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CURIE TEMPERATURES FOR THE ISING MODEL ON ARCHIMEDEAN LATTICES

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Abstract: Critical temperatures for the ferro-paramagnetic transition in the Ising model are evaluated for five Archimedean lattices, basing on Monte Carlo simulations. The obtained Curie temperatures are 1.25, 1.40, 1.45, 2.15 and 2.80 [J/k_{B}] for $(3,12^2)$, $(4,6,12)$, $(4,8^2)$, $(3,4,6,4)$ and $(3^4,6)$ lattices, respectively.

Keywords: computer modelling and simulation, Ising model, phase transition, critical parameter

1. Introduction

The beauty of the Ising model (IM) [1–4] lies in its simplicity. The considered system is a network of N interacting spins, $S_i = \pm 1$, which energy is:

$$E \equiv -J \sum_{(i,j)} S_i S_j, \tag{1}$$

where J is an exchange integral. We assume homogeneous short-range spin interactions, *i.e.* the summation in Equation (1) is performed over (i,j) pairs of the nearest neighbours. The positive sign of $J > 0$ leads to ferromagnetic interactions among spins. The minimisation of energy (1) for temperature $T = 0$ produces spin dynamics which may be described by a deterministic cellular automaton with the following rule:

$$S_i(t+1) = \mathrm{sign}\left(J \sum_j S_j(t) \right), \tag{2}$$

where t denotes discrete time and summation is performed over the nearest neighbours of the i^{th} spin.

For a finite temperature $T > 0$ the deterministic rule (2) is replaced by a probabilistic cellular automaton with a spin update rule, $S_i(t) \to S_i(t+1)$, described

by the Glauber [5] or Metropolis [6] dynamics. Phase transition may then be observed: below the critical temperature, $T < T_C$, spontaneous magnetisation, $m \equiv \sum_{i=1}^{N} S_i/N \neq 0$, is observed, while $m = 0$ for $T > T_C$.

The Ising model has already been investigated in many ways, including Monte Carlo simulation [7–12], series expansion [13–17], the mean-field approach [18–20] or the partition function technique [21–23], and for many systems, such as: antiferromagnets [24–30], frustrated [24, 25, 31], disordered [32, 33] or diluted [34, 35] networks on complex [24, 36–38] or shuffled lattices [39, 40], *etc.* [41].

In this paper, critical temperature T_C is estimated for five two-dimensional lattices, on the basis of the $\langle m(T) \rangle$ dependence, where $\langle \cdots \rangle$ denotes the time average. Archimedean lattices (AL) are vertex transitive graphs that can be embedded in a plane such that every face is a regular polygon. Kepler demonstrated that there is exactly eleven such graphs [42]. The names of the lattices are given according to the sizes of faces incident to a given vertex. The face sizes are listed in order, starting with a face such that the list is the smallest possible in the lexicographical order. In this way, the square latticeis called (4,4,4,4), abbreviated to (4^4), the honeycomb is called (6^3) and Kagomé is (3,6,3,6). Some results concerning IM on AL were presented in Refs. [43–62]; however the Curie temperatures of several AL's are still missing in the literature of which the authors are aware.

Critical properties of these lattices were investigated in terms of site percolation [63, 64] in Ref. [65], where the topologies of all AL's are given as well.

2. Simulation results

We evaluate the Curie temperature, T_C, on the basis of the termal dependence of magnetisation $\langle m \rangle$. The investigated systems contain about $N \approx 6 \cdot 10^4$ spins, which decorate nodes of (3^4,6), (3,4,6,4), (4,6,12), (4,8^2) AL. The Glauber dynamics [5] is applied and the simulation takes $N_{\text{iter}} = 2 \cdot 10^5$ Monte Carlo steps (MCS). One MCS is completed when all N spins are investigated (spin-by-spin in a type-writer order), whether they should flip or not. The time average is performed over the last 10^5 MCS's for an evaluation of $\langle m \rangle$. The results are presented in Figure 1. The temperature at which spontaneous magnetisation $\langle m \rangle$ ceases is accepted to be an estimation of T_C. These estimations are shown in Table 1.

Table 1. AL's and the associated critical temperatures, T_C

z	lattice	T_C [J/k_B]	Ref.
3	(3,12^2)	**1.25**	—
	(4,6,12)	**1.40**	—
	(4,8^2)	**1.45**	—
	(6^3)	1.52	[59]
4	(3,4,6,4)	**2.15**	—
	(4^4)	$2/\operatorname{arcsinh} 1 \approx 2.27$	[60–62]
	(3,6,3,6)	2.27	[17]
5	(3^4,6)	**2.80**	—
	(3^3,4^2)	$2/\ln 2 \approx 2.89$	[58]
	(3^2,4,3,4)	2.93	[58]
6	(3^6)	3.64	[59]

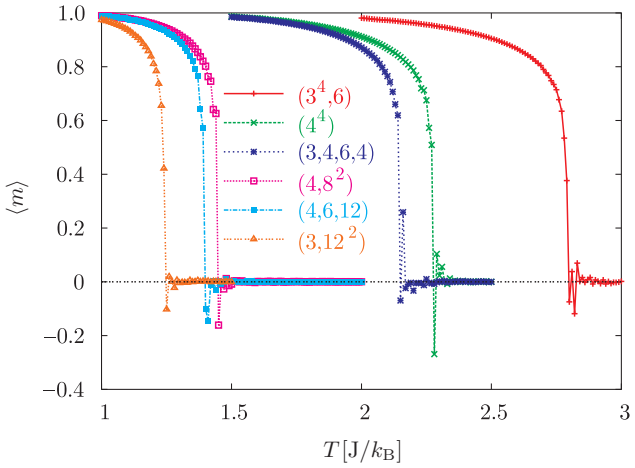

Figure 1. Dependence of average magnetisation $\langle m\rangle$ on temperature T expressed in J/k_B for $(3^4,6)$, $(3,4,6,4)$, $(4,6,12)$, $(4,8^2)$ and (4^4) AL's. Simulations carried out for $N\approx 6\cdot 10^4$ spins during $N_\mathrm{iter}=2\cdot 10^5$ MCS. Magnetisation $\langle m\rangle$ is averaged over the last 10^5 MCS

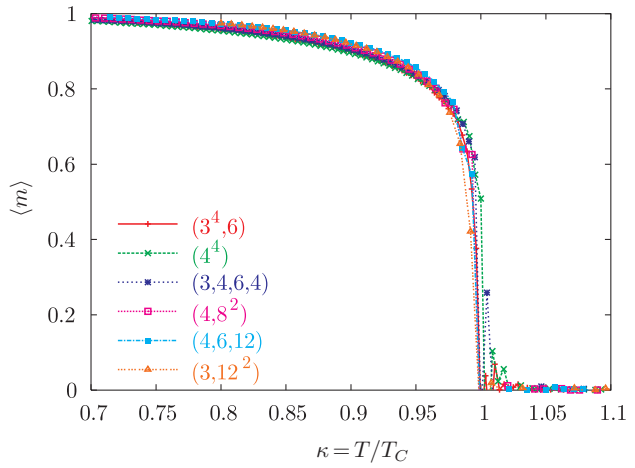

Figure 2. Dependence of average magnetisation $\langle m\rangle$ on normalized dimensionless temperature T/T_C for $(3^4,6)$, $(3,4,6,4)$, $(4,6,12)$, $(4,8^2)$ and (4^4) AL's. Same data as in Figure 1

3. Conclusions

In this paper the Curie temperatures have been collected for IM on all AL's. T_C for $(3,12^2)$, $(4,6,12)$, $(4,8^2)$, $(3,4,6,4)$ and $(3^4,6)$ AL's have been evaluated for the first time with the Monte Carlo simulation.

For all the investigated AL's, the shape of the $m(T/T_C)$ curve (see Figure 2) is roughly the same as for the square lattice. In the latter case, an analytical expression [66] is known:

$$|m(\kappa)| = \sqrt[8]{\frac{\cosh^2(2/\kappa)}{\sinh^4(2/\kappa)}\left(\sinh^2(2/\kappa)-1\right)}\,, \tag{3}$$

where $\kappa \equiv T/T_C$.

In contrast to the Galam-Mauger [67, 68] semi-exact formula for T_C dependence on the system dimensionality, d, and the lattice coordination number, z, we have shown that the critical temperature for IM differs slightly for several AL's (where $d = 2$) with the same values of z. Similarly to the percolation phenomena [63, 64], the dimensionality, d, and the coordination number, z, are not sufficient [69–78] for determining the critical point T_C for IM.

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