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NUMERICAL SIMULATION OF SHOCK WAVE PATTERNS IN SUPERSONIC DIVERGENT SYMMETRIC NOZZLES

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Abstract: This paper presents the results of numerical simulations of supersonic flows with shock waves in a divergent symmetric nozzle of an opening angle ranging from 2° to 6°. At certain Mach number values the shock pattern becomes asymmetric. This asymmetry is analysed here for different values of velocity upstream of the shock wave and for different nozzle divergence angles.

Only the divergent part of the nozzle is considered. Supersonic conditions at the nozzle inlet were prescribed with a chosen Mach number value $\mathrm{Ma} > 1$. The inlet velocity profile included a turbulent boundary layer profile on side walls. The steady flow simulation was applied for nozzle opening angles, α, of 1.877°, 2.5° and 3°, whereas the unsteady approach was necessary for a nozzle of the divergence angle $\alpha = 6.54°$ to obtain a converged solution.

The asymmetry of the shock structure is visible in the unevenness of the heights of both λ-feet. It happens at the same Mach number, at the same boundary layer and with the same geometrical constraints. This is in contradiction with our current understanding of the parameters affecting λ-foot size. The paper provides an explanation of this problem.

Keywords: supersonic flow, supersonic nozzle, shock wave

1. Introduction

The main goal of this paper is to numerically analyse the topography of the shock wave pattern in a symmetric nozzle. The phenomenon is the result of interaction between a normal shock wave and the boundary layer, which produces a λ-foot structure.

Our experimental investigations [1] have shown that for low Mach numbers the feet are of the same size. However, at higher velocities ($\mathrm{Ma} > 1.4$) the λ-feet generated on the upper and lower walls of a symmetric nozzle become different in size. It has also been observed that the tendency towards asymmetry depends on the nozzle divergence angle.

Numerical 2D calculations with the SPARC Navier-Stokes solver [2] were used to analyse the shock wave configuration in a straight divergent nozzle. Nozzles with opening angles, α, of 1.877°, 2.5°, 3.0° and 6.54° and Mach numbers ranging from 1.37 to 1.59 were taken into consideration.

At small opening angles the steady method of calculations was used. However, it turned out that steady calculations were not converging for greater nozzle divergence angles and in these cases the unsteady method was applied to obtain a solution.

2. Flow geometry, mesh and boundary conditions

To calculate supersonic flow in a nozzle it is usually necessary to include the whole flow development: the subsonic part, the throat area and the subsequent supersonic (divergent) part. Large mesh sizes and very long calculation times are usually required to obtain good resolution in the shock wave-boundary layer interaction area. In order to save time by reducing the mesh size, the calculation domain starts downstream of the nozzle throat. The flow is supersonic right from the inlet. This procedure has been verified. The whole nozzle calculations for different pre-shock Mach numbers are presented in Figure 1. The increase in the shock system asymmetry with increasing Mach numbers can be observed.

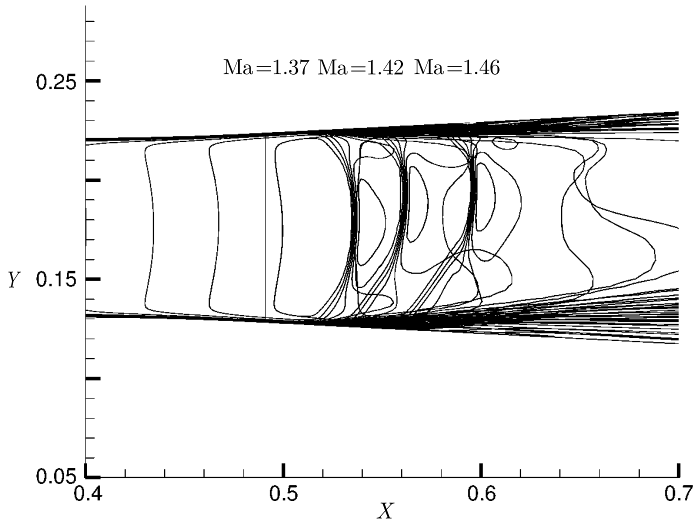

Figure 1. Results for whole geometry

A similar dependence of the shock system's structure on the flow parameters can be observed in the short divergent nozzle shown in Figure 2. The behaviour of the solutions is the same as for the whole nozzle geometry. Therefore, all investigations presented here were carried out using the short nozzle configuration.

The shock waves for a short nozzle are sharper than for a long nozzle due to much higher grid resolution.

In all simulations presented here the channels had the form of a symmetrical nozzle with flat walls where the shock wave was located. Further downstream the channel became parallel and remained so up to the exit plane. This was done in order to shift the outlet plane with a constant static pressure condition far away from the domain of interest. With the supersonic inlet and an appropriate static pressure at the

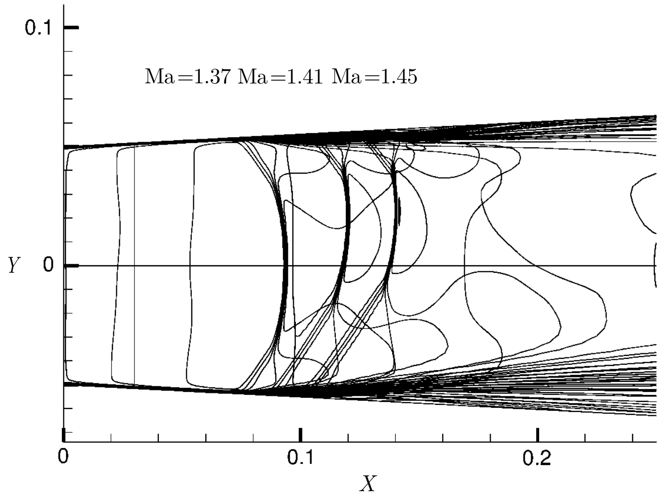

Figure 2. Results for a short divergent supersonic nozzle

outlet one obtains a normal shock wave at a fixed location in the nozzle. Interaction between the normal shock wave and the boundary layers results in forming λ-feet at the walls.

In the simulations, an inlet velocity profile was assumed apart from the usual boundary conditions such as the stagnation parameters (pressure and temperature). Two velocity profiles were applied at the inlet for Mach numbers $\mathrm{Ma} = 1.1$ and $\mathrm{Ma} = 1.28$. The two side walls of the nozzle were assumed to be a "solid wall" condition.

The mesh was generated very precisely to eliminate any incidental asymmetry. As the main aim of these simulations was to explain the asymmetry in the structure of a shock wave with λ-feet, mesh blocks were mirror-reflected across the centre line.

Steady state calculations were made for nozzles with the opening angles, α, of $1.877°$, $2.5°$ and $3.0°$. The dual time stepping method for unsteady calculations was used for the angle of $\alpha = 6.54°$ as it proved impossible to obtain acceptable convergence in the stationary approach.

3. Research goals

It is a well-known and experimentally confirmed fact that for low Mach numbers ($\mathrm{Ma} \leq 1.37$) λ-feet are of the same size at both nozzle walls. However, it has been observed that the λ-feet vary at higher velocities upstream of the shock wave. Thirty years of research [3, 4] have shown that λ-foot size (h) is a function of the Reynolds number (h decreases as Re increases) and the Mach number (h increases as Ma increases). Naturally, the characteristic length scale (boundary layer thickness in the case of interaction) plays an important role. But recent results have shown that the height of the wind channel becomes another characteristic length scale due to the fact that the interaction in internal flows is of a "constrained" type [5].

Taking all the above into account, it is surprising that λ-feet on both sides of a symmetric nozzle may have different sizes even though Re, Ma and both of their length parameters are identical. This is an important challenge for the investigations presented here and the interpretation of this phenomenon justifies the choice of our research goals.

In the simulations carried out, analysis of the problem was developed in two directions:

– the influence of velocity upstream of the shock wave on the degree of the shock system's asymmetry and
– the influence of nozzle divergence on the shock wave structure and its tendency towards asymmetry.

4. Influence of velocity upstream of the shock wave on the shock structure

It was crucial to establish the dependence of the asymmetry of the shock wave pattern on the Mach number upstream of the shock. For this task the nozzle with the smallest divergence was considered ($\alpha = 1.877°$) .

The results were grouped in two categories depending on the Mach number at the nozzle inlet. Two velocity profiles were assumed as inlet boundary conditions. Outside of the boundary layer of the inlet, the velocities were uniform at Ma = 1.1 and Ma = 1.28.

The flow cases for the Ma = 1.1 inlet are presented in Figure 3. Various outlet pressures were used to obtain three shock waves with different Mach numbers in front of them. The outlet pressure variation caused a shift in the shock's location and a change in the corresponding Ma value of the interaction. The chosen pre-shock Mach numbers were 1.37, 1.46 and 1.52. The shock wave structures were altered considerably with the changing Mach numbers.

At the low velocity of Ma = 1.37 the shock configuration is symmetric. The greater the velocity in front of the shock wave, the greater the observed asymmetry. With increasing velocity, the heights of the lower λ-feet grow faster than those of the upper λ-feet. Straight dashed lines link the triple points. The different inclination of these lines indicates the difference in the growth rates of the λ-feet.

Figure 4 shows the numerical solutions for the Mach number of Ma = 1.28 at the inlet.

The three pre-shock Mach number cases of Ma = 1.5, 1.55 and 1.59 confirm the continuation of the tendency (indicated in Figure 3) towards asymmetry increase with increasing pre-shock Mach number values.

All shock configurations in Figure 4 are asymmetric because the velocities in front of them are high. The dashed lines are virtually straight due to the almost linear growth rates of both the upper and lower λ-feet (with a significantly different growth rate).

The appearing asymmetry is connected with the numerical method. The change of the direction of the vertical axis results in changing the sides of the asymmetry.

Figure 5 shows a schlieren picture of the asymmetric shock wave pattern obtained in a physical experiment at Ma = 1.6 to confirm the existence of asymmetric shock structures shown in Figure 4. It is shown here only as a qualitative confirmation of such structures in an experiment with a symmetric nozzle. The side of asymmetry of the shock system may be changed in the experiment.

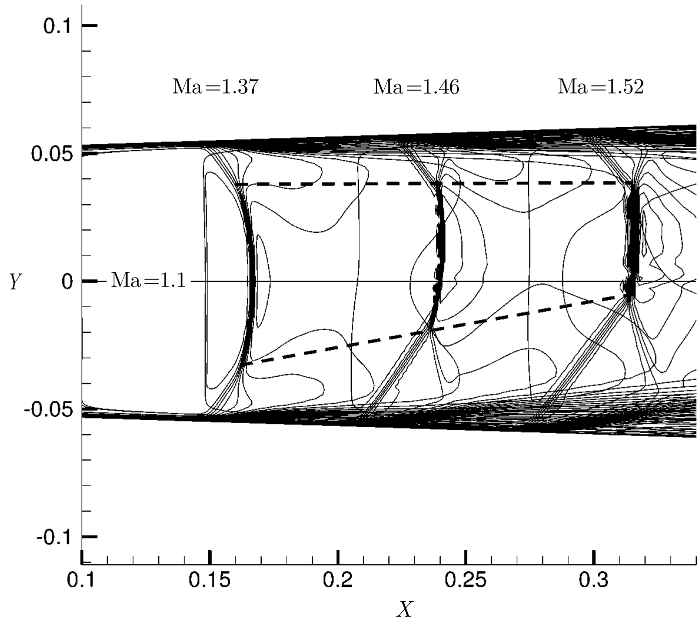

Figure 3. Nozzle divergence angle of 1.877°, inlet Ma = 1.1

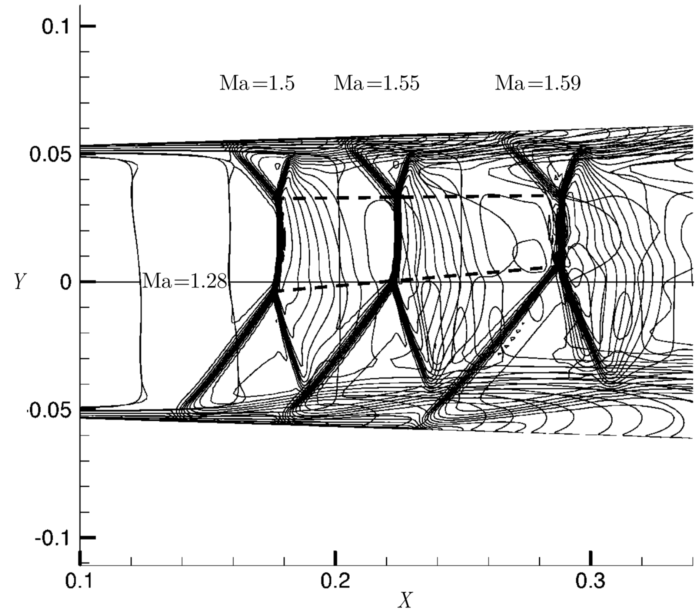

Figure 4. Nozzle divergence angle of 1.877°, inlet Ma = 1.28

5. Influence of nozzle divergence on the shock pattern

Another parameter apparently influencing the shock wave structure and its tendency to become asymmetric is the nozzle divergence angle, α. Three values of the divergence angle were chosen for investigation: $\alpha = 1.877°$, $\alpha = 2.5°$, $\alpha = 3.0°$. Results for the case of Mach number Ma = 1.37 in front of the shock wave are shown

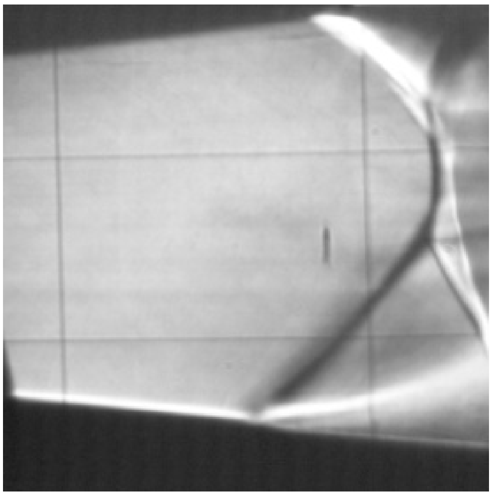

Figure 5. A schlieren picture of an asymmetric shock structure in a nozzle at Ma = 1.6

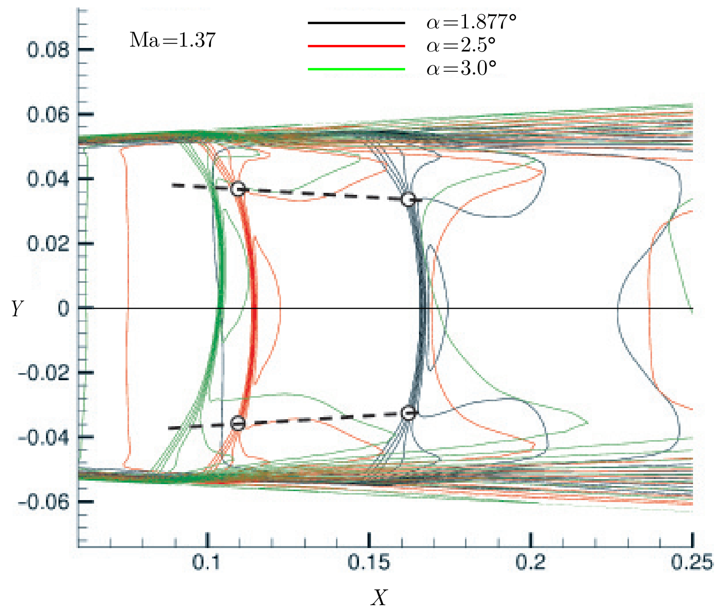

Figure 6. Results for three nozzles, Ma = 1.37 in front of the shocks

in Figure 6. All three nozzles have the same inlet size and they are drawn in Figure 6 over each other.

The inlet Mach number is the same in all three of the nozzles. The simulation results show that the same velocity (Ma = 1.37) upstream of the shock wave occurs at different distances from the inlet, which reflects the uneven differences between divergence angles. The greater the nozzle divergence, the more upstream the location of the shock wave. Greater acceleration takes place in a nozzle with a greater divergence angle and this explains why the shock appears earlier.

The shock wave structure for the nozzles with the smallest and the intermediate opening angles (on the right and in the middle) is symmetric, whereas asymmetry is clearly seen for the shock wave pattern in the nozzle with the largest divergence angle (on the left). Two dashed lines passing through the triple points marked by circles serve as a visual aid.

It should be noted here that in the two symmetric shock cases the λ-feet differ in size, causing the inclination of dashed lines. As the pre-shock Mach number and the Reynolds number are the same, the difference in size must be caused by the characteristic length, which is the boundary layer thickness. The boundary layer is thicker in the nozzle with the smallest divergence angle (far right) because it grows with the distance from the inlet, and thus this λ-foot is larger. At the same time, in this flow case the nozzle height is the smallest, but the difference is so small that its influence is apparently minor.

The investigation was also carried out for a higher velocity. Numerical results for the Mach number of Ma = 1.41 upstream of the shock wave are presented in Figure 7. As previously, the results of numerical simulations from three nozzles are put on top of each other. The inlet nozzle section is of the same size for all three cases. For the smaller divergence the shock is located further downstream from the inlet.

In the case of the smallest nozzle opening angle of $\alpha = 1.877°$ (right) the λ-feet on both sides of the channel are nearly the same. This slight asymmetry increases for $\alpha = 2.5°$ (middle) and becomes pronounced for $\alpha = 3.0°$ (left). The straight dashed lines linking the triple points indicate more clearly the changing asymmetry of the shock wave pattern.

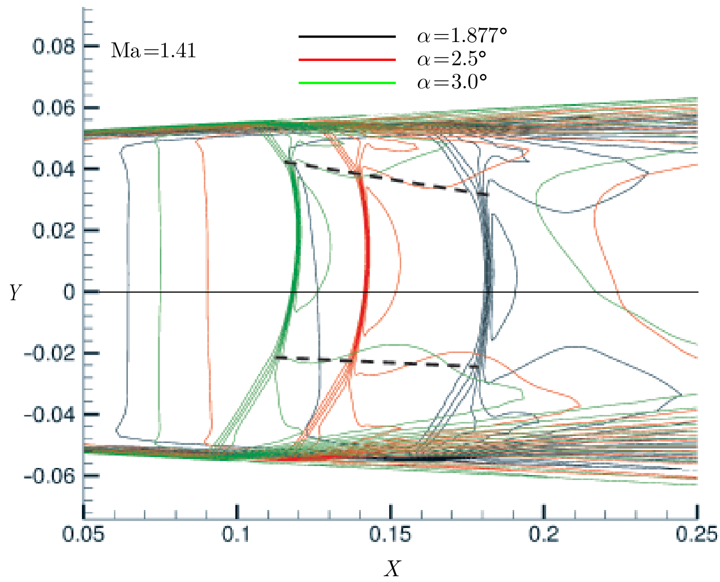

Figure 7. Results for three nozzles, Ma = 1.41 in front of the shocks

The λ-foot height for each single shock case depends on the Mach number in front of the shock wave, the Reynolds number, boundary layer thickness and the size of the channel cross-section where the shock wave occurs [5, 6]. Despite the equality of the above parameters for the upper and lower λ-feet the asymmetry appears in each flow case.

This is surprising and the only possible explanation is that different channel constraints act on each foot. Therefore one should treat the channel as two (parallel) independent parts, each providing a geometrical constraint for one foot. The streamline which passes perpendicularly through the shock wave may be considered the one dividing the channel. The flow deflection on the shock wave is zero at the point where the chosen streamline crosses the shock wave (see Figure 8). In the case of symmetric flow this point is located at the nozzle's axis.

If such a division of the channel contributes to the constraint of the λ-feet, the following relationship should hold: $h_1/a = h_2/b$.

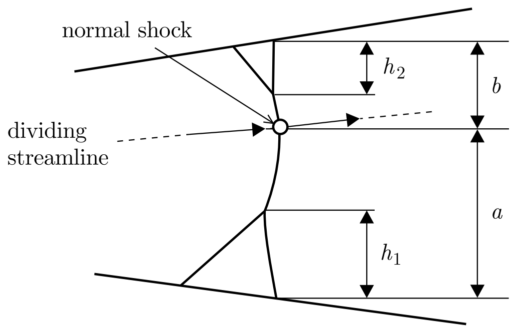

Figure 8. Sketch of shock configuration

Ratios determined from the results of the numerical simulations presented in Figures 6 and 7 are presented in Table 1.

Table 1. Comparison between two cases: Ma = 1.37 and Ma = 1.41

	Solution presented in Figure 6 Ma = 1.37		Solution presented in Figure 7 Ma = 1.41	
α[°]	h_1/a	h_2/b	h_1/a	h_2/b
1.877	0.316	0.311	0.336	0.334
2.5	0.277	0.280	0.447	0.427
3.0	0.350	0.347	0.490	0.466

For both Mach number cases the ratios correspond very well with each other, with a deviation not greater than 1%. This confirms that the asymmetry of flow is expressed by an uneven division of the tunnel into two streamwise parts, limited by the side wall and the streamline at which the main shock is normal, hence introducing no deflection of this streamline. The channel height constraint, affecting the size of the λ-feet, is imposed by these two heights of the channel.

For Ma = 1.37, the relative height of the λ-foot decreases from the case of $\alpha = 1.877$ to $\alpha = 2.5$. This is caused by the decrease of λ-foot height between these two symmetric flow cases. The changeover to asymmetry causes a relative increase in the λ-foot height. The characteristic behaviour of the onset of asymmetry appears to be the minimum of the relative λ-foot size. For Ma = 1.41 flow cases, the relative λ-foot height increases with the increasing divergence angle and the degree of the shock structure's asymmetry. These observations suggest that increasing asymmetry under the same flow conditions leads to an increased relative λ-foot height.

6. The high divergence angle nozzle

With regard to simulations of a nozzle with the opening angle of 6.54°, the unsteady approach led to a good convergence of solutions, whereas the steady approach did not.

Solutions for three different time steps are presented in Figure 9 in order to visualize the unsteadiness of the flow field. The main shock wave position is stationary, but the flow downstream of the nozzle changes significantly in time.

The flow behind the main shock wave is subsonic, but the significant growth of boundary layers along the nozzle walls re-accelerates the subsonic flow to supersonic speeds. The second normal shock wave is generated in the middle part of the stream, quite far downstream behind the main shock wave system. The limited span of this shock shows how thick the boundary layers are, especially the subsonic part.

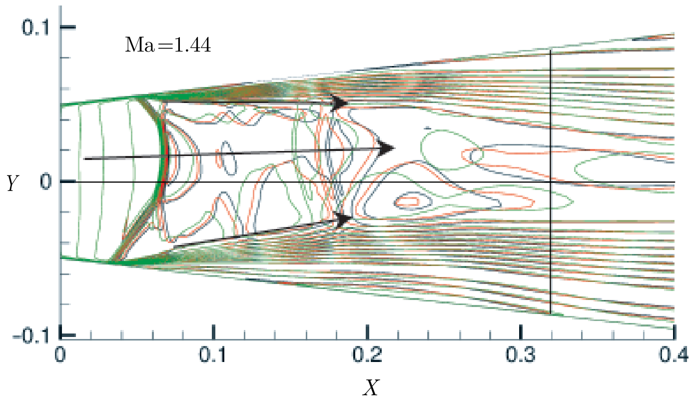

Figure 9. Unsteady flow in a nozzle at different time steps

The Mach number upstream of the shock wave in Figure 9 is Ma = 1.44 and the shock wave pattern is asymmetric. At this strong divergence of the channel it is very well visible that the flow downstream of the shock system is deflected in the upwards direction. The large separation sizes give this freedom of an effective deflection of the stream downstream of the shock system, even though the nozzle is symmetric.

Figure 10 presents the velocity distribution across the channel shown in Figure 9 taken at a distance downstream of the shock.

The velocity distribution can be divided into three parts. The two exterior parts concern the velocity in boundary layers on both sides of the nozzle. They are of different sizes, especially the reverse flow parts. This asymmetry in velocity distribution is linked to the asymmetry of the shock wave pattern and the different thickness of separated boundary layers. There is a "plateau" in the middle of the nozzle. A small dip in the velocity on the "plateau" corresponds to the area where the shock is normal to the flow (see the arrow in Figure 9), inducing the highest losses and therefore the strongest decrease of velocity. This "constant velocity" stream core

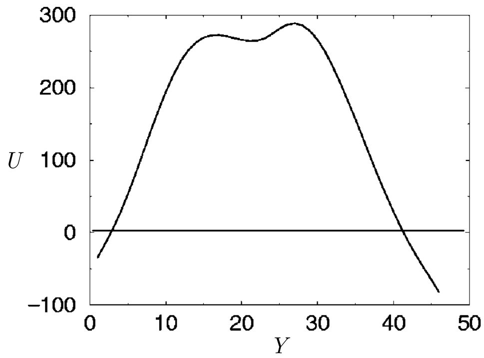

Figure 10. Velocity distribution downstream of the shock wave

between the large separation areas allows for a degree of flow deflection at the shock wave, which is related to the shock pattern's asymmetry.

7. Conclusions

The natural choice of asymmetry in a flow is usually confusing and very often a problem for experimental investigations, as the asymmetry's direction is rather unpredictable. Such flows are especially undesirable in numerical simulations, as the numerical code is mostly blamed for asymmetry appearing in symmetrical geometries. However, if Navier-Stokes equations are to describe the main physical features of the flow they sometimes have to choose asymmetry, just as nature does.

The authors have carefully eliminated all the possible reasons for asymmetry in the approach in order to avoid any suspicion of non-physical reasons for the obtained asymmetric solutions. As the most suspected reason was the numerical grid, the axis of the nozzle was used as the mirror-reflection line. The grid was actually generated in one half of the nozzle and the other part was reflected, so that the symmetry of the nozzle was ideal. However, simulations were also carried out on grids generated in the usual way and no differences in solutions have been found.

The main aim of this paper is to provide more systematic information, which would qualitatively demonstrate the effect of two main reasons for asymmetry, *viz.* the increase in the Mach number and in the nozzle divergence angle.

It has been shown that at some Mach numbers around $\mathrm{Ma} = 1.4$ asymmetry appears in the shock system and increases with increasing Mach numbers.

The Mach number at which asymmetry appears is significantly dependent on the nozzle's divergence angle: the greater the nozzle opening angle, the greater the asymmetry.

The asymmetry of the shock system and the difference in the λ-feet size was very confusing, as all the parameters influencing their height seemed to be the same. Our analysis of the obtained results allowed us to explain this phenomenon. In internal flows the height of the channel is an important constraint influencing the λ-foot size. In the case of an asymmetry, the channel is divided into two independent parts by a streamline, which passes through the shock in the place where it is normal to the flow and induces no deflection of this streamline. It has been proven in this paper that the λ-feet heights' scale is exactly in accordance with this division. It has also been shown that the increasing asymmetry causes an increase in the λ-foot height relative to the channel size under the same flow conditions.

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