FREE VIBRATIONS ANALYSIS OF SHROUDED BLADED DISCS WITH ONE LOOSE BLADE

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Abstract: As a failure of rotor blades in a gas turbine was reported, the rotating-mode shapes of flexible shrouded bladed disc assemblies were calculated using a finite element approach. Rotational effects, such as centrifugal stiffening were accounted for, and all couplings between the flexible parts were allowed. The spin softening effects were neglected. A dynamic analysis of the shrouded bladed disc demonstrated that it was designed with a sufficient safety region of resonance. In case of one blade getting loose, it is vibrating in resonance.

Keywords: blade, shrouded bladed disc, free vibration

1. Introduction

Blade failure is a common problem of steam and gas turbines. On 19 November 1968, RMS "Queen Elizabeth 2" suffered HP turbine blade failures during her voyage [1]. Another major accident occurred on 22 August 1977, after a major overhaul of a 600MW turbo-set in Porcheville [1]. The failure of the last stage of 235MW steam turbine blades in a nuclear power plant in Narora, India, was reported in 1993 [1]. This paper is concerned with a failure of the last stage of the gas turbine of a combat aircraft.

Since blade failures are predominantly vibration-related, the determination of natural frequencies and mode shapes has been studied by several researchers specializing in this problem [2, 3].

In this paper, natural frequencies and mode shapes of an industrial cantilever blade and a shrouded bladed disc with 90 blades are calculated. The calculations demonstrate that when one blade gets loose the system is in resonance and failure occurs.

2. Description of the model

A 3D finite element code was used to calculate the natural frequencies for non-rotating and rotating blades (see Figure 1). The length of a blade was 0.193m. A blade was assumed to be made of EP 109 steel with an Ultimate Tensile Strength of 750MPa and Young's Modulus of 1.63MPa at 850°C. A twenty-node isoparametric solid-element mesh was used in the FEM analysis. Centrifugal effects were included in the free vibration analysis. Gyroscopic effects were neglected, due to the lack of a relevant model in the ANSYS code.

The natural frequencies for a blade fixed in the root cross-section without the shroud and root geometry (see Figure 1) when it is static and when it is rotating at a temperature of 850°C, obtained in our first step, are presented in Table 1. Figure 1 presents the Misses stress caused by rotation with 8400rpm. The maximal value of stress equal to 239MPa is visible in the lower part of the blade.

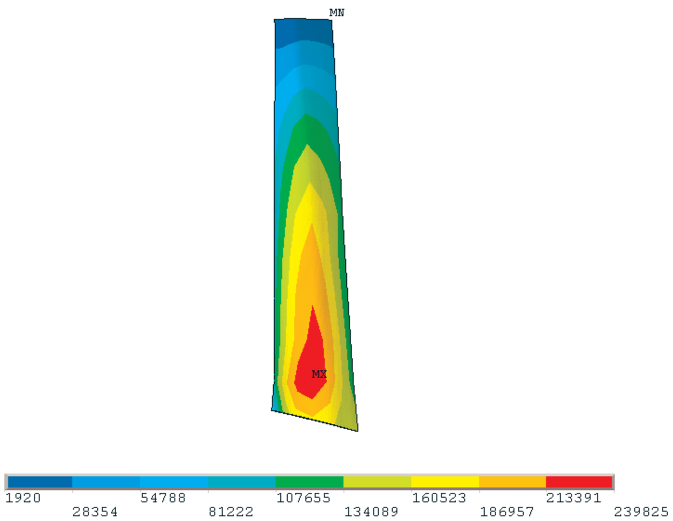

Figure 1. The Huber-von Misses stress in elastic domain

In the next step, the shroud was modeled (see Figure 2). The natural frequencies of a single blade with a part of the shroud were calculated for various angular velocities of the blade and the temperature of 850°C (see Table 2).

Figure 2 presents the von Misses stress caused by rotation with 8400rpm. The maximal value is equal to 417MPa and can be seen in the upper part of the blade.

A considerable difference between the natural frequencies is noticeable in the case of modeling a part of the shroud, as the stiffness of the system was changed. The frequencies decreased. The blade is fixed in the lowest cross-section area of the blade.

The fir-tree root and the bridge portion part were constructed in the third step (see Figure 3).

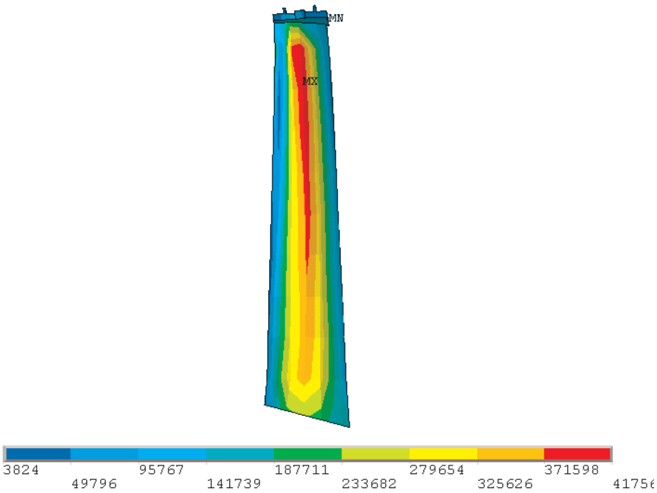

Figure 2. The Huber-von Misses stress in elastic domain

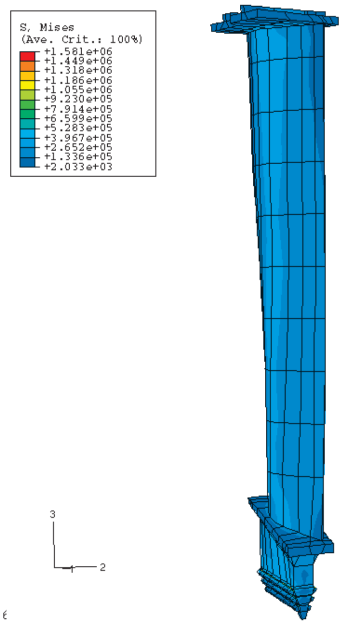

Figure 3. Model of a blade with root and shroud shown in an FE mesh

The natural frequencies of a single blade with a part of the shroud and root were calculated for various angular velocities of the blade and boundary conditions (see Table 3). In one case the blade was assumed to be fixed only in the first root landing and in another – at three root landings. The way in which the blade was fixed in the root area apparently did not affect the frequencies very much, but the

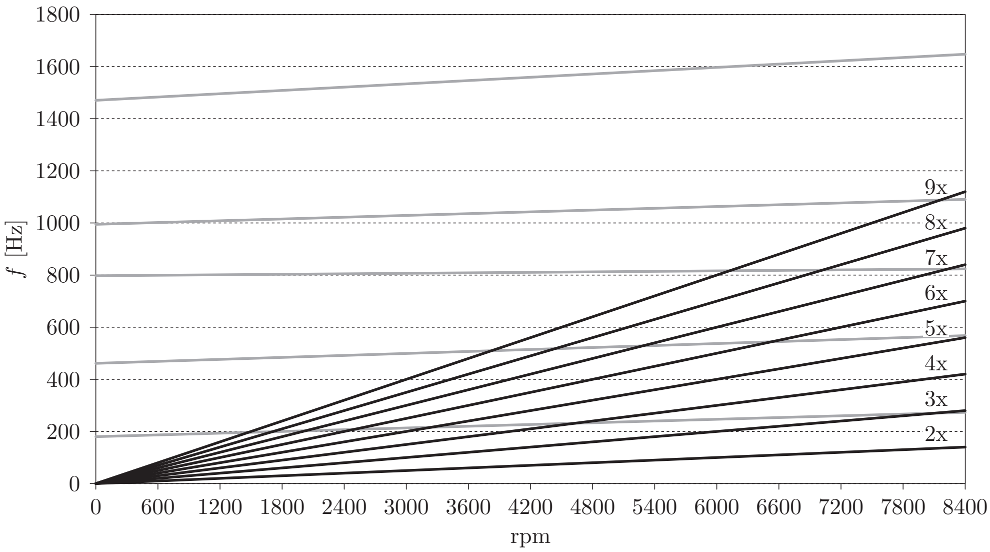

Figure 4. Campbell diagram for a single blade fixed at three root landings

Table 1. Natural frequencies of a single blade without the shroud and root area

No.	0rpm	8400rpm
	$E = 1.63 \cdot 10^8$ MPa	$E = 1.63 \cdot 10^8$ MPa
	[Hz]	[Hz]
1	300.18	408.07
2	776.48	853.16
3	1367.1	1400.6
4	1483.4	1565.8
5	2298.2	2399.6
6	2992.4	3064.0
7	3788.4	3891.6

Table 2. Natural frequencies of a single blade with a part of the shroud

No.	0rpm	8400rpm
	$E = 1.63 \cdot 10^8$ MPa	$E = 1.63 \cdot 10^8$ MPa
	[Hz]	[Hz]
1	205.67	336.54
2	542.20	639.18
3	815.42	839.72
4	1149.0	1282.5
5	1827.4	1963.9
6	2349.8	2424.9
7	3066.3	3190.1
8	3724.2	3826.0
9	4253.9	4331.1
10	5154.7	5277.6

Table 3. Natural frequencies of a cantilever blade with a part of the shroud and root

No.	0rpm	8400rpm
	$E = 1.63 \cdot 10^8$ MPa	$E = 1.63 \cdot 10^8$ MPa
	Fixed at three root landings	Fixed at three root landings
	[Hz]	[Hz]
1	179.91	306.00
2	462.20	570.91
3	801.85	823.82
4	996.91	1111.8
5	1476.1	1637.8
6	2275.3	2327.5
7	2636.9	2784.4

frequencies of a blade with the root area differed considerably from those of the previous cases.

The natural frequencies of a cantilever blade were under resonance conditions $0.9\,kn \le H_k \le 1.1\,kn$, where n is the frequency of rotation and k is the engine order excitation. For the blade frequencies presented in Table 3 and Figure 4, fixed at three root landings, the margins of resonance were $252\,\mathrm{Hz} \le H_2 \le 308\,\mathrm{Hz}$ and $506\,\mathrm{Hz} \le H_4 \le 616\,\mathrm{Hz}$.

Figure 5 presents the Misses stress for a blade fixed on three roots caused by rotation with 8400rpm. The maximal value is equal to 975MPa and found on the first root, near the trailing edge. The increase of stress causes the way of fixing the blade on the small area in the disc.

In the next step, the blade was fixed at three root landings and additionally in the shroud. These boundary conditions correspond to the nominal work of the blade. Natural frequencies for this case are presented in Table 4.

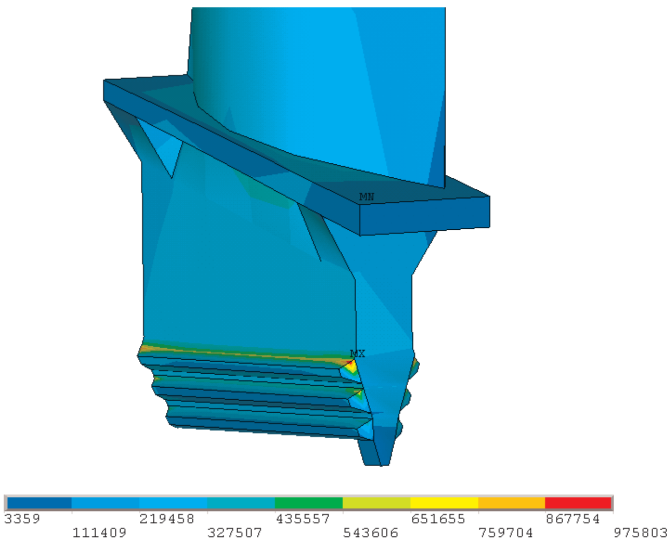

Figure 5. The Huber-von Misses stress for a blade fixed at three root landings

Table 4. Natural frequencies of a single blade fixed at three root landings and in the shroud

No.	0rpm	8400rpm
	[Hz]	[Hz]
1	969.66	1007.5
2	1505.8	1529.6
3	2194.3	2217.8
4	2922.4	2938.8
5	3174.4	3206.5
6	4100.4	4145.1
7	4536.5	4562.8
8	6239.8	6277.7
9	6342.6	6401.8

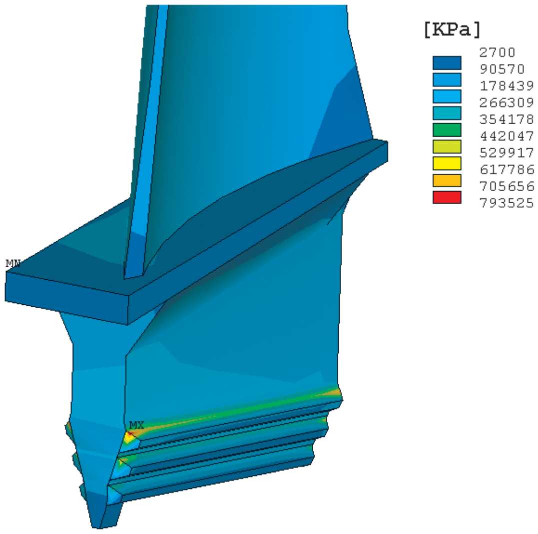

Figure 6. The von Misses stress for a blade fixed at three roots and in the shroud for 8400rpm

The von Misses stress for a blade fixed at three roots and in the shroud is presented in Figure 6. Fixing the shroud decreases the maximal stress due to the centrifugal forces to 793MPa.

The influence of the disc on the dynamic behavior of the blades is very important. The natural frequencies of the disc (see Figure 7) are presented in Table 5.

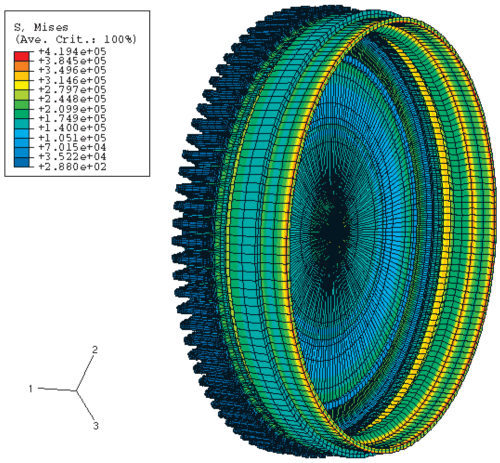

Figure 7. FEM model of the disc

The bladed disc contained 90 shrouded blades (see Figure 8). Its main dimensions were as follows: outer diameter of the disc, $r = 0.2238$m, and blade height, $L = 0.193$m. The blades were staggered and had asymmetrical cross-sections. In the FEM model and free vibration analysis, only $1/90^{\text{th}}$ of the bladed disc assembly was meshed. Isoparametric brick elements with 8 nodes and 3 degrees of freedom per node were used (645 elements for the bladed disc in one sector, 28656 D.O.F.). In our case, disc assemblies had to be analyzed containing 90 tuned turbine blades cir-

Table 5. Natural frequencies of the disc for 0 rpm and 8400 rpm

No.	0 rpm	8400 rpm
	[Hz]	[Hz]
1	1063.7	1091.3
2	1303.8	1328.5
3	2521.5	2548
4	3379.8	3412.6
5	3771.1	3812.9
6	4164.6	4215.6
7	4680.8	4738.2
8	5362.4	5423.1
9	6209.2	6271.1
10	7203.8	7265.7
11	8324.6	8385.5
12	9549.8	9609.3
13	10 859	10 917

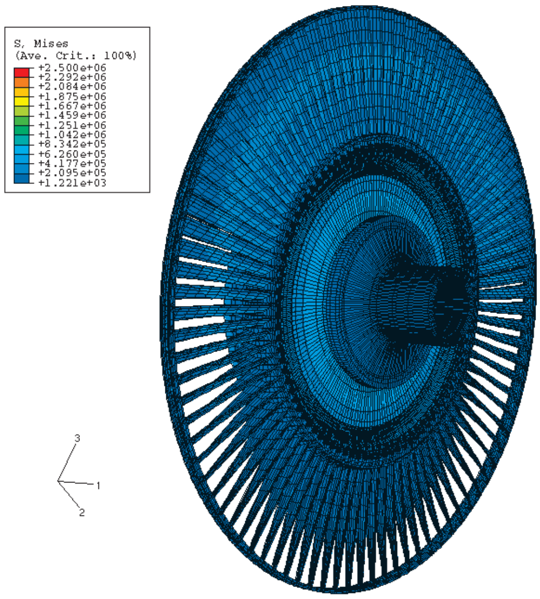

Figure 8. Model of the bladed disc

cumferentially coupled by an elastic rotor and a shroud. Under these conditions, the disc assembly was a rotationally periodic structure of identical blades and the cyclic wave theory could be applied. Centrifugal effects were included in the free vibration analysis. Gyroscopic effects were neglected due to the lack a relevant model in the ANSYS code. The natural frequencies computed for all the possible nodal diameters are shown on the Interference diagram presented in Figure 9 and, partially, in Table 6. The CPU time used for the numerical calculations was 3 hours. The modes of the bladed disc were classified by analogy to axisymmetric modes, which are mainly characterized by nodal lines lying along the structure's diameters and constant angular spacing. There were zero ($k=0$) (see Figure 10, on the left), one ($k=1$) (see Figure 10, in the middle), two ($k=2$) (see Figure 10, on the right), or more ($k>2$) nodal diameter bending or torsion modes. Series 1 is associated with the first natural frequency of a single blade fixed at the root and in the shroud (see Table 1). Series 2 is associated with the second natural frequency of a single blade fixed at the root and in the shroud, and so on (k is the number of nodal diameters).

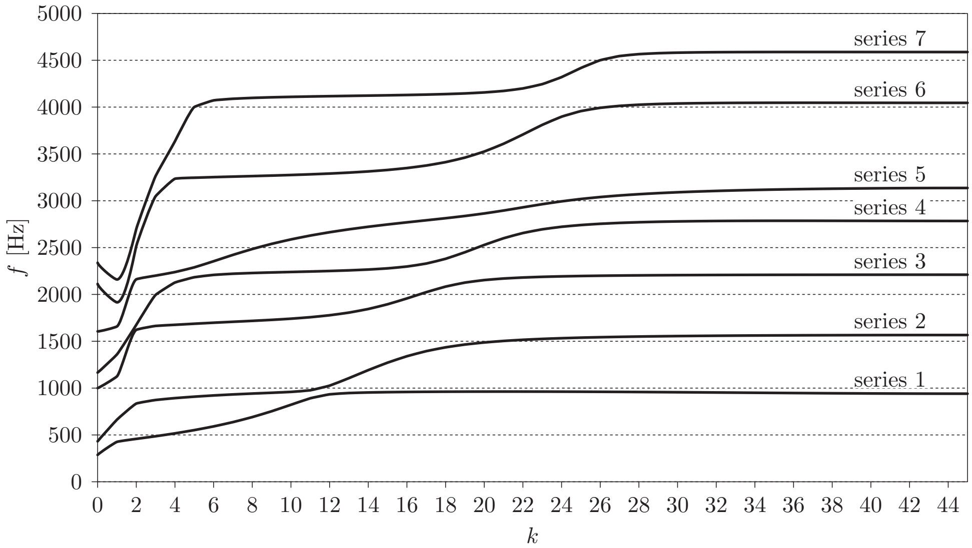

Figure 9. Natural frequencies of a bladed disc of various nodal diameters

Table 6. Natural frequencies of the bladed disc

No.	Full shroud	Nodal diameters	Shroud with one discontinuity	Nodal diameters	With one loose blade	Nodal diameters
1	–	–	–		304.12	Blade
2	321.73	0	323.25	0	323.27	0
3	440.68	1	440.56	1	440.57	1
4	440.68	1	441.69	1	441.57	1
5	448.71	0, 2 series	450.05	0, 2 series	449.94	0, 2 series
6	473.77	2	472.44	2	472.28	2
7	473.77	2	472.69	2	472.46	2
8			487.68	New mode shape	490.31	New mode shape
9	503.50	3	498.76	3	498.01	3
10	503.50	3	499.14	3	499.04	3
11	537.03	4	526.72	4	525.15	4
12	537.03	4	527.72	4	527.22	4
13	–		–		551.42	Blade

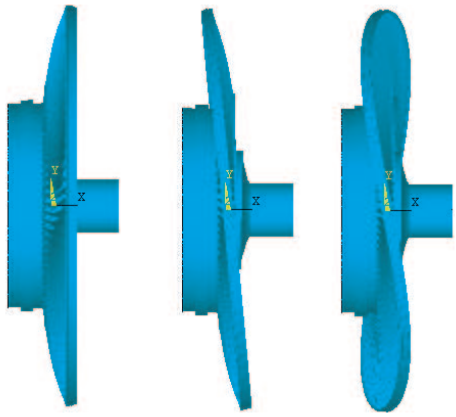

Figure 10. Zero, first and second nodal diameter form

It is apparent from the presented results that the bladed discs' natural frequencies for the first two series are not in resonance.

The blade was fixed at all three root-landing positions in all three translational degrees of freedom. The Huber-von Misses stress field obtained near the root region is shown in Figure 11. The element near the notch root of the first landing was subjected to severe stress. Here, the maximum Huber-von Misses stress was 1450MPa – well beyond the ultimate tensile strength. It occured in a narrow, 3mm zone, which is almost the depth of one element. Beyond this depth the stress field is elastic. This value is higher than in the case of a single blade (793MPa), when the stiffness of the disc was not taken into consideration.

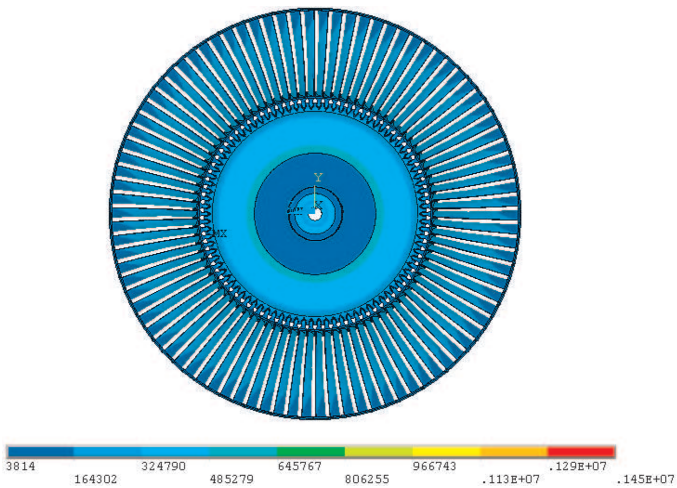

Figure 11. The Huber-von Misses stress in elastic domain

Two cases were considered in order to model the blade failure. In one, the blades' shroud was loosened along one surface (see Figure 12), while in the other the shroud was loosened in two places, so that one blade of the shroud was a cantilever (see Figure 13). Dynamic calculations of the systems were made for the case of loosening

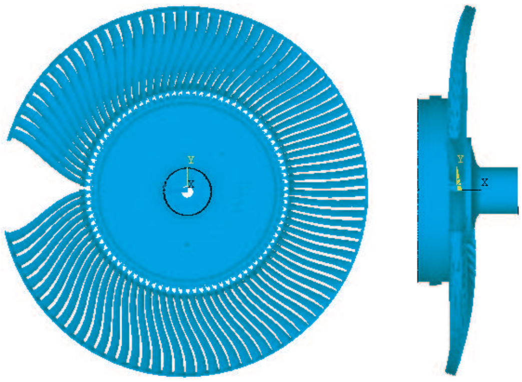

Figure 12. Natural frequencies of the bladed disc with one discontinuity in the shroud at a frequency of 487.68Hz

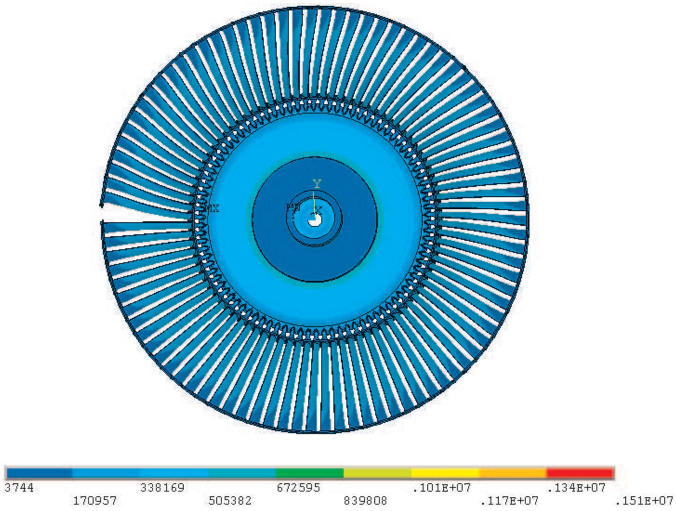

Figure 13. The von Misses stress for the bladed disc with one discontinuity at 8400rpm

of the entire shroud. The natural frequencies of the bladed disc with continuous and discontinuous shroud are presented in Table 6.

In the case of a break in the shroud in just one place, the natural frequencies from the first series are smaller than for the disc with a continuous shroud. Shroud discontinuity introduces a new mode shape of 487.68Hz (see Figure 12). The maximal centrifugal stress of 1510MPa (see Figure 13) is visible in the blade close to the break in the shroud and is smaller than that of the continuous shroud (1450MPa, see Figure 11). Shroud discontinuity in two places introduces another frequency of 304.12Hz, which corresponds to the first cantilever blade frequency (see Figure 14). The mode shape of the 490.31Hz frequency (Figure 14) remains within the disc's plane and is connected with the mode shape of the 487.68Hz frequency of the bladed disc with one discontinuity in the shroud (see Table 6).

The frequency of 323.27Hz (Figure 15 and Table 6) is connected with the 0 nodal diameter mode of the bladed disc with one blade remaining motionless. The

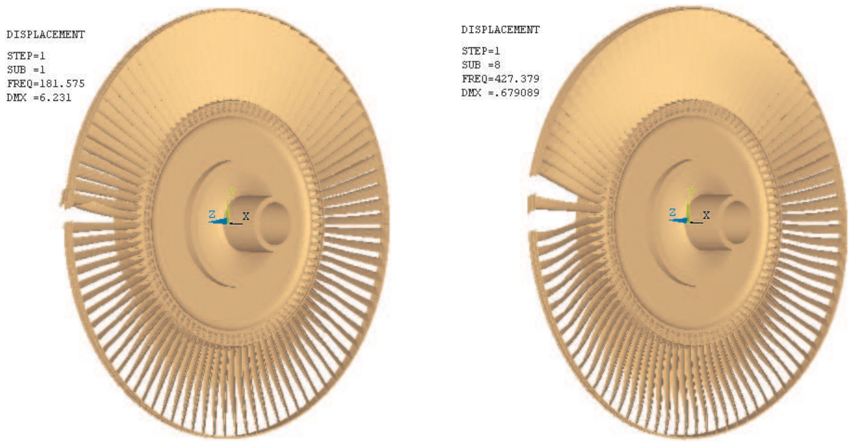

Figure 14. Natural frequencies of a bladed disc with one loose blade at frequencies of 304.12 Hz and 430.31 Hz

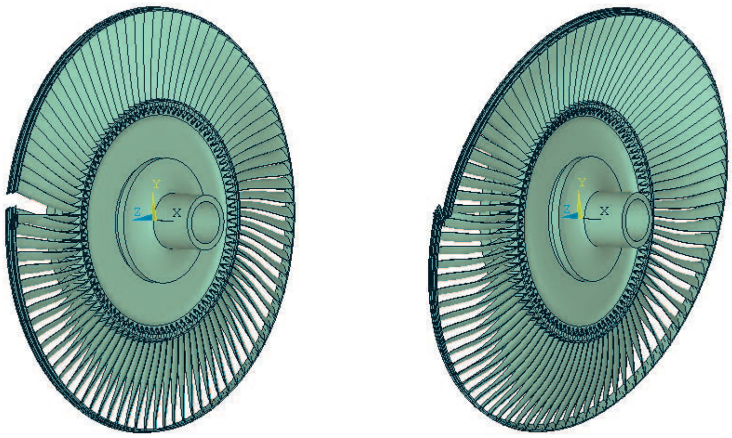

Figure 15. Natural frequencies of a bladed disc with one loose blade at frequencies of 323.27 Hz and 440.57 Hz

bladed disc's mode shape at 440.57 Hz has one nodal diameter with one blade remaining motionless.

The loosening of the shroud decreases the system's frequency values (from the first series) and introduces the splitting of double modes as in the case of blades' mistuning. For example, a system with a continuous shroud has a frequencies of 440.68 Hz (mode shapes with one nodal diameter, see Table 6). If it has one discontinuity in the shroud, these frequencies are split into 440.56 Hz and 441.69 Hz. If it has two discontinuities in the shroud, the split is into 440.57 Hz and 441.57 Hz (see Table 6). The maximal splitting can be observed for the 10 nodal diameters' mode (see Table 6). The discontinuities of the shroud introduce asymmetry into the system, so the splitting of the modes resembles a mistuned system. In the considered case, the interference diagram changes only in the region of disc-dominated modes (number of nodal diameters from 2 to 16, see Figure 16). Diamonds in Figure 16

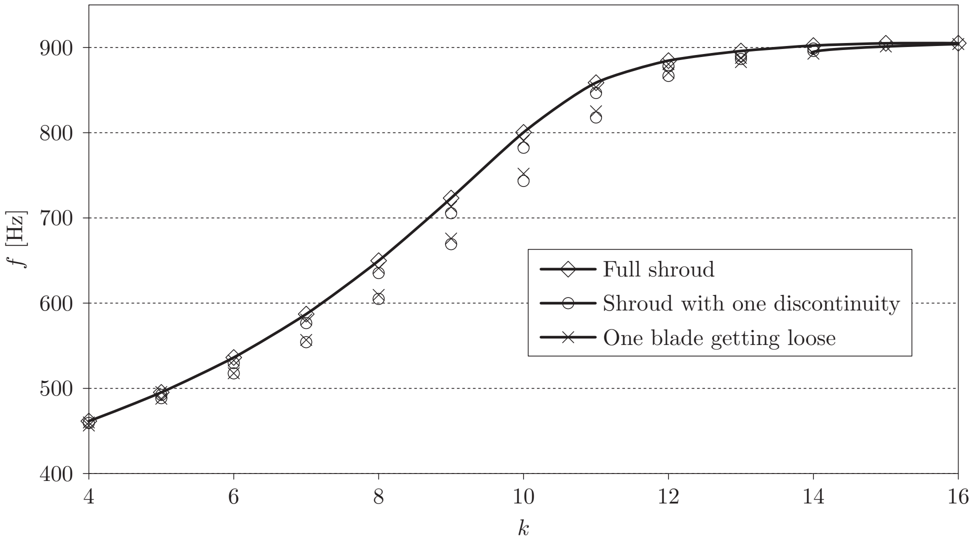

Figure 16. Natural frequencies of the bladed disc with various nodal diameters

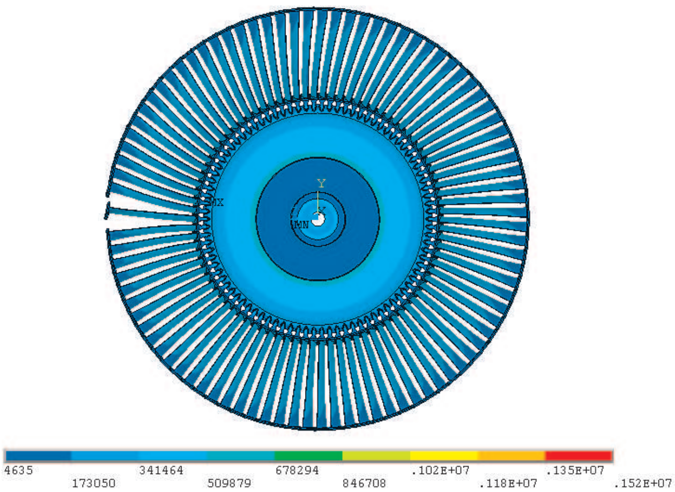

Figure 17. The Huber-von Misses stress in the bladed disc with one loose blade

stand for a continuous shroud, circles represent one discontinuity and crosses – two discontinuities.

The loosening of a blade in the shroud increases the maximal stress in that blade's root to 1520 MPa (Figure 17).

3. Conclusions

This paper is concerned with a failure of the last stage of a gas turbine in a combat aircraft. The natural frequencies, mode shapes and stresses caused by the rotation of an industrial cantilever blade and a shrouded bladed disc with 90 blades were calculated. The calculation has shown that in the case of one blade getting loose the system is in resonance and failure is predictable.

The paper describes general implications of coupling on the free vibration of a bladed discs-shaft system. The calculations have shown the influence of the shaft on the natural frequencies and mode shapes of the discs and the importance of dynamic calculations for a single blade.

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