MODELLING AND SIMULATION OF FLUID FLOW AND HEAT PROCESSES IN A REGENERATOR WITH CERAMIC CHIMNEY BLOCK CHECKER WORK

LILIANA ZASHKOVA AND NINA PENKOVA

Department of Silicate Technology,
University of Chemical Technology and Metallurgy,
Kl. Ohridsky 8, Sofia, Bulgaria
{zashkova, nina}@uctm.edu

(Received 17 May 2004; revised manuscript received 25 June 2004)

Abstract: Selected results of mathematical modelling and computer simulation of fluid flow and heat transfer processes in a glass furnace regenerator are reported. The conjugate heat transfer problem is solved in 3D using the ANSYS 8.0 / FLOTRAN programme. The regenerator's geometry, finite element mesh, thermal loads and boundary conditions are presented. The momentum, continuity and energy equations are solved.

Keywords: glass furnace regenerator, modelling, simulation, FEM, conjugate heat transfer, cyclic equilibrium

Nomenclature

c_p, c – specific heat, $[\mathrm{kJ \cdot g^{-1} \cdot K^{-1}}]$
T – temperature, [K]
V_x, V_y, V_z – velocity in the x, y, z directions, $[\mathrm{m \cdot s^{-1}}]$
p – pressure, [Pa]
t – time, [s], [min]
K – conductivity, $[\mathrm{W \cdot m^{-1} \cdot K^{-1}}]$
K' – "artificial" conductivity, $[\mathrm{W \cdot m^{-1} \cdot K^{-1}}]$
k – turbulent kinetic energy, $[\mathrm{J \cdot kg^{-1}}]$
h_r – radiation heat transfer (film) coefficient, $[\mathrm{W \cdot m^{-2} \cdot K^{-1}}]$
h_c – convection heat transfer (film) coefficient, $[\mathrm{W \cdot m^{-2} \cdot K^{-1}}$
h – total heat transfer (film) coefficient, $[\mathrm{W \cdot m^{-2} \cdot K^{-1}}]$
T_0 – reference temperature, $T_0 = 273.15\mathrm{K}$
T_m – average temperature, [K]
d – width of a checker block channel, [m]
h_e – height of a checker block, [m]
H_p – height of a checker, [m]
y^+ – outer boundary of the laminar sublayer's distance from the wall, dimensionless

q – heat flux, $[\mathrm{W \cdot m^{-2}}]$
g – gravity acceleration, $[\mathrm{m \cdot s^{-2}}]$
Re – the Reynolds number, $\mathrm{Re} = V \cdot d \cdot \rho_f \cdot \mu^{-1}$
Nu – the Nusselt number, $\mathrm{Nu} = h_c \cdot d \cdot K_f^{-1}$
Gr – the Grasshof number, $\mathrm{Gr} = d^3 \cdot g \cdot \beta \cdot \rho^2 \cdot (To - T) \cdot \mu^{-2}$,
Pr – the Prandtl number, $\mathrm{Pr} = \mu \cdot c_f \cdot K_f^{-1}$

Greek symbols

ρ – density, $[\mathrm{kg \cdot m^{-3}}]$
ε – turbulent kinetic energy dissipation rate, $[\mathrm{W \cdot kg^{-1}}]$
ε_g – gas emissivity, dimensionless
ε_w – gas absorptivity, dimensionless
μ – laminar viscosity, $[\mathrm{Pa \cdot s}]$
σ_k – the Schmidt number of turbulent kinetic energy, dimensionless
σ_ε – the Schmidt number of turbulent kinetic energy dissipation rate, dimensionless
δ – width of a checker block wall, [m]
τ_w – shear stress, $[\mathrm{N \cdot m^{-2}}]$
β – coefficient of volume expansion, $[\mathrm{K^{-1}}]$
σ – the Stefan-Boltzmann constant, $[\mathrm{W \cdot m^{-2} \cdot K^{-4}}]$

Subscripts

i – node number, $i = 1, 2, \ldots, 28236$
1, 2, 3 – material number
$z = 1, 2, \ldots$ – period number
c – checker blocks
f – fluid
in – inlet
g – gas
w – wall
out – outlet

Superscripts

c – cooling period
h – heating period

1. Introduction

The processes taking place in regenerative heat exchangers have significant effect on glass furnaces' thermal work. Mathematical modelling and computer simulation enable us to analyse the heat transfer and fluid flow processes in different kinds of regenerators. Model equations for regenerator heat transfer processes and their analytical and numerical solutions have already been reported [1–6]. They are based on Fourier's law, energy balance equations and Newton's law of cooling. However, no results obtained from the solution of the conjugate heat transfer problem in regenerators have been available published yet. This approach is successful in cases of changing surface temperature during gas-solid heat transfer [7].

The aim of the present work is to analyse transient heat transfer in regenerator and checker work efficiency rating by solving the conjugate transient heat transfer problem. To achieve this aim, the following problems have been resolved:

1. to build a mathematical model of transient heat and fluid flow processes in the investigated regenerator;

2. to simulate the consecutive checker air cooling and flue-gas heating periods to achieve a cyclic equilibrium of the regenerator;
3. to obtain and analyse the following transient fields for both the air cooling and flue-gas heating periods at the regenerator's cyclic equilibrium:
   - local velocities of fluid flows in the checker,
   - fluid pressure,
   - local fluid and checker block (solid) temperatures,
   - local heat fluxes and heat transfer (film) coefficients.

2. Regenerator parameters

The regenerator investigated in this work, is vertical and contraflow. Its checker is composed of semicylindrical ceramic chimney blocks [8], shown in Figure 1a, made of three materials, up to the height of the regenerator.

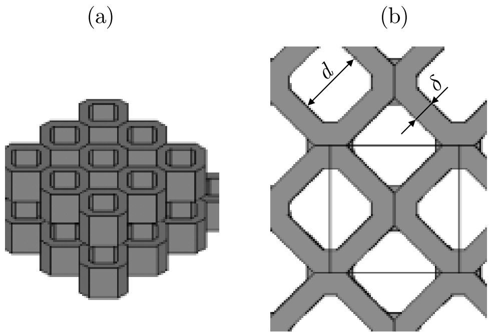

Figure 1. Ceramic chimney blocks checker work:
(a) checker work; (b) object of modelling in the checker's centre

The main checker geometry parameters are its height, $H_p = 5.5\,\mathrm{m}$, block diameter, $d = 120\,\mathrm{mm}$, specific heating surface, $f = 18.25\,\mathrm{m}^2/\mathrm{m}^3$, free flow cross-section, $f_f = 55.4\%$, and checker density, $\rho_c = 1317\,\mathrm{kg/m}^3$.

The length of periods of heating the checker with flue gases and cooling it with air is $\Delta t = 30\,\mathrm{min}$ (1800s) each. The hot flue gases' inlet temperature at the top of the regenerator is 1300°C, while the air inlet temperature at the bottom of the regenerator is 50°C.

3. Mathematical modelling with ANSYS 8.0 / FLOTRAN

The model has been solved three-dimensionally (3D) because of the complicated geometry of the checker work. The object of modelling is a self-similar region in the centre of the checker, with symmetry boundaries shown in Figure 1b. All of the checker height is covered. Heat losses into the environment are not taken into account. The flue-gas and air flows are assumed to be turbulent [9].

The thermal fluid flow problem in the Cartesian coordinate system is defined by thefollowing equations (described in detail in the ANSYS Theory Reference [10]):

– the continuity equation,
– the momentum equations (in the x, y, z directions) for the turbulent case,

– the uncompressible energy equation,
– the two-equation standard turbulent k-ε model with the following constants: $C_\mu = 0.09,\ C_1 = 1.44,\ C_2 = 1.92,\ \sigma_k = 1,\ \sigma_\varepsilon = 1.3,\ \sigma_t = 1,\ C_3 = 1,\ C_4 = 0,$ and $\beta = 0$.

The boundary layer parameters are determined in terms of the Van Driest conductivity model [11] with selected constants, $A = 26$, $E = 9$, $\chi = 0$, 4.

The above equations are discretized by a finite elements-based technique, using standard ANSYS elements of FLUID 142. In this case, Galerkin's method of weighted residuals is used to form element integrals. The main degrees of freedom (DOF) are p, V_x, V_y, V_z, T, k and ε for the fluid elements and T (solid temperature) for the non-fluid elements. The output derived values include q, h, τ_w, y^+.

Since the ANSYS programme cannot make models and calculate the radiation heat transfer between gas and solid directly, we have modelled the radiation heat transfer between flue gases and checker bricks according to the following approach:

1. the radiation heat flux between flue gases and checker blocks and the radiation heat transfer coefficient, $h_r = \sigma\cdot(\varepsilon_g\cdot T_g^4 - \varepsilon_w\cdot T_w^4)/(T_g - T_w)$, have been determined using one of the well-known methods (described by Hausen [1]);
2. a suitable dimensionless equation has been chosen to calculate the convection heat transfer coefficient, h_c:

$$\mathrm{Nu} = 0.255\cdot\mathrm{Gr}^{0.25}\cdot\mathrm{Re}^{0.07}\cdot\mathrm{Pr}^{0.37}; \tag{1}$$

3. flue gas conductivity, K', referred to in the present work as "artificial", has been calculated on the basis of Equation (1), as follows:

   - Equation (1) may be expressed as:

$$\frac{h_c\cdot d}{K} = 0.255\cdot\mathrm{Gr}^{0.25}\cdot\mathrm{Re}^{0.07}\cdot\left(\nu\cdot\rho\cdot c\cdot K^{-1}\right)^{0.37}, \tag{2}$$

   - after introducing $P = 0.255\cdot\mathrm{Gr}^{0.25}\cdot\mathrm{Re}^{0.07}\cdot(\nu\cdot\rho\cdot c)^{0.37}\cdot d^{-1}$ and rearranging Equation (2) we have:

$$h_c = P\cdot K^{0.63}, \tag{3}$$

   - the total heat transfer coefficient, h, is given by the sum of the radiation coefficient, h_r, and the convective coefficient, h_c [1]:

$$h = h_c + h_r, \tag{4}$$

   - by substituting the right-hand side of Equation (4) into Equation (3) we obtain:

$$h_c + h_r = P\cdot {K'}^{0.63}, \tag{5}$$

   where K' is the "artificial" flue gas conductivity,

- after dividing Equation (5) by Equation (3), we obtain:

$$\frac{h_c+h_r}{h_c}=\left(\frac{K'}{K}\right)^{0.63}, \tag{6}$$

- the $h_c=n\cdot h_r$ correlation is determined by h_c and h_r calculated in advance for the concrete model. Its substitution in Equation (6) gives:

$$\frac{n\cdot h_c+h_c}{h_c}=\left(\frac{K'}{K}\right)^{0.63}. \tag{7}$$

  Simplifying this, we obtain:

$$K'=K\cdot(n+1)^{1.5873}, \tag{8}$$

- the "artificial" gas conductivity, K', may be calculated from Equation (8). For the investigated regenerator we have obtained $n\approx 1$ ($h_c\approx h_r$), so:

$$K'=K\cdot 2^{1.5873}\approx 3\cdot K; \tag{9}$$

4. the "artificial" conductivity, K', is used to specify the flue gases properties in ANSYS.

As information about local temperatures in different checker works at the cyclic equilibrium of regenerators cannot be found in the literature, consecutive cooling and heating periods must be simulated until achieving the regenerator cyclic equilibrium [3].

4. Geometrical model and finite element mesh

The geometry of the model and the coordinate system (Cartesian) are shown in Figure 2.

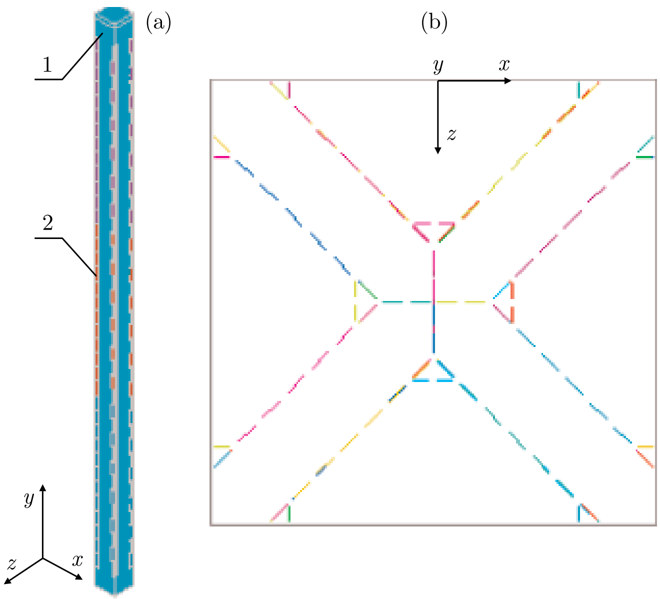

Figure 2. Geometry and coordinate system of the model:
(a) isometric view (1 – fluid volume, 2 – non-fluid volume); (b) top view

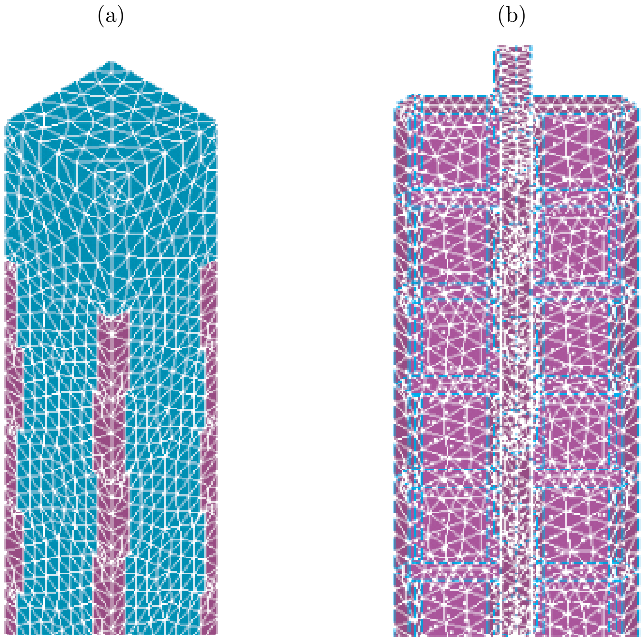

Figure 3. Finite element mesh: (a) fluid and non-fluid volumes; (b) non-fluid volumes

The fluid and non-fluid volumes (checker material) are marked with different colours, as shown in Figure 2a.

An enlarged view of the finite element mesh of the model is shown in Figure 3. In all, 142 791 tetrahedral finite elements are generated: 78 425 fluid and 64 366 non-fluid elements. The number of nodes is 28 236.

5. Loads and boundary conditions

5.1. Fluid properties

Temperature variations of gas conductivity and viscosity are expressed and specified by the relationships, proposed by Sutherland [7, 10]. The ideal gas equation is used to specify density, while linear functions of temperature are used for specific heat. The fluid property relationships for the cooling and heating periods are summarized in Table 1.

Table 1. Fluid properties' relationships

Air	Flue gases
$\rho = 1.293 \cdot \frac{p}{T} \cdot \frac{273.15}{101\,325}$	$\rho = 1.293 \cdot \frac{p}{T} \cdot \frac{273.15}{101\,325}$
$c_p = 1006 + 0.18 \cdot T$	$c_p = 1042 + 0.264 \cdot T$
$K = 0.02454 \cdot \left(\frac{T}{T_0}\right)^{1.5} \cdot \left(\frac{T_0 + 147.7}{T + 147.7}\right)$	$K = 0.0628 \cdot \left(\frac{T}{T_0}\right)^{1.5} \cdot \left(\frac{T_0 + 534.34}{T + 534.34}\right)$ (Correlation obtained using the "artificial" conductivity, $K' = 3 \cdot K$)
$\mu = 17.1 \cdot 10^{-6} \cdot \left(\frac{T}{T_0}\right)^{1.5} \cdot \left(\frac{T_0 + 89.77}{T + 89.77}\right)$	$\mu = 15.8 \cdot 10^{-6} \cdot \left(\frac{T}{T_0}\right)^{1.5} \cdot \left(\frac{T_0 + 138}{T + 138}\right)$

Table 2. Material properties' relationships

"Radex SG(D)" (material N2)	"Radex GV" (material N3)	"Radex GZ" (material N3)
$c=913.3+0.208\cdot T$	$c=913.3+0.208\cdot T$	$c=853.216+0.208\cdot T$
$K_{xx}=K_{yy}=K_{zz}=$ $=10.15-0.0042\cdot T$	$K_{xx}=K_{yy}=K_{zz}=$ $=10.15-0.0042\cdot T$	$K_{xx}=K_{yy}=K_{zz}=$ $=9.655-0.0042\cdot T$
$\rho=3000\,\mathrm{kg/m^3}$	$\rho=2950\,\mathrm{kg/m^3}$	$\rho=3000\,\mathrm{kg/m^3}$

5.2. Non-fluid material properties

Three kinds of checker materials are used, from top to bottom: "Radex SG(D)", "Radex GV" and "Radex GZ". Temperature variations of the specified material properties are given in Table 2.

5.3. Initial conditions

Non-fluid element nodes

Consecutive cooling of the checker with air and heating it with flue gases is simulated until achieving the regenerator's cyclic equilibrium, determined by:

$$\Delta T=T_{m,z}-T_{m,z-1}=3\mathrm{K}, \tag{10}$$

where $T_{m,z}$ is an average checker blocks (solid) temperature at the end of z^{th} cooling period, $T_{m,z-1}$ – average checker blocks (solid) temperature at the end of $(z-1)^{\mathrm{th}}$ cooling period.

To achieve this aim, the following initial conditions are specified:

- the first checker heating period: $T^h_{1,i}(x,y,z,0)=293\mathrm{K}$;
- for each next period, the previous period's end temperature is given as initial temperature of checker blocks:
  $T^c_{1,i}(x,y,z,0)=T^h_{1,i}(x,y,z,1800)$,
  $T^h_{2,i}(x,y,z,0)=T^c_{1,i}(x,y,z,1800)$,
  ...
  $T^h_{z,i}(x,y,z,0)=T^c_{z-1,i}(x,y,z,1800)$,
  $T^c_{z,i}(x,y,z,0)=T^h_{z,i}(x,y,z,1800)$.

Fluid element nodes

$V_{x,i}(x,y,z,0)=V_{y,i}(x,y,z,0)=V_{z,i}(x,y,z,0)=0$;

$p_{x,i}(x,y,z,0)=0,\ \ k_{x,i}(x,y,z,0)=0,\ \ \varepsilon_{x,i}(x,y,z,0)=0$;

Checker heating period: $T^h_{z,i}(x,y,z,0)=T_{in}=1573\mathrm{K}$;

Checker cooling period: $1\le z\le 10,\ \ T^c_{z,i}(x,y,z,0)=T_{in}=313\mathrm{K}$;

$z\ge 10,\ \ T^c_{z,i}(x,y,z,0)=T_{in}=323\mathrm{K}$.

5.4. Boundary conditions

$V_x=0$, $V_y=0$, $V_z=0$ for checker areas surrounded by fluids. The surrounded areas are of four types, expressed by the following equations:

1. $z\pm x\pm\frac{1}{2}\sqrt{2}d=0$, bounded in the x direction $x_{1,i}(d,\delta)\le x_{2,i}(d,\delta)$,
2. $z+x-\frac{1}{2}\sqrt{2}(d+2\delta)=0$, bounded in the x direction $x_{1,i}(d,\delta)\le x_{2,i}(d,\delta)$,

3. $z = z(d,\delta) = const$, bounded in the x direction $x_{1,i}(d,\delta) \le x_{2,i}(d,\delta)$,

4. $x = x(d,\delta) = const$, bounded in the z direction $z_{1,i}(d,\delta) \le z_{2,i}(d,\delta)$.

In the y direction, the above areas are bounded:
$1+2nh_e \le y \le 1+h_e(2n+1)$ – even checker rows,
$1+h_e(2n+1) \le y \le 1+h_e(2n+2)$ – uneven checker rows.

Symmetry boundary

$-\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ 0.8\le y\le 1+37h_e+0.2,\ z=0,\ \frac{\partial t}{\partial z}=0, V_z=0;$
$-\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ 0.8\le y\le 1+37h_e+0.2,\ z=\sqrt{2}(d+\delta),\ \frac{\partial t}{\partial z}=0,\ V_z=0;$
$0\le z\le\sqrt{2}(d+\delta),\ 0.8\le y\le 1+37h_e+0.2,\ x=\frac{1}{2}\sqrt{2}(d+\delta),\ \frac{\partial t}{\partial x}=0,\ V_x=0;$
$0\le z\le\sqrt{2}(d+\delta),\ 0.8\le y\le 1+37h_e+0.2,\ x=\frac{1}{2}\sqrt{2}(d+\delta),\ \frac{\partial t}{\partial x}=0,\ V_x=0.$

Fluid velocity, turbulent kinetic energy and turbulent kinetic energy dissipation rate are specified at the regenerator's inlet, pressure is specified at the regenerator's outlet:

Checker heating period

$-\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \ y=1+37h_e+0.2,\ \ 0\le z\le\sqrt{2}(d+\delta),$
$V_x=0,\ V_y=-2.09\,\mathrm{m\cdot s^{-1}},\ V_z=0,\ k_{in}=6.55\cdot10^{-4}\,\mathrm{J\cdot kg^{-1}},\ \varepsilon_{in}=7.7\cdot10^{-4}\,\mathrm{W\cdot kg^{-1}};$
$-\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \ y=0.8,\ \ 0\le z\le\sqrt{2}(d+\delta),\ \ p_{out}=100\,\mathrm{Pa}.$

Checker cooling period

$-\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \ y=0.8,\ \ 0\le z\le\sqrt{2}(d+\delta);$
$V_x=0,\ V_y=0.35\,\mathrm{m\cdot s^{-1}},\ V_z=0,\ k_{in}=1.88\cdot10^{-5}\,\mathrm{J\cdot kg^{-1}},\ \varepsilon_{in}=3.74\cdot10^{-6}\,\mathrm{W\cdot kg^{-1}};$
$-\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \ y=1+37h_e+0.2,\ \ 0\le z\le\sqrt{2}(d+\delta),\ \ p_{out}=25\,\mathrm{Pa}.$

Checker blocks' contact surfaces

As checker blocks are of the same material, ideal contact is assumed. The blocks' volumes are added and three solid blocks are thus formed (see Figure 2).

Various materials of checker blocks are assumed:

$y=1+12h_e,\ -\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \frac{1}{2}\sqrt{2}d-x\le z\le\frac{1}{2}\sqrt{2(d+2\delta)}-x,$
$$-K_1\frac{\partial t}{\partial y}\bigg|_{y=1+12h_e}=-K_2\frac{\partial t}{\partial y}\bigg|_{y=-(1+12h_e)},\qquad t\bigg|_{y=1+12h_e}=t\bigg|_{y=-(1+12h_e)};$$

$y=1+12h_e,\ -\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \frac{1}{2}\sqrt{2}d+x\le z\le\frac{1}{2}\sqrt{2(d+2\delta)}+x,$
$$-K_1\frac{\partial t}{\partial y}\bigg|_{y=1+12h_e}=-K_2\frac{\partial t}{\partial y}\bigg|_{y=-(1+12h_e)},\qquad t\bigg|_{y=1+12h_e}=t\bigg|_{y=-(1+12h_e)};$$

$y=1+24h_e,\ -\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \frac{1}{2}\sqrt{2}d-x\le z\le\frac{1}{2}\sqrt{2(d+2\delta)}-x,$
$$-K_2\frac{\partial t}{\partial y}\bigg|_{y=1+24h_e}=-K_3\frac{\partial t}{\partial y}\bigg|_{y=-(1+24h_e)},\qquad t\bigg|_{y=1+24h_e}=t\bigg|_{y=-(1+24h_e)};$$

$y=1+24h_e,\ -\frac{1}{2}\sqrt{2}(d+\delta)\le x\le\frac{1}{2}\sqrt{2}(d+\delta),\ \frac{1}{2}\sqrt{2}d+x\le z\le\frac{1}{2}\sqrt{2(d+2\delta)}+x,$
$$-K_2\frac{\partial t}{\partial y}\bigg|_{y=1+24h_e}=-K_3\frac{\partial t}{\partial y}\bigg|_{y=-(1+24h_e)},\qquad t\bigg|_{y=1+24h_e}=t\bigg|_{y=-(1+24h_e)}.$$

6. Results

Figure 4 shows the temperature changes in the regenerator until attaining a cyclic equilibrium. The equilibrium was obtained after 17 cycles of checker heating and cooling.

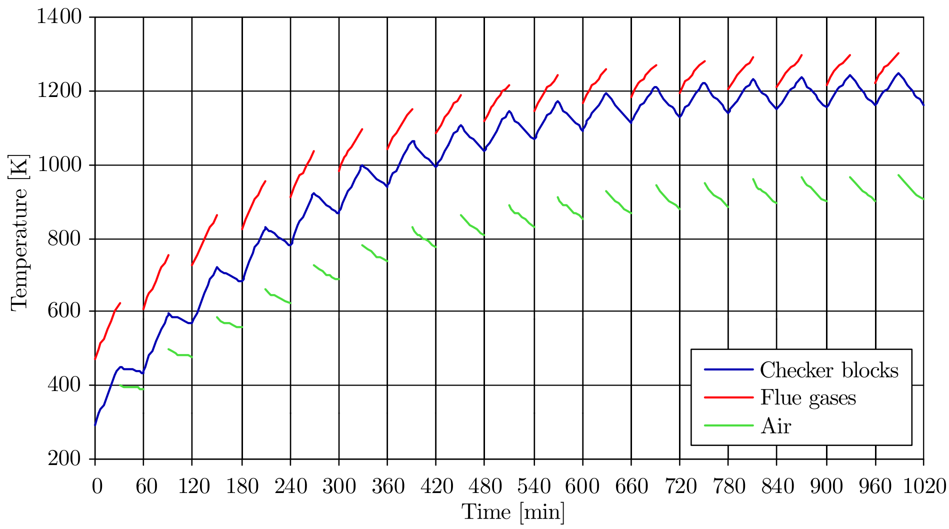

Figure 4. Change of the average temperatures of air, flue gases and checker blocks until the regenerator's cyclic equilibrium is obtained

The local DOF for every step in time of the periods (p, V_x, V_y, V_z, T, k, ε and solid temperature) and the output derived values (q, h, τ_w, y^+) have been saved in $*$.rfl files. The data are available upon request.

7. Conclusion

1. According to the present study, the following average temperature differences have been observed in the cyclic equilibrium of the regenerator:
   (i) checker blocks–flue gases: $\Delta T_m^h = T_{m,f} - T_{m,c} \approx 60\mathrm{K}$ and,
   (ii) checker blocks–air: $\Delta T_m^c = T_{m,c} - T_{m,f} \approx 265\mathrm{K}$.

   It has been established that $\Delta T_m^h < \Delta T_m^c$, because of radiation and convection heat transfer between flue gases and checker blocks in the heating period. There is only convection heat transfer between air and checker blocks in the cooling period.
2. The obtained results make possible a future thermal analysis of regenerator's cyclic equilibrium according to the aims of our work.

References

[1] Hausen H 1976 *Warmeurbertragung im gegenstrom, gleichstrom und kreuzstrom*, Springer-Verlag, Berlin/Heidelberg
[2] Razelos P and Benjamin M K 1978 *Int. Journal of Heat Mass Transfer* **21** 735
[3] Willmott A J and Duggan R C 1980 *Int. J. of Heat and Mass Transfer* **23** 655
[4] Romie F E 1993 *ASME J. of Heat Transfer* **115** 1064

[5] Muralikrishna S 1999 *Trans. IChemE* **77A** (March) 131

[6] Muralidhar K and Suzuki K 2001 *Int. J. of Heat and Mass Transfer* **44** 2493

[7] Likov A V 1978 *Heat and Mass Transfer*, Moscow, Energia (in Russian)

[8] Radex 1996 *Radex in the Glass Industry* **6** 3

[9] Zashkova L and Penkova N 2003 *Science Works*, Vol. XXX, University of Food Technology, Plovdiv, pp. 29–33 (in Bulgarian)

[10] *ANSYS Theory Reference*, 1998, Release 5.5, 9$^{\rm th}$ Edition, SAS IP

[11] Bradshaw P, Cebeci T and Whitelaw J H 1981 *Engineering Calculation Methods for Turbulent Flow*, London, Academic Press, pp. 211–255