HEAT TRANSFER PROCESSES IN A BIOGAS-REACTOR

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Abstract: Visualizations and numerical values are presented of some fluid flow and heat transfer parameters in a biogas reactor with and without a stirring frame. Results obtained through mathematical modeling and computer simulation with ANSYS 8.0/Flotran are used for a heat transfer calculation method for the investigated bioreactor type.

Keywords: mathematical modeling, bioreactor, stirring process, heat transfer, temperature and velocity fields, Bingham viscosity

Nomenclature:

W – angular velocity [rad·s^{-1}],
n – revolution [s^{-1}], [min^{-1}],
h_c – convection heat transfer (film) coefficient [W·m^{-1}·K^{-1}],
τ_0 – yield stress (plastic stress) [N·m^{-2}],
μ_0 – plastic viscosity [Pa·s],
μ_n – Newtonian viscosity [Pa·s],
D – strain rates [s^{-1}],
h_p – propeller's height [m],
D_r, R_r – reactor's diameter and radius [m],
D_p, r_p – propeller's diameter and radius [m],
r, φ, y – directions of a cylindrical coordinate system,
R', r' [m]; α [rad] – epicycloid parameters,
d – stirrer spread side diameter [m],
a – hexagon side length in a propeller's center [m],
H – bioreactor's height [m],
ρ – density [kg·m^{-3}],
λ – conductivity [W·m^{-1}·K^{-1}],
c_p – specific heat [kJ·kg^{-1}·K^{-1},
Re – the Reynolds number, $\mathrm{Re} = \dfrac{V \cdot D_r \cdot \rho_f}{\mu}$,

Nu – the Nusselt number, $\mathrm{Nu} = \dfrac{h_c \cdot D_r}{K_f}$.

Subscripts

m – average value.

1. Introduction

Biogas production has been the subject of a large number of scientific projects aimed at increasing the energy efficiency of biogas installations. The main factors influencing the quality and quantity of biogas production are heat transfer and uniform temperature field in bioreactors. According to many researches [1–3], these factors depend on biomass stirring and the type of stirring frame used. However, the research published until now is still in progress and hardly comprehensive, due to the diversity of biomass used and bioreactor designs. During our preliminary research on biogas production from biomass of milk whey and lime fleshing, an optimal biomass composition has been established (milk whey 40%, lime fleshing 60%) for maximum biogas production [4]. Biomass of these ingredients has not been used for biogas production until now. Consequently, the stirring and heat transfer processes in a bioreactor with this kind of biomass have not been investigated either. In view of the above, the main aims of the present paper have been formulated as follows:

1. Investigation of fluid flow and heat transfer processes in a bioreactor with biomass composition as above and a stirring frame through mathematical modeling and computer simulation.
2. Formulating an equation for heat transfer calculation in the investigated object with the obtained results.

2. The object of investigation and conditions of numerical experiments

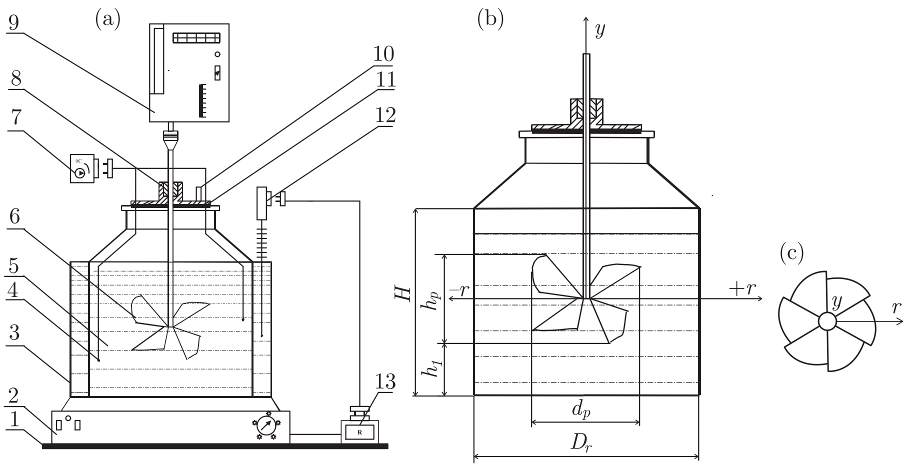

Figure 1. 1 – padding; 2 – hotplate; 3 – water mantle; 4 – thermostat; 5 – reactor; 6 – agitator; 7 – digital multimetre DT-838; 8 – gasket; 9 – electromotor; 10 – gas outlet; 11 – gasket; 12 – contact thermometer; 13 – electrorelay

The investigated bioreactor is a part of a laboratory installation (see Figure 1a) and is geometrically similar to a working bioreactor. The stirring frame is a propeller of the Nartsis type [3]. The projections of round edges at the propeller blades in the x-z plane, crossing the center of the coordinate system, are described by an epicycloid equation (see Figure 1c).

The bioreactor's dimension ratios (see Figure 1b), $d_p/D_r = a_1$, $h_p/H = a_2$, $h_1/H = a_3$, $D_r = H$ have been assumed according to recommendations of various authors [3] working in this field: $a_1 = 0.33$, $a_2 = 0.5$, $a_3 = 0.3$. 70% of the bioreactor's volume is filled with the treated biomass. The bioreactor's dimensions, $H = D_r = 0.12\,\mathrm{m}$, $h_p = 0.05\,\mathrm{m}$, $h_1 = 0.038\,\mathrm{m}$, $d_p = 0.4\,\mathrm{m}$, have been used to create the geometry model.

Empirical and numerical experiments have been conducted for the biomass composition of 60% lime fleshing and 40% milk whey and the following conditions:

1. biomass temperature: t=37°C [4];
2. angular velocities: $w = 0\,\mathrm{rad \cdot s^{-1}}$ to evaluate the influence of stirring on heat transfer in the bioreactor, and $w_1 = 5\,\mathrm{rad \cdot s^{-1}}$, $w_2 = 7.5\,\mathrm{rad \cdot s^{-1}}$, $w_3 = 10\,\mathrm{rad \cdot s^{-1}}$, $w_4 = 12.5\,\mathrm{rad \cdot s^{-1}}$, $w_5 = 15\,\mathrm{rad \cdot s^{-1}}$.

The latter angular velocities were determined experimentally as the most suitable for laboratory bioreactor stirring.

3. Mathematical modelling using ANSYS 8.0/Flotran

A system of the following partial differential equations (described in detail in ANSYS Theory Reference [5]) is solved in a rotating coordinate system in order to model the heat transfer and stirring of the biomass:

1. the continuity equation;
2. the momentum equation for a turbulent case;
3. the uncompressible energy equation;
4. two equations for the RNG-k-ε turbulence model.

The motion of the biomass in the reactor is assumed to be turbulent and the RNG-k-ε turbulence model is used, for the following reasons:

– some turbulence has been observed experimentally during laboratory stirring of the biomass, which is a hydraulic mixture of H_2O and insoluble fragments;
– the RNG-k-ε turbulence model is recommended to model rotating flows with whirling motions and circulation areas. Similar motion has been observed experimentally in the laboratory bioreactor.

The above equations are discretized with a technique based on finite elements, with standard ANSYS element FLUID 142. In this case Galerkin's method of weighted residuals is used to form element integrals. The boundary layer parameters are determined according to the Van Driest conductivity model [6], constants $A = 39.18$ and $\chi = 0.58$ are calculated according to relationships, suitable for low Re numbers [4, 7]: $\chi = 0.40 + 0.19/(1 + 0.49 z_2^2)$, $A = 26 + 14/(1 + z_2^2)$, $z_2 = 10^{-3} \cdot \mathrm{Re} > 0.3$.

The biomass was homogenized in advance. There is no mass transfer in the biomass during stirring and heating, as the differences in temperature (≈ 2°C) and

Figure 2. Initial model geometry

concentration field in the fluid volume were minimal. Thus, there is no diffusion-reaction equation in the above system of partial differential equations.

The system above was solved in terms of the following conditions and loads [3].

(A) *Geometrical model and finite element mesh*

The initial model geometry was created in a cylindrical coordinate system (see Figure 2). The geometry was simplified according to observations made during experiments with the laboratory bioreactor. It was found that at certain stirrer revolutions it is not necessary to model the following:

1. the propeller shaft, which does not influence the stirring process,
2. the free surface of the biomass, as it is not deflected in the horizontal plane in the axial direction by the revolutions of the propeller shaft or the stirrer, or
3. the thickness of the blades of the propeller stirrer. (It is enough for the propeller to be represented in the model by the blade surfaces.)

The final volume geometry consists of two separated fluid volumes (volume 1 and volume 2) with contact areas, as shown in Figure 3b. The finite element mesh is presented in Figure 4.

(B) *Fluid properties*

The characteristics of the investigated biomass, determined experimentally [7], are applied: $\rho = 1.0002\,\mathrm{kg\cdot m^{-3}}$, $\lambda = 0.619\,\mathrm{W\cdot m^{-1}\cdot K^{-1}}$, $c_p = 4\,\mathrm{kJ\cdot kg^{-1}\cdot K^{-1}}$.

Research in the field of hydraulic composites has shown that these mixtures are of the non-Newtonian fluid viscosity type, because of the persistent yield stress (plastic stress), τ_0 [2]. It has been proved experimentally, that the subject biomass is of the non-Newtonian-Bingham fluid viscosity type as well [7]. The biomass' tendency to precipitation is the probable source of τ_0 and it has a negative influence on the convective heat transfer and the biogas production process. The Bingham biomass parameters $\mu_0 = 0.016\,\mathrm{Pa\cdot s}$ and $\tau_0 = 20\,\mathrm{N\cdot m^{-2}}$ were determined at the working temperature ($t = 37^\circ\mathrm{C}$).

The "bi-viscosity" Bingham model [5, 1] was used in order to model the biomass' Bingham viscosity:

$$\mu = \begin{cases} \mu_0 + \dfrac{\tau_0}{D} & \text{if} \quad D > \dfrac{\tau_0}{\mu_n - \mu_0}, \\ \mu_n & \text{otherwise.} \end{cases} \tag{1}$$

The μ_n value must be chosen to at list an order of magnitude larger than μ_0 [5] for modeling the areas in the real Bingham fluids, where $\mu = \infty$ at $\tau < \tau_0$; $\mu_n = 10\,000\,\mathrm{Pa\cdot s}$ is applied in this work.

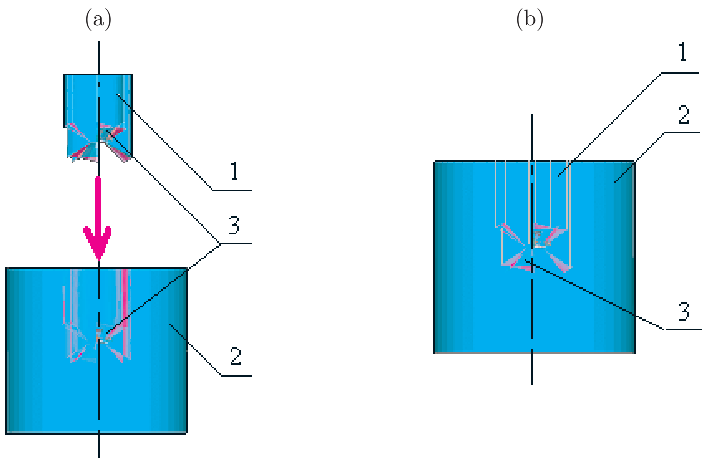

Figure 3. Model geometry creating: (a) separated fluid volumes; (b) final geometry (1 – volume no. 1, 2 – volume no. 2, 3 – contact areas)

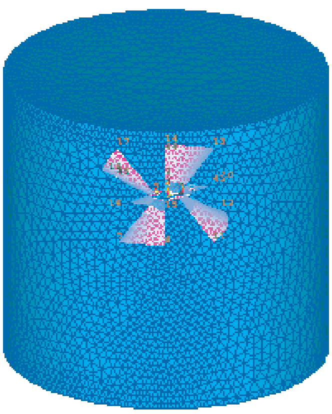

Figure 4. Finite element mesh

(C) *Boundary conditions*

1. Bioreactor bottom:

$y = -\frac{1}{2}h_b - h_1;\ \ 0^\circ \le \alpha \le 360^\circ;\ \ 0 \le r \le R_r;\ \ V_x = V_y = V_z = 0;$

$$w = \begin{cases} 0 \\ w_1 \\ \dots \\ w_5 \end{cases} \text{(relevant to the revolutions in the experiments);}$$

$T = 273 + 39 = \text{const}.$

2. Biomass surface:

$y = 0.8 \cdot H - \frac{1}{2}h_b - h_1;\ \ 0^\circ \le \alpha \le 360^\circ;\ \ 0 \le r \le R_r;\ \ V_z = 0;\ \ T = 273 + 35 = \text{const}.$

3. Bioreactor walls:

$r = R;\ \ 0^\circ \le \alpha \le 360^\circ;\ \ -\frac{1}{2}h_b - h_1 \le y \le H - \frac{1}{2}h_b - h_1;\ \ V_x = V_y = V_z = 0;$

$$w=\begin{cases}0\\ w_1\\ \dots\\ w_5\end{cases}\quad \text{(relevant to the revolutions in the experiments);}$$

$T=273+37.5=\text{const}.$

4. Surfaces of the propeller blades:

   $\frac{\pi}{4}\le\alpha\le 0.475\pi;\;\; g\frac{\pi}{6}\le\varphi\le g\frac{\pi}{6}+\frac{\pi}{3};\;\; r_{\le}\le r\le r_2;$

   $g=1,5,9{:}\; (r-r_1)\sqrt{\dfrac{d-r_1}{r_2-r_1}-1}-y=0;\quad g=3,7,11{:}\; (r-r_1)\sqrt{\dfrac{d-r_1}{r_2-r_1}-1}+y=0;$

   the epicycloid restrictions about r are:

   $$r_1=\frac{a\sqrt{3}}{2\sin(\frac{\pi}{6}(2-n)+\varphi)};\quad r_2=\frac{1}{\sin\varphi}\left[(R'+r')\sin\alpha-r'\sin\left(\frac{R'+r'}{r'}\alpha\right)\right];$$

   $V_x=V_y=V_z=0.$

5. Biomass volume:

   $0\le r\le R_r;\;\; -\frac{1}{2}h_b-0.05\le y\le H-\frac{1}{2}h_b-h_1;\;\; 0^{\circ}\le\alpha\le 360^{\circ};\;\; T=273+37=\text{const}$ (reference temperature).

   A steady-state process of stirring and heat transfer is modeled due to the extremely short time for reaching the steady-state regime.

4. Results of numerical experiments

Visualizations of heat and velocity fields in the bioreactor, obtained for some rotation velocity of the stirrer $(w_1,w_3,w_5,w=0)$ are shown in Figures 5–18.

The average values of convection heat transfer coefficients, h_c, from the surroundings the bioreactor walls towards the biomass volume and the average viscosity values, μ_m, are presented in Table 1. The Nu number and Re number have been determined according to the aims of the present paper.

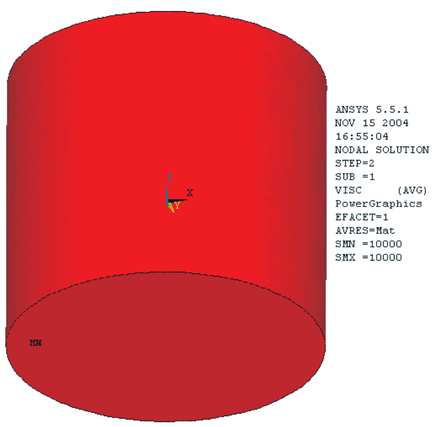

Figure 5. Viscosity, μ, distribution in the fluid volume, $w=0\,\text{rad}\cdot\text{s}^{-1}$

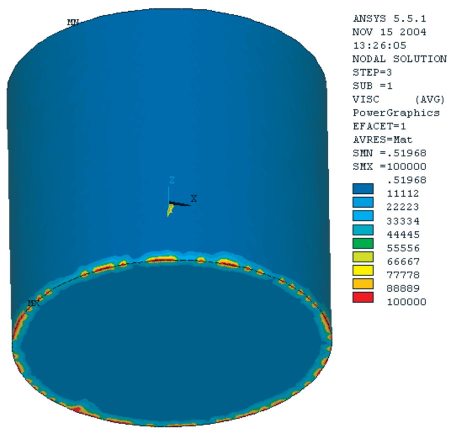

Figure 6. Viscosity, μ, distribution in the fluid volume, $w_1=5\,\text{rad}\cdot\text{s}^{-1}$

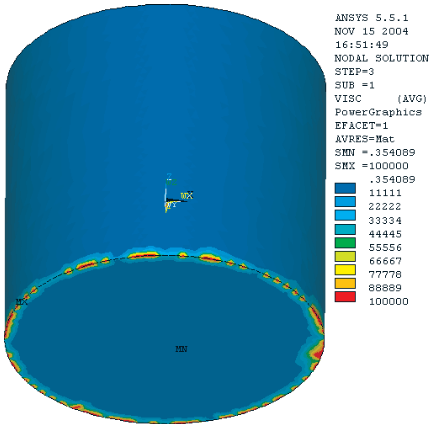

Figure 7. Viscosity, μ, distribution in the fluid volume, $w_3 = 10\,\mathrm{rad\cdot s^{-1}}$

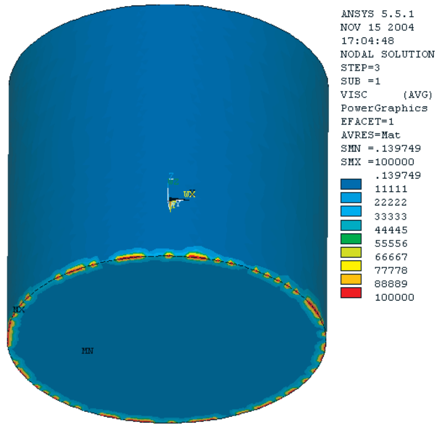

Figure 8. Viscosity, μ, distribution in the fluid volume, $w_5 = 15\,\mathrm{rad\cdot s^{-1}}$

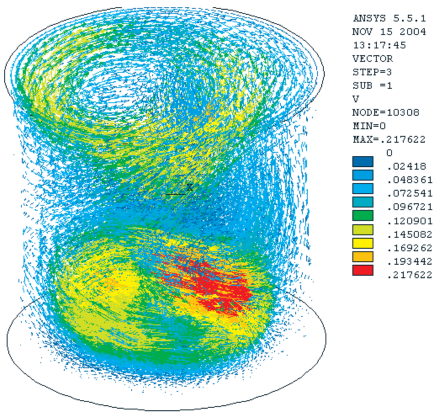

Figure 9. Velocity vector field, $w_1 = 5\,\mathrm{rad\cdot s^{-1}}$

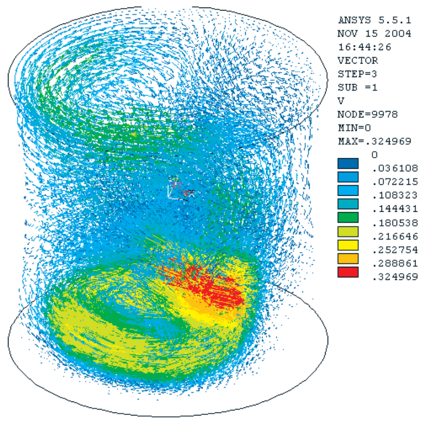

Figure 10. Velocity vector field, $w_3 = 10\,\mathrm{rad\cdot s^{-1}}$

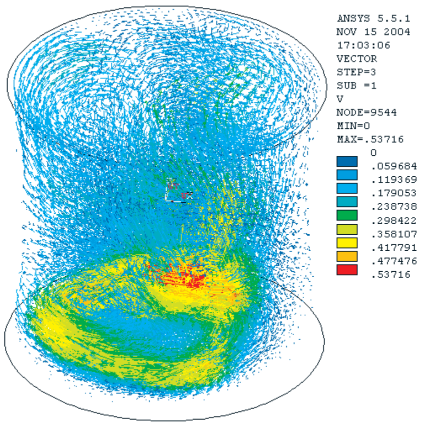

Figure 11. Velocity vector field, $w_5 = 15\,\mathrm{rad\cdot s^{-1}}$

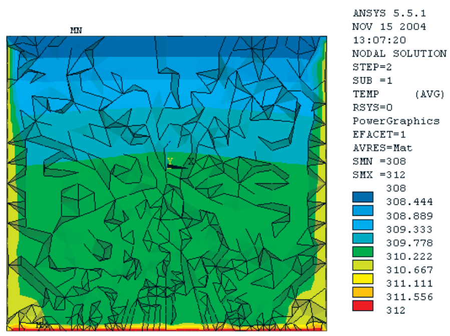

Figure 12. Temperature field in a vertical cross-section, $w = 0\,\mathrm{rad\cdot s^{-1}}$

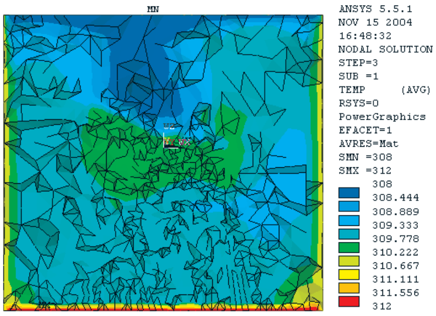

Figure 13. Temperature field in a vertical cross-section, $w_3 = 10\,\mathrm{rad\cdot s^{-1}}$

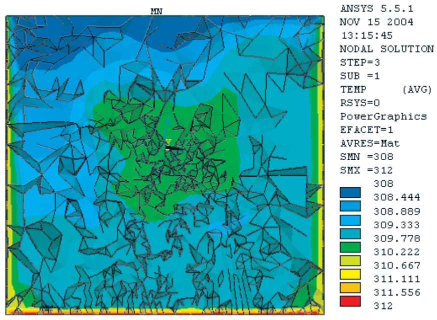

Figure 14. Temperature field in a vertical cross-section, $w_1 = 5\,\mathrm{rad\cdot s^{-1}}$

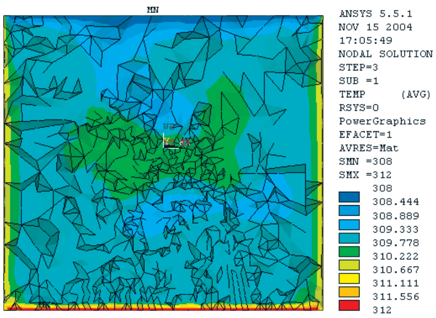

Figure 15. Temperature field in a vertical cross–section, $w_5 = 15\,\mathrm{rad\cdot s^{-1}}$

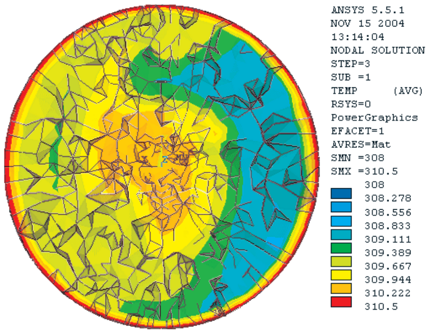

Figure 16. Temperature field in a horizontal cross-section, $y = 0$, $w_1 = 5\,\mathrm{rad\cdot s^{-1}}$

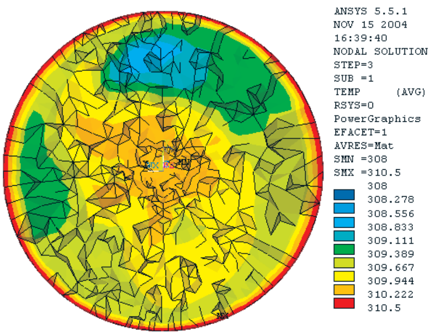

Figure 17. Temperature field in a horizontal cross-section, $y = 0$, $w_3 = 10\,\mathrm{rad\cdot s^{-1}}$

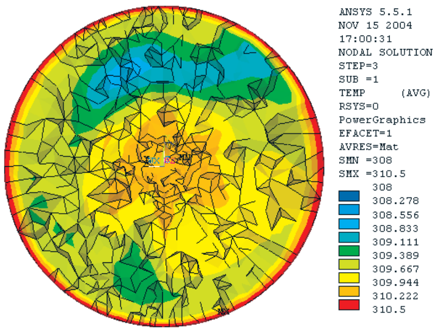

Figure 18. Temperature field in a horizontal cross section, $y = 0$, $w_5 = 15\,\mathrm{rad\cdot s^{-1}}$

The following inferences and conclusions have been made, based on the results of the numerical investigation:

1. A lack of natural convection ($V_x = V_y = V_z = 0$) and zero value of the convection heat transfer coefficient, h_c, (Table 1) have been ascertained in the case without

Table 1. Heat and fluid flow parameters

Revolution [min^{-1}]	Re	h_c [$\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$]	Nu	μ_m [Pa·s]
0	0	0	0	10000
50	1500	659	127.73	5.3
75	2250	681	132.11	3.75
100	3000	700	135.71	2.92
125	3750	715	138.5	2.2
150	4500	731	141.8	1.06

biomass stirring under determined temperature boundary conditions, due to the non-Newtonian nature of the biomass. When the stress is below the plastic level ($\tau < \tau_0$), the biomass' viscosity $\mu \rightarrow \infty$ under real conditions, and μ assumes the value of Newtonian viscosity from the "bi-viscosity" Bingham model: $\mu = \mu_n$ in the numerical model (see Figure 5).

2. High viscosity values (Figures 6–8) at the reactor's bottom have been observed at certain stirrer revolutions. They are due to the low strain rates (velocity gradients) near the bottom. These parts of the biomass volume are the model of the precipitation and settling zones observed in real bioreactors.
3. The volumes of settling zones decrease with increased stirring velocities (Figures 6–8). This relation is observed both in the model and in the laboratory biogas installation.
4. The highest velocities have been obtained in the reactor areas near the scoop-up parts of the propellers in the experiments with biomass stirring. Lower velocities have been obtained near the reactor walls (Figures 9–11).
5. The convection heat transfer coefficients walls/fluid increase with the stirring velocity (see Table 1). This is a result of increased biomass velocities and decreased biomass precipitation ability when the stirrer revolutions increase.
6. The temperature field in the biomass volume is differs from experiment to experiment. A significant temperature gradient directed from the heat source to the unheated surface has been observed in one of experiments without stirring (see Figure 12). The uniformity of temperature distribution in the biomass volume increases with the rotation velocity during the stirring experiments (see Figures 13–18).
7. The stirring process has a significant influence upon the temperature field and heat transfer in the bioreactor. This is apparent both from the lack of natural convection and the unsteady heat field observed in the biomass volume during experiments without stirring.

Experimental data obtained from the laboratory biogas installation are used for verification of the model and comparisons are made with:

– temperature values at fixed points of the biomass volume in the mathematical model and the laboratory bioreactor,
– parts of the biomass volume where precipitation exists in the laboratory bioreactor and $\mu \approx \mu_n$ in the model.

Reliability of the mathematical model has been confirmed by the implemented qualitative and quantitative verification and the results from Table 1 have been used to create a calculation method conformity with the initial investigation aims.

5. A criterial equation for convective heat transfer calculation

A relationship between the stirrer revolutions n and the convection heat transfer coefficients h_c is worked out, based on the above positive estimation of the biomass stirring, and some similar views, given by another authors [1, 3]. According to various authors' investigations into stirring of non-Newtonian, Bingham fluids the expression below is accepted as such a relationship:

$$\mathrm{Nu} = c_1 \cdot \mathrm{Re}^{c_2} \cdot \left(\frac{\mu_0}{\mu}\right)^{c_3}. \tag{2}$$

The constants, $c_1 = 75.854$, $c_2 = 0.08$, $c_3 = 0.011$, are determined by the results in Table 1:

$$\mathrm{Nu} = 75.854 \cdot \mathrm{Re}^{0.08} \cdot \left(\frac{\mu_0}{\mu}\right)^{0.011}. \tag{3}$$

The analytical viscosity calculation $\mu = \mu_0 + \tau_0/D$ in Equations (2) and (3) is a difficult one, as the velocity components in the different directions do not equal 0 (Figures 9–11) and calculation of the average strain rate, D, for the biomass volume is quite complicated.

The biomass velocities are: $V = 0$ along the bioreactor's walls and bottom according to the initial boundary conditions, and $V = V_{\max} = f(D_r)$ along the surrounding edges of blades. For every part of the biomass volume in the radial and axial direction D may be determined approximately from the following expression:

$$D_{ri} = \frac{V_{ri\max} - 0}{\Delta r_i}, \quad D_{yi} = \frac{V_{yi\max} - 0}{\Delta y_i}. \tag{4}$$

However, all dimensions of the investigated bioreactor (Figure 1) are related with bioreactor diameter D_r by coefficients a_1, a_2 and a_3. After using Equation (4) for the various parts of the biomass volume and some mathematical transformations, the expression below, written in a generalized form, is obtained for strain rate calculation:

$$D_i = n \cdot f(a_1, a_2, a_3). \tag{5}$$

For example, in the radial direction, when $-\frac{1}{2}h_p \le y \le \frac{1}{2}h_p$:

$$f(a_1, a_2, a_3) = \begin{cases} \pi & \text{if } 0 \le r \le r_p, \\ \dfrac{2\pi a_1}{1 - a_1} & \text{if } r_p \le r \le R_r. \end{cases} \tag{6}$$

It follows from Equation (5) that the average velocity gradients for the biomass volume depend on the stirrer's revolutions, n, and values chosen for a_1, a_2 and a_3, but it does not depend on the bioreactor's dimensions:

$$D = n \cdot f'(a_1, a_2, a_3). \tag{7}$$

The latter relation enables us to develop an expression from the data in Table 1, $\mu = f(n)$, which can be used to calculate viscosity in bioreactors of geometries similar to that discussed in the present paper.

It follows from expressions (1) and (7) and, additionally, the following conditions:

$$\lim_{n\to 0}\left(\mu_0+\frac{\tau_0}{D}\right)=\infty,$$
$$\lim_{n\to\infty}\left(\mu_0+\frac{\tau_0}{D}\right)=\mu_0, \tag{8}$$

that the $\mu=f(n)$ relations can be assumed as:

$$\mu=\frac{b}{n}+\mu_0. \tag{9}$$

Such an expression is determined by using the results of Table 1 and regression:

$$\mu=\frac{270.5}{n}+0.16. \tag{10}$$

The last relationship is shown graphically in Figure 19.

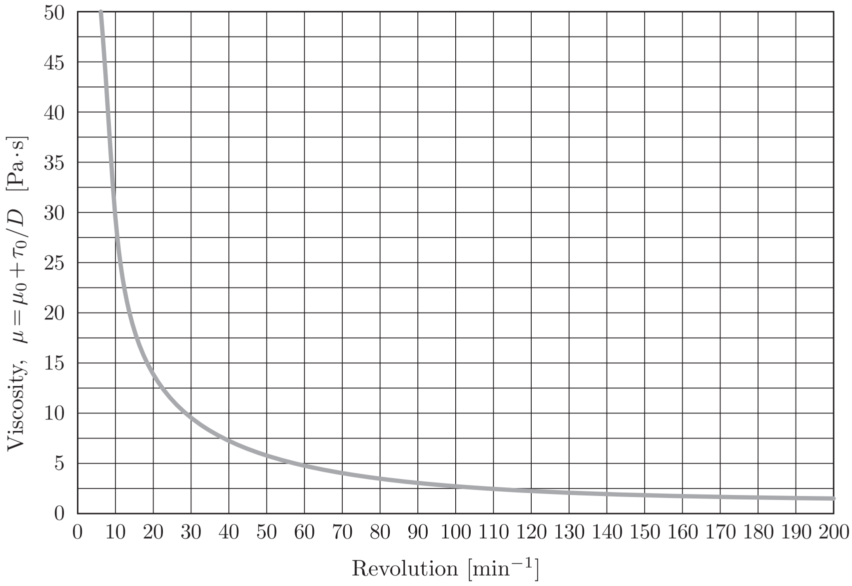

Figure 19. Relationship between the stirring revolution and biomass viscosity

The difference between viscosity values listed in Table 1 and those calculated with expression (10) is less than 5%. Expression (10) and the graph in Figure 19 are convenient for easy and correct Bingham viscosity μ determination. The value of μ thus determined and expression (3) can be used for convective heat transfer calculations in bioreactors of geometries similar to that discussed in the present paper.

6. Conclusions

1. The positive influence of stirring on heat transfer and the temperature field in biomass, with a tendency to precipitation, have been proven by mathematical modeling and computer simulation of heat transfer and fluid flow processes in a bioreactor.
2. A heat transfer calculation method suitable for bioreactors with a stirring frame and biomass composed of lime fleshing and milk whey has been developed using the results of the numerical investigation.

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