ANALYSIS OF NON-LINEAR ELASTIC MATERIAL PROPERTIES OF PVC-COATED PANAMA FABRIC

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Abstract: The aim of the present paper is to propose a method of laboratory tests to determine non-linear elastic properties of coated Panama fabric. The material parameters are specified on the basis of uniaxial tensile tests in the warp and weft directions. Techniques based on the least squares' methods are used in the determination process. Reduction concepts of the strength parameters are proposed in order to take into account rheological effects in the fabric. Numerical applications to MSC.Marc, a commercial program, are presented. A user-defined HOOKLW subroutine is used to introduce the non-linear elastic properties of the PVC-coated fabric into the MSC.Marc system.

Keywords: non-linear elastic modeling, PVC-coated fabric, MSC.Marc

1. Introduction

A number of theoretical models describing PVC-coated fabrics have been developed (described *e.g.* in paper [1]). Different concepts of threads' properties for warp, weft and coating have been assumed in these models. Some of them assume that the polymer structures of the fabric exhibit viscoelastic [2], viscoplastic [3] or elastic [4] characteristics. The choice of the model is conditioned by the type of coated fabrics, but is always disputable.

In this paper, it is assumed that PVC-coated Panama fabrics generally posses non-linear elastic properties. However, omission of all the effects connected with the fabric's rheological characteristics, which appear in long-term usage, is not permissible. A method of taking these effects into account is proposed here.

Manufacturers of PVC-coated fabrics usually provide only principal strength parameters, such as tensile strength or tear strength. In practical civil engineering projects, the usage of these immediate strength parameters is not admitted since structures are usually designed for a specified lifetime. It is essential to take these aspects into account in the design process by reducing the immediate strength parameters. For instance, the ultimate tensile strength (UTS) specifies only the tensile strength of 5cm wide fabric band, but in no way determines the long-term durability of coated fabrics.

Based on the experience of geosynthetics' designers (see *e.g.* [5]), the author proposes to apply to coated fabrics the reduction of the ultimate tensile strength parameter according to the following equation:

$$LTS = UTS/(\gamma_1 \cdot \gamma_2 \cdot \gamma_3). \tag{1}$$

In Equation (1), LTS and UTS are the values of the long-term tensile strength parameter and the immediate strength factor, which is subject to reduction, respectively. The γ_1 material parameter takes into account the rheological phenomenon, $\gamma_2 \geq 1.05$ makes allowance for environmental influence (*e.g.* UV radiation), and $\gamma_3 \geq 1.05$ describes individual work conditions of the fabric's built-in structures. Factors γ_1 and γ_2 are determined on the basis of laboratory tests, while the γ_3 parameter is specified by the designer depending on the character and lifetime of the structures utilization.

Another fundamental and disputable problem is the choice of range and kind of laboratory tests in which the material parameters will be determined. The author has chosen the dense net model [6] to describe the behavior of PVC-coated Panama fabric. Based on the assumptions of this model, which requires only identification of the longitudinal modulus in the weft, F_2, and warp, F_1, directions, uniaxial tension tests have been chosen. In the case of biaxial tension tests, the main problem lies in selecting the ratio of the tensile stresses (forces) in the warp and weft directions. The method of the ratio's determination in laboratory tests is questionable, since there are different stress ratios in real structures at every point of construction. As a matter of course, constitutive models are developed in which biaxial tensile tests are necessary to determine material parameters. For example, results of biaxial tests for the stress proportion $\sigma_{\text{warp}} : \sigma_{\text{weft}}$ equal to $1:1$, $5:1$, $1:5$ are presented in [7]. At the same time, biaxial tensile tests are performed for the assumed strain proportion in the warp and weft directions. For instance, results of biaxial tests for the strain proportion $\varepsilon_{\text{warp}} : \varepsilon_{\text{weft}} = 2$ are given in [8].

2. The laboratory tests

The elastic characteristics of PVC-coated Panama fabric were determined on the basis of laboratory tests. The tests were carried out at the Department of Hydraulic Structures and Water Resources Management of Gdansk University of Technology's Faculty of Civil and Environmental Engineering (see [9] for details). In this work, the fabric's inelastic properties are determined for viscoplastic Chaboche's [10] and Bodner–Partom's [11] constitutive models. The author proposes a new identification concept, in which the fabric's non-linear elastic properties are precisely investigated. Rheological characteristics in this variant will be taken into consideration with the concept of reduction of the immediate strength parameters.

The investigated coated Panama fabric was manufactured as polyester threads (PES) on woven P 2/2 double-sided PVC coating (according to the DIN 60001 standard). In the laboratory tests, a computer-operated strength-testing machine of

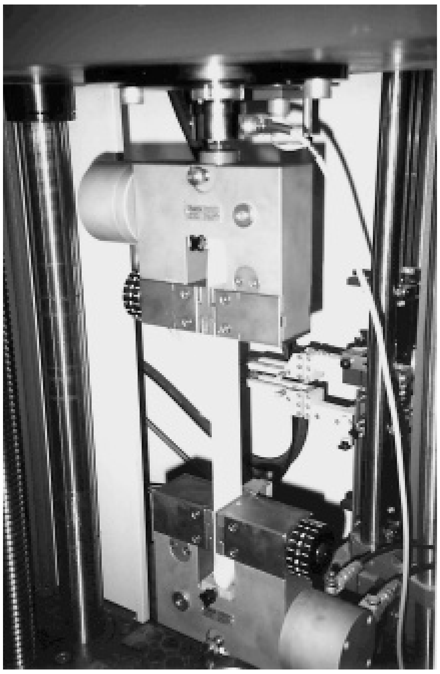

Figure 1. The gripping jaws of the testing machine [9]

the Zwick 147670 type was used (see Figure 1). Specimens of same-batch Panama fabric were cut in the warp and weft directions. The specimens' width of 0.0500 ± 0.0005m and active length (gripping distance) of 0.2000 ± 0.0005m were taken. The manufacturer specified the fabric's weight as 870 ± 30g/m^2, thicknesses $t_t = 0.00075 \pm 0.00005$m, and weave angle $\alpha_0 = 90^\circ$.

The specimens were subjected to tension in the directions of warp and weft threads with a constant strain rate of $1 \cdot 10^{-5}$s^{-1}, $1 \cdot 10^{-4}$s^{-1}, $1 \cdot 10^{-3}$s^{-1} and $5 \cdot 10^{-3}$s^{-1}. The full program consisted of about 40 tests in the warp direction (some of which served the purpose of testing the correctness of specimens' seizure in the machine's grippers) and 25 tests in the weft direction. The tests for the two lowest strain rates were carried out to 5% deformation for $1 \cdot 10^{-4}$s^{-1} and 3% deformation for $1 \cdot 10^{-5}$s^{-1}. Due to the assumed concept of determining non-elastic properties in a wide range of deformations, tests for the $1 \cdot 10^{-5}$s^{-1} strain rate were omitted. Results obtained directly from the tests are presented in Figures 2–4 for weft and in Figures 5–7 for warp. It is worth pointing out that the curves have similar characteristics and shapes.

3. The concept

Firstly, a transformation of the data obtained from the strength machine, *i.e.* displacement-force to strain-stress curves (see Figures 8–10) is required. It may be noted that the specific expression for "stress" [kN/m]:

$$\sigma = F \cdot \varepsilon, \tag{2}$$

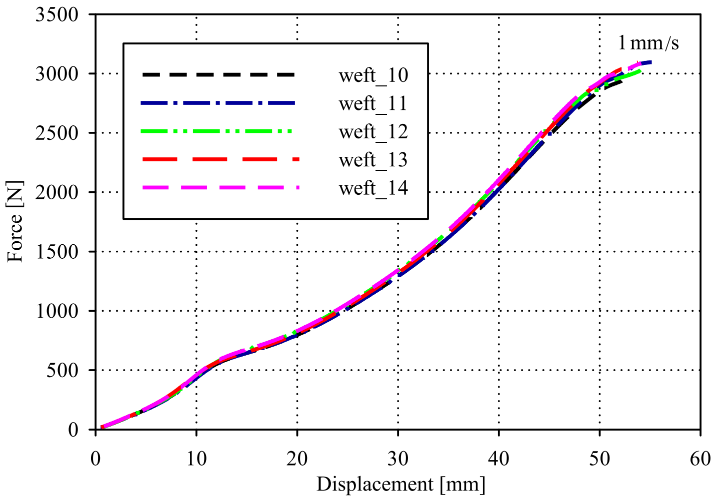

Figure 2. Tests results: weft, gripper displacement rate of 1mm/s [9]

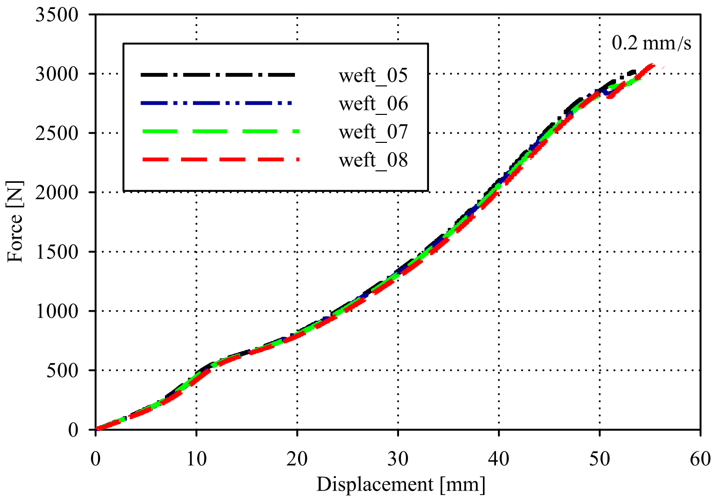

Figure 3. Tests results: weft, gripper displacement rate of 0.2mm/s [9]

has been accepted in accordance with the nomenclature generally used for fabrics. The F [kN/m] symbol in Equation (2) stands for the longitudinal modulus and can be expressed by the following formula:

$$F = E \cdot t, \tag{3}$$

where E is the modulus of elasticity and t is the coated fabric's total thickness.

After analysis of the strain-stress curves for an individual strain rate it is possible to observe for warp and weft two characteristic points of district curvature. The author proposes that the three straight line coefficients $F2A$, $F2B$, $F2C$ and $F1A$, $F1B$, $F1C$ (see Figure 11) specify the longitudinal modulus for weft and warp in the admissible range of strain.

Having found the curves' coefficients in the assumed range of strain (see Table 1), intersection points ε_{P1}, ε_{P2} for weft and warp were determined. These

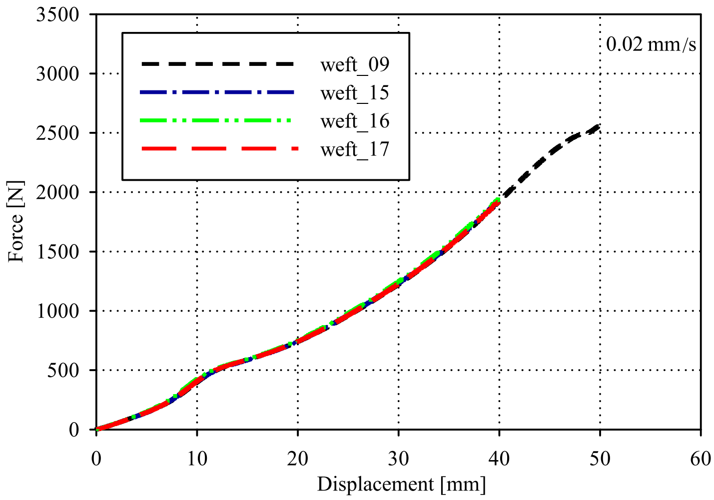

Figure 4. Tests results: weft, gripper displacement rate of 0.02mm/s [9]

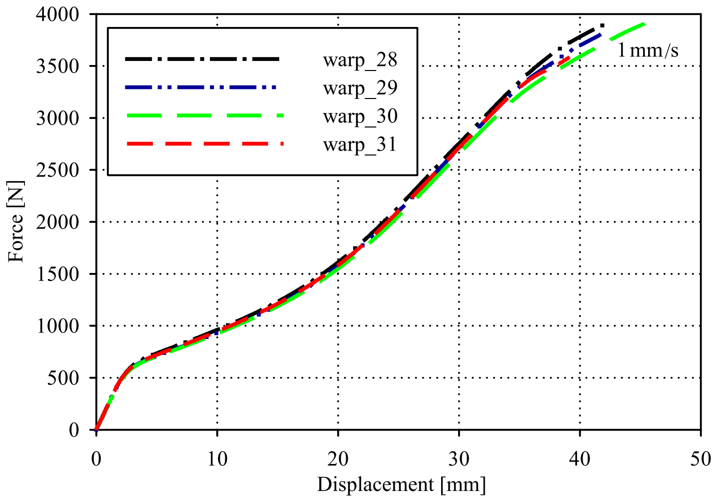

Figure 5. Tests results: warp, gripper displacement rate of 1mm/s [9]

characteristic points specify the range of applicability of individual parameters. The original strain point ($\varepsilon_{P0} = 0.0$) and the final strain point ($\varepsilon_{P3} = 0.18$ for warp, $\varepsilon_{P3} = 0.24$ for weft) were pre-established. Beyond the final points, the weakening of material increased, as a consequence of which a rapture was observed in the specimens.

4. The procedure

A concept of determination of the elastic properties of coated Panama fabric has been presented in the preceding section. The dependence between stress and strain in the adequate strain range was assumed as piece-wise linear relations. The least squares' regression was chosen to determine these relations. For that purpose the SigmaPlot 2002 commercial program's version 8.02 [12] was applied. The SigmaPlot curve filter uses the Marquardt-Levensberg algorithm (see *e.g.* [13]) to find the coefficients of the independent variables that give the best fit between

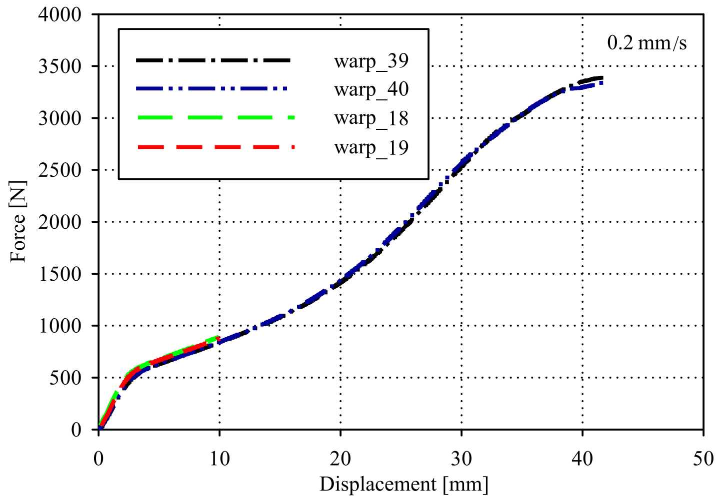

Figure 6. Tests results: warp, gripper displacement rate of 0.2mm/s [9]

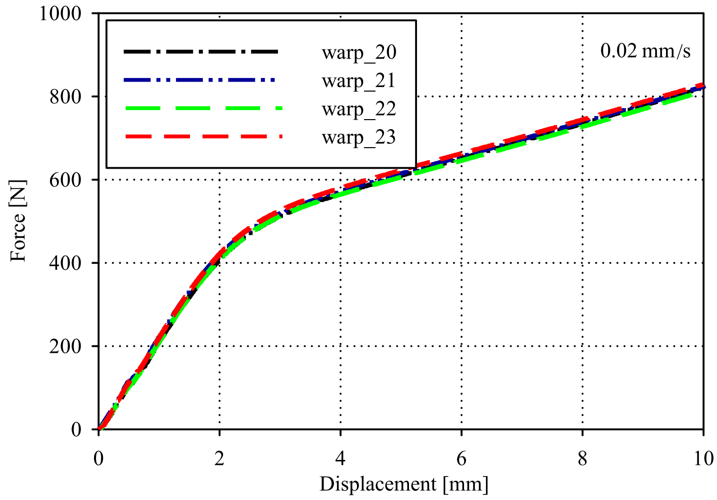

Figure 7. Tests results: warp, gripper displacement rate of 0.02mm/s [9]

the equation and the data. The *F1A*, *F1B*, *F1C* and *F1A*, *F1B*, *F1C* coefficients for weft and warp, respectively, were determined on the basis of this numerical algorithm for individual tests. The results of piece-wise linear approximation are given in Tables 2 and 3.

5. The final results

A proper presentation of laboratory results is the best approximation of the measured quantity and the range in which the quantity is found (see *e.g.* [14]). Hence, the results obtained for warp (Table 2) and weft (Table 3) require an appropriate statistical error analysis. After determining the arithmetic mean, defined as:

$$\overline{x} = \sum \frac{x_i}{N}, \tag{4}$$

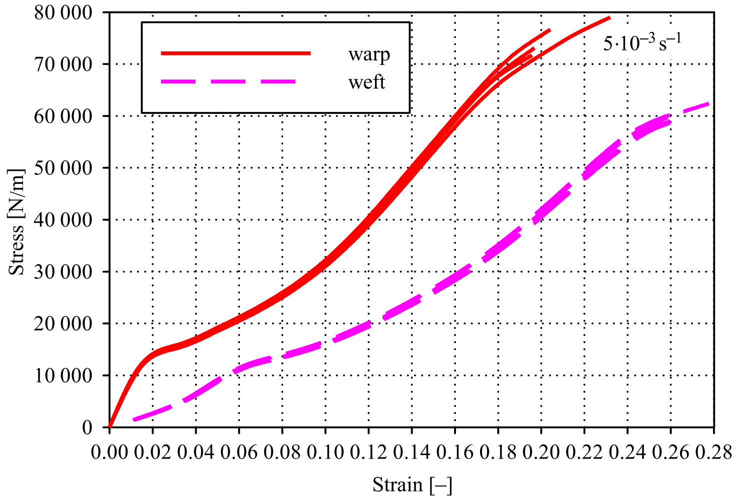

Figure 8. Strain-stress curve for $\dot{\varepsilon} = 5\cdot 10^{-3}\,\mathrm{s}^{-1}$

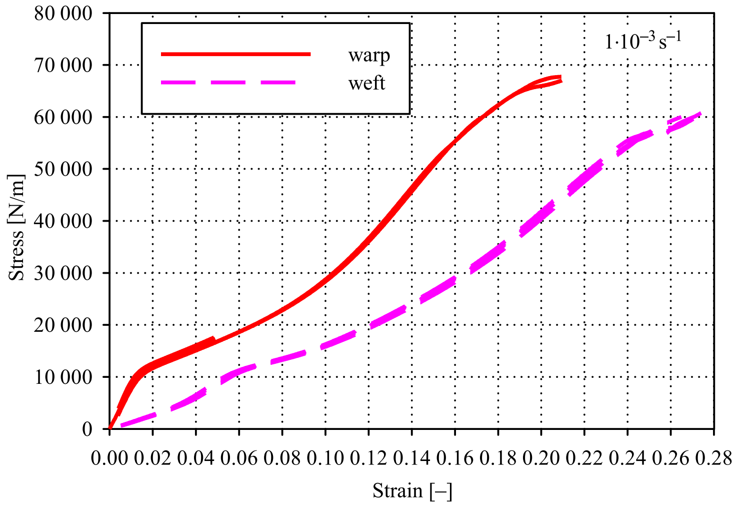

Figure 9. Strain-stress curve for $\dot{\varepsilon} = 1\cdot 10^{-3}\,\mathrm{s}^{-1}$

and the standard error of the mean of the specified range, defined by:

$$\overline{s}_x = \frac{s_x}{\sqrt{N}} = \frac{\sqrt{\frac{1}{N-1}\sum(x_i - \overline{x})^2}}{\sqrt{N}}, \tag{5}$$

for the individual columns of Tables 2 and 3, the results have been collected in Table 4. Evaluation of the permissible limit (of the acceptable values of stress and strain) for coated Panama fabric, shown in Table 4, is based on the concept of reduction of the immediate strength parameters. The values of the ultimate tensile strength for warp, $UTS_{\mathrm{warp}} = 73 \pm 2\,\mathrm{kN/m}$, and weft, $UTS_{\mathrm{weft}} = 60 \pm 1\,\mathrm{kN/m}$, have been determined. For example, Sattler [15], manufacturers of coated Panama fabric, give the values of ultimate tensile strength of $UTS_{\mathrm{warp}} = 84\,\mathrm{kN/m}$ and $UTS_{\mathrm{weft}} = 80\,\mathrm{kN/m}$ and the limit strain of 0.15 in the warp direction and 0.20 in the weft direction.

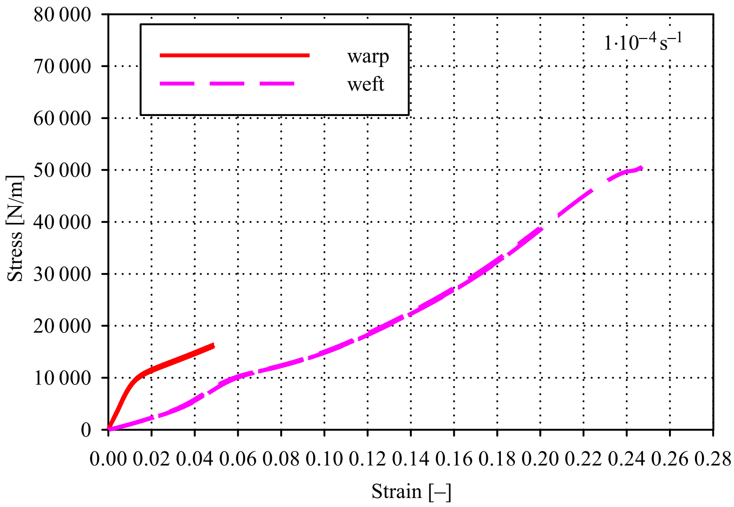

Figure 10. Strain-stress curve for $\dot{\varepsilon} = 1 \cdot 10^{-4}\,\mathrm{s}^{-1}$

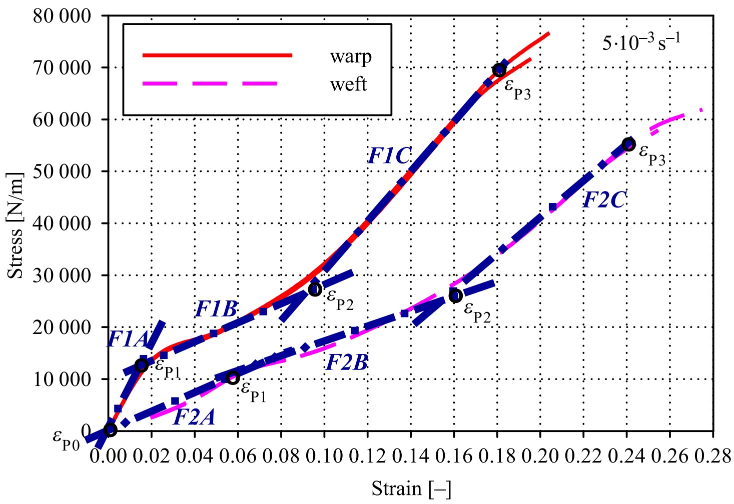

Figure 11. Graphical concept of identification

Table 1. Strain range assumed for F coefficients

	F coefficients	range of strain ε [–]
warp	*F1A*	0.00–0.01
	F1B	0.02–0.07
	F1C	0.11–0.18
weft	*F2A*	0.00–0.06
	F2B	0.06–0.13
	F2C	0.18–0.24

Next, it is necessary to specify the γ_i parameters for reduction. On the basis of rheological laboratory tests [9] it has been found that the studied coated fabric can be applied in structures where stresses in the fabric are of the order of 50%

Table 2. The warp coefficients

$\dot{\varepsilon}$ [s^{-1}]	Test name	*F1A* [N/m]	*F1B* [N/m]	*F1C* [N/m]	ε_{P1} [–]	ε_{P2} [–]	*UTS* [N/m]
$5\cdot10^{-3}$	warp_28.dat	976787	189810	480634	0.0125	0.0926	78588
	warp_29.dat	964787	185906	478075	0.0125	0.0953	76702
	warp_30.dat	946987	184755	464787	0.0123	0.0935	79230
	warp_31.dat	974716	188303	474735	0.0122	0.0933	71689
$1\cdot10^{-3}$	warp_39.dat	800830	174734	457322	0.0131	0.0938	67798
	warp_40.dat	814886	176128	442327	0.0111	0.0899	67010
	warp_18.dat	932844	173452	—	0.0120	—	—
	warp_19.dat	910581	167933	—	0.0124	—	—
$1\cdot10^{-4}$	warp_20.dat	869059	168980	—	0.0114	—	—
	warp_21.dat	887692	173681	—	0.0110	—	—
	warp_22.dat	870723	164155	—	0.0114	—	—
	warp_23.dat	899719	164804	—	0.0113	—	—

Table 3. The weft coefficients

$\dot{\varepsilon}$ [s^{-1}]	Test name	*F2A* [N/m]	*F2B* [N/m]	*F2C* [N/m]	ε_{P1} [–]	ε_{P2} [–]	*UTS* [N/m]
$5\cdot10^{-3}$	weft_10.dat	188027	142482	351480	0.0459	0.1520	59412
	weft_11.dat	187307	152511	360040	0.0467	0.1537	52477
	weft_12.dat	193361	146430	356155	0.0496	0.1513	61023
	weft_13.dat	195312	146954	361372	0.0402	0.1524	62149
	weft_14.dat	197345	147377	362843	0.0461	0.1517	61885
$1\cdot10^{-3}$	weft_05.dat	196349	150406	350299	0.0369	0.1493	60358
	weft_06.dat	190712	148941	342284	0.0396	0.1481	59675
	weft_07.dat	191062	147866	344270	0.0387	0.1494	59315
	weft_08.dat	182656	143639	348528	0.0469	0.1517	61800
$1\cdot10^{-4}$	weft_09.dat	173772	140591	306700	0.0345	0.1414	51228
	weft_15.dat	174585	143525	314585	0.0280	0.1473	—
	weft_16.dat	181083	144624	310801	0.0270	0.1457	—
	weft_17.dat	176325	141010	310076	0.0322	0.1461	—

of the immediate ultimate tensile strength. The creep process is stable in this stress range and does not lead to the destruction of the specimen. Therefore, the material parameter $\gamma_1 = 2.0$ has been assumed here. It should be noted that it is recommended in [5] to use $\gamma_1 = 2.5$ where detailed data are not available. Additionally, material parameters $\gamma_2 = 1.05$ and $\gamma_3 = 1.05$ have been assumed here. According to Equation (1), the values of the long-term tensile strength are defined by:

$$LTS_{\text{warp}} = \frac{73\,\text{kN/m}}{2.0\cdot1.05\cdot1.05} \approx 33\,\text{kN/m}$$

$$LTS_{\text{weft}} = \frac{60\,\text{kN/m}}{2.0\cdot1.05\cdot1.05} \approx 27\,\text{kN/m}$$

Table 4. Non-linear elastic properties of coated Panama fabric

	F [kN/m]	ε [–]	
		from	to
warp	904 ± 17	0	0.0119 ± 0.0002
warp	176 ± 3	0.0119 ± 0.0002	0.093 ± 0.002
warp	471 ± 4	0.093 ± 0.002	**0.110**
weft	187 ± 2	0	0.039 ± 0.002
weft	146 ± 1	0.039 ± 0.002	0.1495 ± 0.0008
weft	340 ± 6	0.1495 ± 0.0008	**0.160**

Based on LTS_{warp} and LTS_{weft}, characteristic strain limits $\varepsilon^{\mathrm{ch}}_{\mathrm{warp}} = 0.11$ and $\varepsilon^{\mathrm{ch}}_{\mathrm{weft}} = 0.16$ have been determined.

6. Numerical applications

The material parameters determined in the previous section can be applied directly to the commercial program. From a number of available commercial programs applicable to the analysis of membrane structures, the author has chosen the MSC.Marc system. The dense net model [6] has been chosen to describe the coated fabric's behavior. The HOOKLW user-defined subroutine [16] was used to introduce the dense net model into the MSC.Marc system.

Numerical calculations were performed for a simple 3D structure (see Figure 12) in order to compare the obtained results with those of the uniaxial tension test described in Section 2. A numerical simulation of tension was performed in the directions of the warp and weft threads. Four-node, isoparametric membrane elements (Element 18, see [17]) were used in the calculations. The following geometrical parameters were assumed: $a = 200.0$ mm (length), $b = 50.0$ mm (width) and $t = 0.75$ mm (thickness).

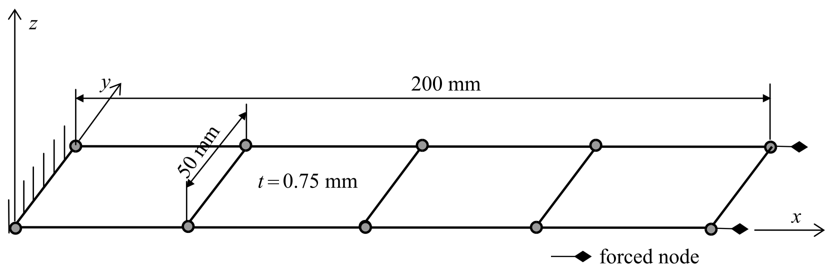

Figure 12. Visualization of the analyzed structure

The results obtained from the MSC.Marc system, with the non-linear elastic properties for warp and weft applied according to Table 4, were compared with the laboratory tests, warp_39 (see Figure 6) and weft_08 (see Figure 3), described in detail in Section 2. The results of these numerical calculations are shown in Figure 13. The author has noticed good agreement of the stress-strain relations obtained both the MSC.Marc calculations and the laboratory tests. These simple tests have confirmed the correctness of the obtained material parameters.

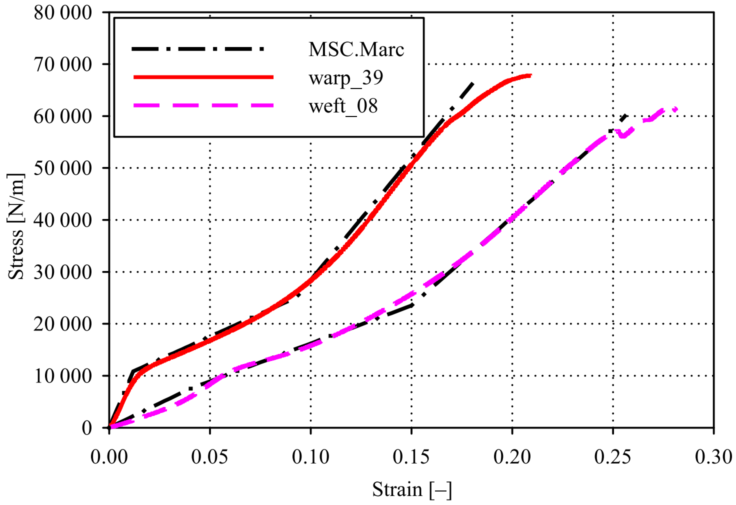

Figure 13. Calculation results: comparison with laboratory tests

(a) (b)

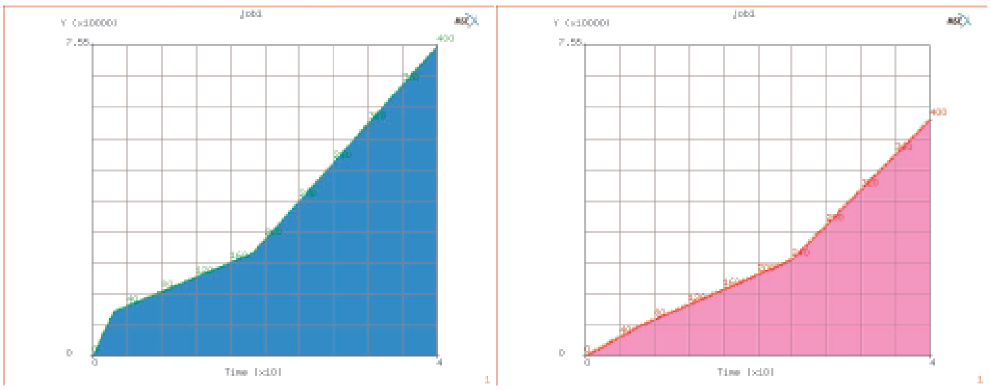

Figure 14. MSC.Marc – stress results in the time domain: a) for warp; b) for weft

For clear presentation, the results obtained directly from the MSC.Marc system, 2003 version, are shown in Figure 14. The graphs present numerical simulations of the uniaxial tension test for warp (Figure 14a) and weft (Figure 14b).

7. Conclusions

Identification of non-linear elastic models has been successfully performed on the basis of uniaxial tension tests for the dense net model used to describe PVC-coated Panama fabric. The author has proposed a simple and practical non-linear elastic model describing the warp and weft threads' behavior during a deformation process up to 45% of the immediate *UTS*. The model is suggested to be applicable in a wide range of engineering problems. The material parameters determined above can be used directly in FEM to analyze structures made of PVC-coated fabrics, where geometric non-linearity is accompanied by the fabric's physical non-linearity. By applying users' constitutive model subroutines linked to MSC.Marc, the material properties of PVC-coated fabrics can be applied directly in this system.

Acknowledgements

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