\frac{u(i,j,k+1)-2u(i,j,k)+u(i,j,k-1)}{\Delta t^2}=\\ &\frac{1}{h}\left\{\left[\frac{\alpha^2_{i+1,j}+\alpha^2_{i,j}}{2}\right]\left[\frac{u(i+1,j,k)-u(i,j,k)}{h}\right]-\left[\frac{\alpha^2_{i,j}+\alpha^2_{i-1,j}}{2}\right]\left[\frac{u(i,j,k)-u(i-1,j,k)}{h}\right]\right\}\\ &+\frac{1}{2h}\left\{\alpha^2_{i+1,j}\left[\frac{w(i+1,j+1,k)-w(i+1,j-1,k)}{2h}\right]-\alpha^2_{i-1,j}\left[\frac{w(i-1,j+1,k)-w(i-1,j-1,k)}{2h}\right]\right\}\\ &-\frac{1}{2h}\left\{\beta^2_{i+1,j}\left[\frac{w(i+1,j+1,k)-w(i+1,j-1,k)}{2h}\right]-\beta^2_{i-1,j}\left[\frac{w(i-1,j+1,k)-w(i-1,j-1,k)}{2h}\right]\right\}\\ &+\frac{1}{2h}\left\{\beta^2_{i,j+1}\left[\frac{w(i+1,j+1,k)-w(i-1,j+1,k)}{2h}\right]-\beta^2_{i,j-1}\left[\frac{w(i+1,j-1,k)-w(i-1,j-1,k)}{2h}\right]\right\}\\ &+\frac{1}{h}\left\{\left[\frac{\beta^2_{i,j+1}+\beta^2_{i,j}}{2}\right]\left[\frac{u(i,j+1,k)-u(i,j,k)}{h}\right]-\left[\frac{\beta^2_{i,j}+\beta^2_{i,j-1}}{2}\right]\left[\frac{u(i,j,k)-u(i,j-1,k)}{h}\right]\right\},\\ &\frac{w(i,j,k+1)-2w(i,j,k)+w(i,j,k-1)}{\Delta t^2}=\\ &\frac{1}{2h}\left\{\alpha^2_{i,j+1}\left[\frac{u(i+1,j+1,k)-u(i+1,j-1,k)}{2h}\right]-\alpha^2_{i,j-1}\left[\frac{u(i+1,j-1,k)-w(i-1,j-1,k)}{2h}\right]\right\}\\ &+\frac{1}{h}\left\{\left[\frac{\alpha^2_{i+1,j}+\alpha^2_{i,j}}{2}\right]\left[\frac{w(i,j+1,k)-u(i,j,k)}{h}\right]-\left[\frac{\alpha^2_{i,j}+\alpha^2_{i,j-1}}{2}\right]\left[\frac{w(i,j,k)-w(i,j-1,k)}{h}\right]\right\}\\ &-\frac{1}{2h}\left\{\beta^2_{i,j+1}\left[\frac{u(i+1,j+1,k)-u(i-1,j+1,k)}{2h}\right]-\beta^2_{i,j-1}\left[\frac{u(i+1,j-1,k)-u(i-1,j-1,k)}{2h}\right]\right\}\\ &+\frac{1}{h}\left\{\left[\frac{\beta^2_{i+1,j}+\beta^2_{i,j}}{2}\right]\left[\frac{w(i+1,j,k)-w(i,j,k)}{h}\right]-\left[\frac{\beta^2_{i,j}+\beta^2_{i-1,j}}{2}\right]\left[\frac{w(i,j,k)-w(i-1,j,k)}{h}\right]\right\}\\ &+\frac{1}{2h}\left\{\beta^2_{i+1,j}\left[\frac{u(i+1,j+1,k)-u(i+1,j-1,k)}{2h}\right]-\beta^2_{i-1,j}\left[\frac{u(i-1,j+1,k)-u(i-1,j-1,k)}{2h}\right]\right\},\end{aligned} \tag{10}$$

we have decided to use the absorbant boundaries model also in this case.

3. Decomposition of computations

A standard serial wave-field modeling algorithm can be accelerated by parallelization, which here means using many computers for one computational task, with different data processed at the same time. This operation is called decomposition. There are two kinds of decomposition which are useful in wave-field modeling: homogenous (when computations are performed on different computers without communication between them) and heterogenous (when communication between cluster nodes is necessary).

Both methods were used in parallelization of wave-field modeling. In the homogenous splitting case (which we called *one PC-one shoot point* decomposition), we calculated a wave field for the whole model on one PC. The only difference between data on every computer in the cluster was in localization of the source point along the seismic profile. In the heterogenous case, we divided the calculation grid into as many subsections as we had computers in the cluster. Calculations were made separately for each section, but values for border grid points were exchanged. In this case, all computers in the cluster were involved in modeling for the same source point at the same time. This type of heterogenous parallelization is often called domain decomposition.

The most important difference between homo- and heterogenous decomposition is their granularity [7]. This parameter is defined as a relation between computational complexity and communicational complexity. In the *one PC-one shoot point* case we have an ultra coarse-grained situation, where each sub-problem is completely independent from all others. Domain decomposition is fine-grained because it needs to exchange information between subsections after each time step.

4. Implementation

Both approaches were tested on a PC cluster of 20 computers with AMD Duron 900MHz processors and 256MB memory. All computers belonged to the Faculty of Geology, Geophysics and Environmental Protection's 100Mb peer-to-peer network and were localized in computer laboratory 8. All machines were working under the Linux operating system. Computations were performed for a two-layer medium. The values of geophysical parameters used are typical for shallow sedimentary rocks. The modeling parameters are shown in Table 1.

During the experiment we measured the relation between the time of computation and the number of computers in the cluster. PVM clusters are easily scalable, which means that a user can add or delete computers thus increasing or decreasing the computational power of the virtual machine. Results of modeling for a shoot point localized in the central part of the model are shown in Figure 1.

5. Results and discussion

In the experiment we compared computational time needed by both of the analyzed methods of decomposition for modeling a wave field for 20 shoot points. The relation between this time in the acoustic case and the number of computers in the cluster is shown in Figure 2a. The same relation in the case of elastic modeling is shown in Figure 2b. The difference between times for one-processor computations

Table 1. Model parameters

Parameters	Values
Model dimensions [m]	500×500
Spatial grid step [m]	1
Depth to layer border [m]	250
Velocity of acoustic waves in the upper layer α_1 [m/s]	1000
Velocity of acoustic waves in the bottom layer α_2 [m/s]	2000
Localization of the first shoot point [m]	$x=200,\ z=20$
Distance between shoot points [m]	$\Delta x=10,\ \Delta z=0$
Number of shoot points	20
End time [s]	0.5
Time step [s]	0.002
α/β relation (elastic variant only)	2.0

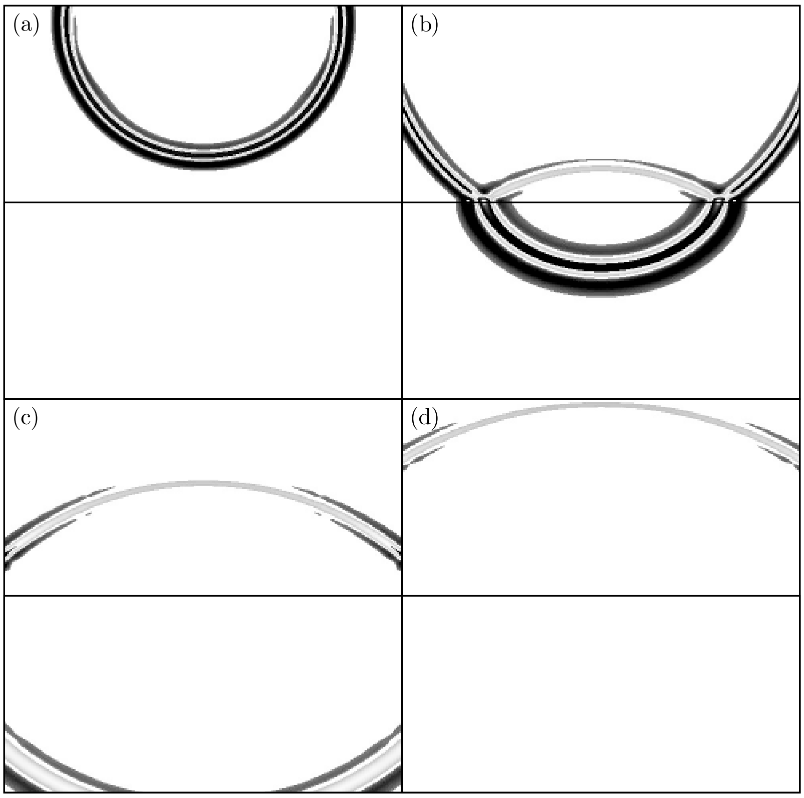

Figure 1. Results of acoustic wave-field modeling:
(a) after 0.2s; (b) after 0.3s; (c) after 0.4s; (d) after 0.5s

is due to using different compilers. In other cases (from 2 to 20 processors), we used the same compiler.

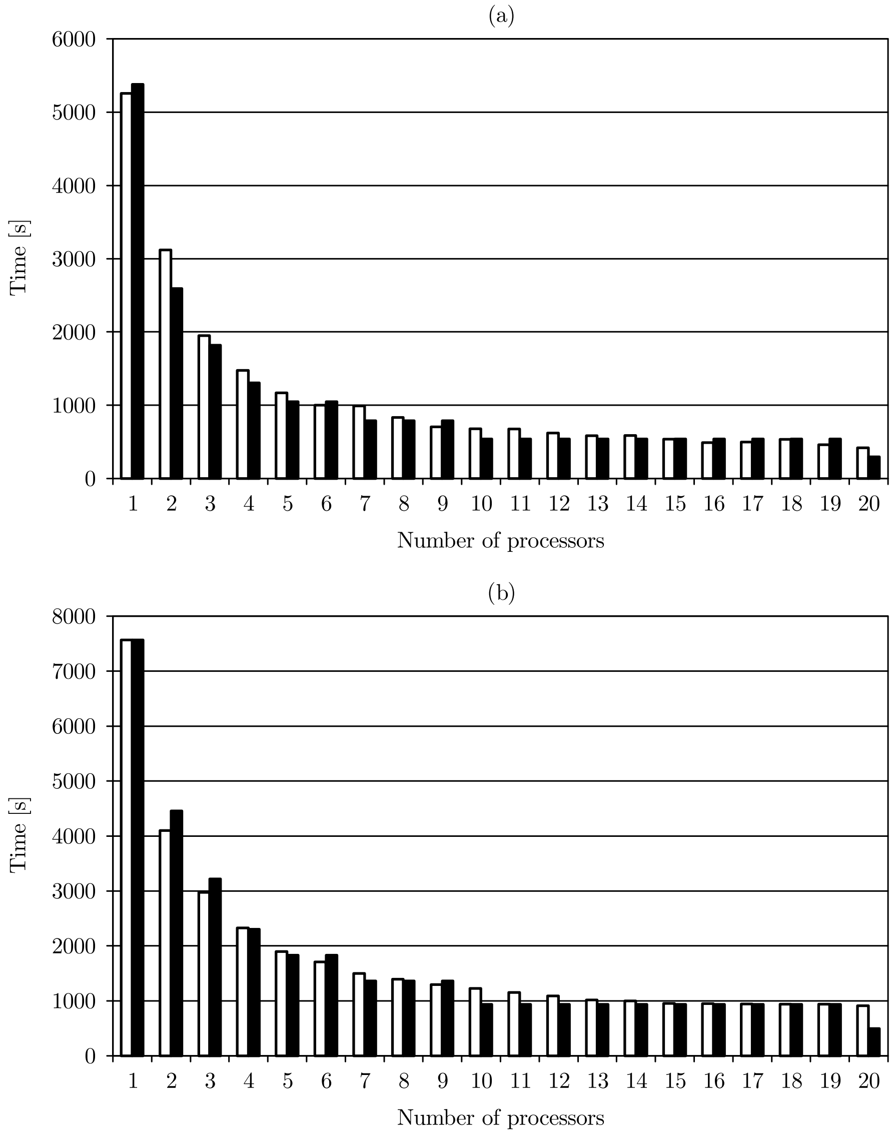

Figure 2. Relation between the computation time and the number of computers in: (a) the acoustic case; (b) the elastic case. White bars represent domain decomposition, black bars represent *one PC-one shoot point* decomposition

A good measure of the advantages of parallelization is acceleration [7] defined as the relation between the time needed by one processor and the time needed by P processors to perform the same computations:

$$S(P) = \frac{T(1)}{T(P)}, \tag{11}$$

where S is acceleration and T is time.

Acceleration increases monotonically for domain decomposition, while it does so suddenly for *one PC-one shoot point* acceleration when the number of computational cycles changes. When we have 20 shoot points and 20 processors, we have 1

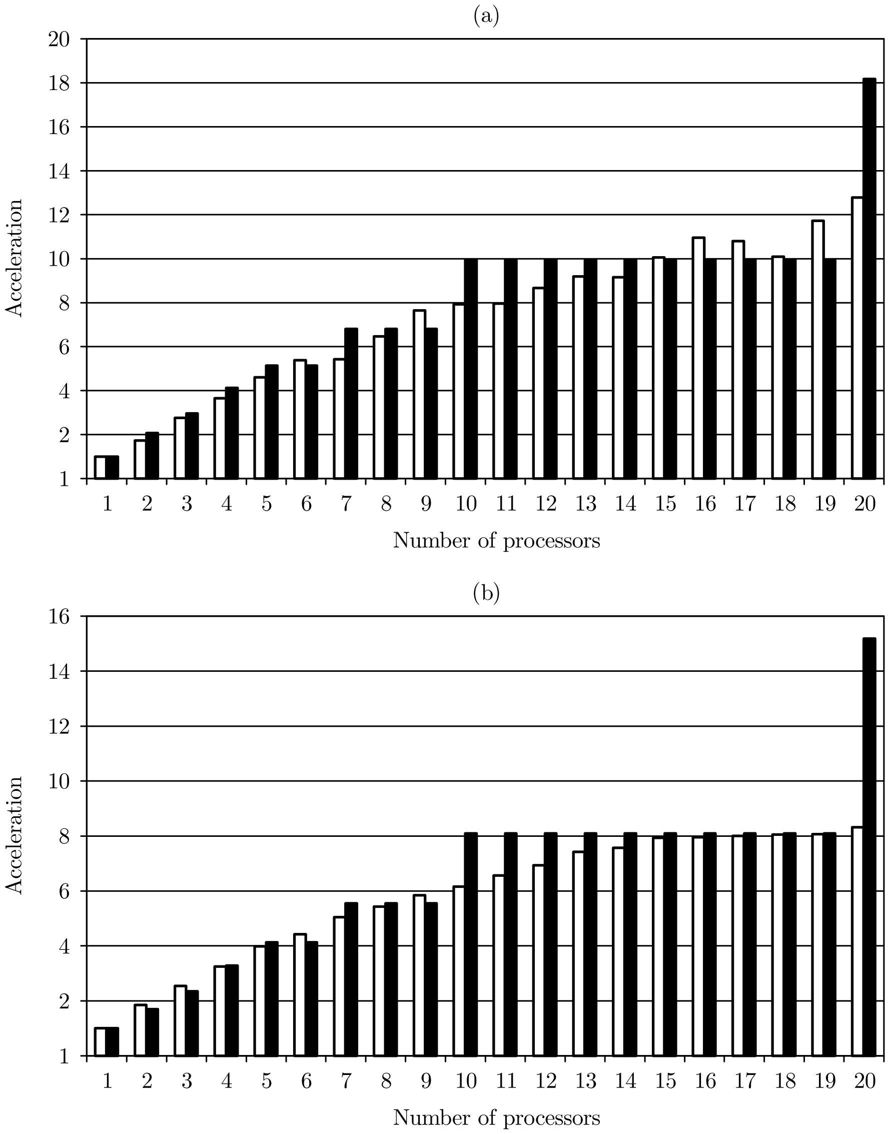

Figure 3. Relation between acceleration and the number of computers in: (a) the acoustic case; (b) the elastic case. White bars represent domain decomposition, black bars represent *one PC-one shoot point* decomposition

computational cycle. When we have 10 processors, we have 2 cycles. But when we have 11 to 19 processors, we still have 2 cycles. Figures 3a and 3b show the acceleration of acoustic and elastic wave-field modeling.

Another useful parameter describing the advantages of parallelization is effectivity [7] (see Figures 4a and 4b). This parameter is defined similarly to acceleration, but $T(P)$ is additionally multiplied by the number of processors, P:

$$e(P) = \frac{T(1)}{P \cdot T(P)}. \tag{12}$$

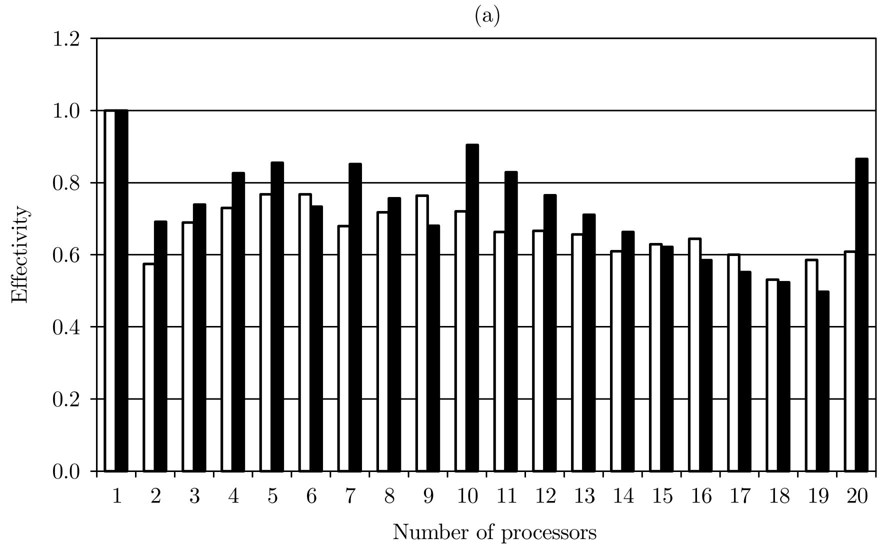

Figure 4. Relation between effectivity and the number of computers in: (a) the acoustic case; (b) the elastic case. White bars represent domain decomposition, black bars represent *one PC-one shoot point* decomposition

All charts for acoustic wave-field modeling in the case of domain decomposition are affected by a phenomenon which we have called the *modulo effect*. This effect is visible as an unexpected increase of computational time (or decreasing acceleration) for certain numbers of processors. It happens when the number of grid points is not divisible by the number of computers and one of the computers has to perform computations for a few more grid points than the others. This situation results in slowing down the whole cluster, as all the other processors have to wait for their border information. This example shows how a repetition of one little slow-down can affect global efficiency. In the case of elastic modeling the *modulo effect* was eliminated by even distribution of grid points among the computers.

6. Summary

In this paper we have presented two kinds of parallelization of wave-field modeling. In both cases a decrease of computation time with increasing of number of processors was observed. The differences between the obtained results were small. The computational time of domain decomposition decreased exponentially as the number of processors increased. Increasing the computational power of the virtual machine decreases its granulation, as the number of grid points calculated at each node of the cluster is smaller, while the number of gird points which have to be exchanged is greater.

Domain decomposition is much more error-prone, because problems with one computer slow down the whole cluster, while during *one PC-one shoot point* decomposition even a shutdown of some of the computers does not stop the others. Of course the advantages of *one PC-one shoot point* decomposition are relevant only when we have to calculate many shoot points along a seismic profile. When we want to model one wave-field, domain decomposition is a better solution.

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