THE ELASTO-VISCOPLASTIC CHABOCHE MODEL

ANDRZEJ AMBROZIAK AND PAWEŁ KŁOSOWSKI

Department of Structural Mechanics,
Faculty of Civil and Environmental Engineering,
Gdansk University of Technology,
Narutowicza 11/12, 80-952 Gdansk, Poland
{ambrozan, klosow}@pg.gda.pl

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Abstract: The elasto-viscoplastic constitutive equations of the Chaboche model [1] have been developed and modified many times. The aim of the present paper is to present the existing Chaboche model variants and describe its scientific and engineering applications. A compact review of literature on these applications is given, focussed on publications offering a wider and more comprehensive view of the elasto-viscoplastic Chaboche model.

The article is an introduction to a comprehensive investigation of the elasto-viscoplastic Chaboche model.

Keywords: elasto-viscoplastic, constitutive model, Chaboche model

1. Introduction

Computation of structures beyond the yield limit requires taking into account materials' viscosity and hardening properties. Developing realistic mathematical constitutive equations to describe the elasto-viscoplastic material behaviour has been an important objective of research in constitutive modelling for the last decades. Woznica in his work [2], gave a detailed description of the twelve elasto-viscoplastic laws proposed by Aubertin [3], Bodner-Partom [4], Chaboche [1], Freed-Verrilli [5], Krieg-Swearengen-Jones [6], Krempl [7], Korhonen-Hannula-Le [8], Lehmann-Imatani [9], Miller [10], Perzyna [11], Tanimura [12], and Walker [13]. Rapid advances in computational power made it possible to implement these constitutive equations in engineering applications.

2. Applications of the Chaboche model

The finite element method (FEM) has recently become the most powerful approach in structural analysis, applicable in engineering calculations under various conditions. FEM developed rapidly in the last decade and is nowadays a fundamental tool for various problems of the engineering science. Argyris *et al.* presented in their pioneering work [14] a computational technique of solving the viscous flow approach with an Eulerian description of the velocity field using a finite element mesh fixed in space, and solving the solid mechanics approach with a Lagrangian description of the displacement field using an updated material mesh to trace the motion of each particle. Both formulations are illustrated with two forming problems: the extrusion of an aluminium billet through a curved nozzle and forming the head of a steel bolt. Argyris and Doltsinis [15] extended their considerations published in an earlier paper [16] concerning large-strain inelastic phenomena in the dynamic domain. In a homogeneous, natural presentation of the strain and stress states, description of the material's behaviour was discussed on the basis of thermodynamics. An extension of the application range of the simple triangular shell element introduced in [17] to the domain of large inelastic deformations was presented by Argyris *et al.* in [18]. The element is partly based on ideas of physical lumping with a simple mechanical interpretation. Argyris and Doltsinis (see [19] and [20]) examined the numerical properties of an approximation scheme for incremental inelastic stress-strain relations and proposed an integration procedure for a time dependent or independent material response.

The finite element structure computations under cyclic viscoplasticity by Chaboche [21] were discussed on the basis of several actual examples, including notched specimens under repeated loads including hold periods and a tube under tension and a cyclic thermal gradient. Based on an overstress model for elastic-viscoplastic materials, an incrementally formulated FE algorithm was described by Pitzer [22]. The field equations, not translated to the weak formulation, were expressed in the function of the velocity field components and became the groundwork for discretisation. Regarding the precise rendering of stress distributions, the above-mentioned methods were compared to each other with two types of elements: plate (for the plane-stress problems) and axisymmetric elements. Imatani [23] presented the fundamental investigation of the inelastic constitutive relationship for high-temperature materials and its application to finite element analysis. His work was divided into three parts. The first introduced the theoretical formulation of the inelastic constitutive relationship. Inelastic behaviour of high-temperature materials under the biaxial stress state was described in the second. A finite element implementation with the constitutive models employed was carried out in the third.

Kłosowski *et al.* [24] studied the problem of the elasto-viscoplastic dynamic behaviour of geometrically non-linear plates and shells under the assumption of small strains and moderate rotations. A nine-node isoparametric shell element was applied in the finite element algorithm. The Chaboche and Bodner-Partom models were chosen from several types of constitutive laws. To avoid calculating the stiffness matrix, an effective procedure has been applied using the central difference method of solving the equations of motion. The trapezoidal method was used to integrate the constitutive viscoplastic laws. Chellapandi *et al.* [25] examined the modified structural material 9Cr 1Mo (RCC-MR). One of the important material parameters necessary for the use of simplified rules given in RCC-MR was the symmetrisation coefficient, K_S, not yet included in RCC-MR. The K_S values were established from numerous stress-strain cyclic data generated theoretically using the Chaboche viscoplastic model and recommended for application in the RCC-MR. The Chaboche model used 20 material parameters which are identified on the basis of uniaxial monotonic and cyclic data. The published data and ample uniaxial monotonic, cyclic, creep data were compared with the predictions.

The paper [26] by Hartmann *et al.* dealt with two main topics. One of them was the equivalence of stress algorithms, based on a Backward-Euler-step applied to viscoplastic models of the Chaboche type, and their elastoplastic counterpart. The other concerned a special constitutive relation based on a kinematic hardening model using a sum of Armstrong-Frederick terms, equivalent to a multi-surface plasticity model. Furthermore, only the viscoplastic algorithm had to be implemented, since it included the elastoplastic constitutive model as a special case. Furukawa and Yagawa [27] presented a method of identifying the parameter set of inelastic constitutive equations based on an evolutionary algorithm. The method's advantage is that appropriate parameters can be identified even when the measured data are subject to considerable errors and the model equations are inaccurate. Experiments were described with respect to parameter identification of the Chaboche material model under uniaxial loading and stationary temperature conditions. In a finite element viscoplastic analysis program with the Chaboche model, the non-iterative and self-correcting solution method, proposed by Tanaka and Miller [28] was implemented by Chellapand and Alwar in [29]. The computational efficiency of this model was demonstrated by solving a variety of benchmark problems over a wide range of strain sensitivity domain under complex monotonic and cyclic loading histories related to fast breeder reactor applications.

Two parameter identification procedures for linear viscoelastic materials were presented by Ohkami [30]. One method used the incremental constitutive relation for linear viscoelastic materials, whilst the other applied the elastic-viscoelastic correspondence principle. Part of back analysis in both methods was formulated on the basis of the boundary control concept. Two numerical examples were presented to compare the efficiency of both methods. An evaluation of material parameters for the viscoplastic Chaboche and the Bodner-Partom formulations was carried out by Woznica and Kłosowski [31]. In this work, the authors proposed supporting tensile test experiments with numerical simulations. A set of parameters for each formulation was identified for steel and used to calculate the dynamic behaviour of circular plates. The results were compared with experimental data concerning steel plates. Modelling the dynamic behaviour of elasto-viscoplastic structural elements using the Chaboche and Bodner-Partom models was studied by Kłosowski and Woznica [32].

The problem of simulating cyclic loadings made of long-life components was investigated by Kiewel *et al.* [33]. To overcome this problem, the authors extrapolated a complete set of internal variables over a certain number of cycles. To demonstrate the capabilities of the scheme, failure analysis was carried out for a ring combustor of a gas turbine. The material model used was based on the Chaboche model in combination with Kachanov's damage model [34].

Alwar *et al.* [35] described elastic and axisymmetric inelastic analysis. The authors used the classical kinematic hardening model, ORNL, and Chaboche viscoplastic models to assess the creep-fatigue damage for the inner vessel of a pool-type fast reactor. The material properties of 316LN stainless steel at various temperatures were used in their analysis. The results were used to assess creep-fatigue damage and ratcheting. Chaboche [36] analysed aspects of the thermodynamics of local state, focussing on two complementary aspects: the generalized standard models and the micro-macro approach to elastoplasticity. The author revised some of the thermodynamic properties with an internal variable and discussed some properties of constitutive equations deduced from macro-homogenisation tools in the light of thermodynamic aspects.

Kłosowski *et al.* [37] presented the results of experiments on the Panama technical fabric, carried out in order to identify the inelastic properties of the warp and weft, as well as identification techniques based on the least-squares method. The dense net type of a finite element was proposed to mimic the fabric's behaviour in the FEM analysis. The Bodner-Partom and Chaboche viscoplastic models were applied in the description of the warp and weft properties. Material parameters were calculated on the basis of an uniaxial tension test in the warp and weft directions.

These are only a few of the many engineering applications of the Chaboche model. A limited number of applications are available, as they are usually subject to intellectual property restrictions.

3. The Chaboche model equations

In this section a brief review is given of variants of the Chaboche constitutive equations most often used for material modelling in practical engineering applications. It should be noted that the Chaboche model belongs to a group of isotropic constitutive models which can describe the elasto-viscoplastic behaviour of materials.

3.1. Variants of equations of the Chaboche model without damage

The inelastic strain rate, $\dot{\boldsymbol{E}}^I$, of the simple variant of the Chaboche model can be written as:

$$\dot{\boldsymbol{E}}^I = \frac{3}{2}\dot{p}\frac{\boldsymbol{S}' - \boldsymbol{X}'}{J(\boldsymbol{S}' - \boldsymbol{X}')}, \tag{1}$$

where $\dot{p}$ describes the rate of the equivalent plastic strain and has the following form:

$$\dot{p} = \left\langle \frac{J(\boldsymbol{S}' - \boldsymbol{X}') - R - k}{K} \right\rangle^n. \tag{2}$$

The k, R and K, n constants are the initial yield stress, isotropic hardening and two material parameters, respectively. Tensors $\boldsymbol{S}'$ and $\boldsymbol{X}'$ are the deviatoric parts of stress and back stress tensors. The $J(\boldsymbol{S}' - \boldsymbol{X}')$ invariant is calculated form the following formula:

$$J(\boldsymbol{S}' - \boldsymbol{X}') = \sqrt{\frac{3}{2}(\boldsymbol{S}' - \boldsymbol{X}') : (\boldsymbol{S}' - \boldsymbol{X}')}. \tag{3}$$

The evolution of the kinematic hardening rate, $\dot{\boldsymbol{X}}$, is defined by:

$$\dot{\boldsymbol{X}} = \frac{2}{3}\, a\, \dot{\boldsymbol{E}}^I - c\, \boldsymbol{X}\, \dot{p}, \tag{4}$$

while $\dot{R}$ is the isotropic hardening rate calculated form the following equation:

$$\dot{R} = b(R_1 - R)\dot{p}. \tag{5}$$

Applications of this basic variant of the Chaboche model in the MSC.Marc commercial program are presented in the papers [38] and [39]. In both papers, the

UVSCPL [40] user's subroutine is used to introduce the elastic-viscoplastic Chaboche equations into the MSC.Marc system.

It should be noted that the variants of the Chaboche model presented in this paper, are based on the assumption of additive strain rates,

$$\dot{\boldsymbol{E}} = \dot{\boldsymbol{E}}^{E} + \dot{\boldsymbol{E}}^{I}. \tag{6}$$

Material parameters can be found in the literature, *e.g.* for steel (Table 1).

Table 1. Parameters of steel for the Chaboche mode

	Steel 400°C [27]	St37 20°C [41]	St37 20°C [42]	Steel 316 20°C [43]	Steel 316 20°C [44]	Steel 20°C [45]
E [MPa]	160 000.0	168 600.0	113 066.0	196 000.0	200 000.0	223 000.0
ν [–]	0.3	0.3	0.3	0.3	0.3	0.3
k [MPa]	96.0	167.88	180.0	82.0	80.0	210.15
n [–]	5.0	4.22	8.15	24.0	4.55	9.51
K [MPa·s$^{1/n}$]	50.0	63.12	11.45	151.0	85.2	14.085
b [–]	100.0	0.0	0.0	8.0	21.3	16.74
R_1 [MPa]	0.05	0.0	0.0	60.0	436.0	–138.48
a [MPa]	2000.0	2500.0	98 939.30	162.4	93.57	611 700.0
c [–]	300.0	20.3	1533.41	2800.0	843.0	38 840.0

The proposed modification of the Chaboche model's basic variant was described by Imatani in [23], where the kinematic and isotropic hardening equations were verified. In this concept, the kinematic back stress, $\boldsymbol{X}$, was divided in two elements:

$$\boldsymbol{X} = \sum_{k=1}^{2} \boldsymbol{X}_{(k)} = \boldsymbol{X}_{(1)} + \boldsymbol{X}_{(2)}. \tag{7}$$

Evolution for the $\boldsymbol{X}_{(1)}$ and $\boldsymbol{X}_{(2)}$ elements was given as follows:

$$\begin{aligned} \dot{\boldsymbol{X}}_{(1)} &= \frac{2}{3}\, a_1\, \dot{\boldsymbol{E}}^{I} - c_1\, \dot{p}\, \boldsymbol{X}_{(1)} - \beta_1 \left(J_2\left(\boldsymbol{X}_{(1)}\right)\right)^{r_1-1} \boldsymbol{X}_{(1)}, \\ \dot{\boldsymbol{X}}_{(2)} &= \frac{2}{3}\, a_2\, \dot{\boldsymbol{E}}^{I}, \end{aligned} \tag{8}$$

while the isotropic variable was assumed to be:

$$\dot{R} = b\,(R_1 - R)\,\dot{p} - q_1\, R^{q_2}. \tag{9}$$

The material parameters for stainless steel at 650°C and for $2\,\frac{1}{4}\mathrm{Cr}-1\mathrm{Mo}$ steel at 600°C can be found in [23] and are shown in Table 2.

The next modification of the Chaboche model was proposed by Yaguchi *et al.* [46], where alternative forms of the equivalent plastic strain and kinematic hardening equations were introduced. In this case, the equivalent plastic strain was specified as:

$$\dot{p} = \left\langle \frac{J\,(\boldsymbol{S}' - \boldsymbol{X}')}{K} \right\rangle^{n}, \tag{10}$$

and kinematic hardening was described as:

$$\dot{\boldsymbol{X}} = \frac{2}{3}\; a\, \dot{\boldsymbol{E}}^{I} - c\, \boldsymbol{X}\, \dot{p} - \beta_1\, \left(J_2\,(\boldsymbol{X})\right)^{r_1-1}\, \boldsymbol{X}, \tag{11}$$

Table 2. Parameters for the Chaboche model [23]

	Stainless steel 650°C	$2\frac{1}{4}$Cr−1Mo 600°C
E [MPa]	145 000	155 000
ν [–]	0.3	0.3
k [MPa]	129.0	75.0
n [–]	8.03	8.54
K [MPa·s$^{1/n}$]	103.0	230.0
b [–]	25.0	63.0
R_1 [MPa]	73.6	–12.0
q_1 [–]	$1.1\cdot10^{-18}$	0.0
q_2 [–]	8.03	0.0
a_1 [–]	11 743.9	148 025.0
a_2 [–]	491.0	400.0
c_1 [–]	133.0	155.0

where β_1 and r_1 were material constants. The last term of the above equation depicted the static recovery property of the back stress using the power law function. In the same work, the authors considered the anisotropic deformation property and proposed a second rank tensor, $\boldsymbol{Y}$, acting on the back stress in the form of:

$$\dot{\boldsymbol{X}} = \frac{2}{3}\, a\, \dot{\boldsymbol{E}}^I - c\,(\boldsymbol{X}-\boldsymbol{Y})\,\dot{p} - \beta_1\,(J_2(\boldsymbol{X}))^{r_1-1}\,\boldsymbol{X}. \tag{12}$$

It should be noted that Equation (12) is the same as the kinematic hardening rule first proposed by Chaboche and Nouailhas in [47] in order to improve the descriptive capacity in the case of ratchetting. In this case, the evolution of the $\boldsymbol{Y}$ variable $\boldsymbol{Y}$ was expressed as:

$$\boldsymbol{Y} = -\alpha\left(Y_{\rm st}\frac{\boldsymbol{X}}{J(\boldsymbol{X})} + \boldsymbol{Y}\right)(J(\boldsymbol{X}))^{r_1}, \tag{13}$$

where α and $Y_{\rm st}$ were additional material constants, expressing the evolution rate of the $\boldsymbol{Y}$ variable and saturation of $\boldsymbol{Y}$, respectively. The material parameters for IN738LC at 950°C and IN738LC at 950°C are given in Table 3.

Another variant of the Chaboche model was presented by Nouailhas [48] and described by Chellapandi and Alwar [29]. The general expressions of this model introduced a modification to the basic Chaboche equations. In this case, the inelastic strain rate had the form:

$$\dot{\boldsymbol{E}}^I = \frac{3}{2}\,\dot{p}\,\exp\left(\alpha\left\langle\frac{J(\boldsymbol{S}'-\boldsymbol{X}')-R^*-k}{K(R)}\right\rangle^{n+1}\right)\frac{\boldsymbol{S}'-\boldsymbol{X}'}{J(\boldsymbol{S}'-\mathbf{X}')}, \tag{14}$$

while the equivalent plastic strain rate was expressed as:

$$\dot{p} = \left\langle\frac{J(\boldsymbol{S}'-\boldsymbol{X}')-R^*-k}{K(R)}\right\rangle^{n}, \tag{15}$$

where R^* was the product of material constant α_R and isotropic hardening, R:

$$R^* = \alpha_R R. \tag{16}$$

Table 3. Parameters for the Chaboche model [46]

	IN738LC 850°C	IN738LC 950°C
E [MPa]	164 000.0	164 000.0
ν [–]	0.3	0.3
n [–]	4.75	5.645
K [MPa$\cdot$s$^{1/n}$]	1510.0	1156.0
a [MPa]	175 000	175 000
c [–]	500.0	500.0
β_1 [–]	$3.54\cdot10^{-18}$	$5.507\cdot10^{-14}$
r_1 [–]	6.08	4.275
α [–]	—	$5.507\cdot10^{-15}$
Y_{st} [MPa]	—	100

The isotropic hardening rate ($R(t=0)=0$) had the following form:

$$\dot{R}=b(Q-R)\dot{p}+\gamma\,|Q-R|^m\,\mathrm{sign}(Q_R-R), \tag{17}$$

where

$$Q_R=Q-Q_R^*\left(1-\left(\frac{Q_{\max}-Q}{Q_{\max}}\right)^2\right). \tag{18}$$

The material parameter $K(R)$ was determined as:

$$K(R)=K_0+\alpha_K R. \tag{19}$$

Additionally, the plastic strain memory had the form:

$$\dot{Q}=2\,\mu\,(Q_{\max}-Q)\,\dot{q}, \tag{20}$$

where $Q(t=0)=Q_0$ and $\dot{q}$ was the internal variable corresponding to the radius of the memory surface, F, and its centre, $\boldsymbol{\xi}$, as:

$$F=I\left(\dot{\boldsymbol{E}}^I-\boldsymbol{\xi}\right)-q\le 0, \tag{21}$$

where

$$I\left(\dot{\boldsymbol{E}}^I-\boldsymbol{\xi}\right)=\sqrt{\frac{2}{3}\left(\dot{\boldsymbol{E}}^I-\boldsymbol{\xi}\right):\left(\dot{\boldsymbol{E}}^I-\boldsymbol{\xi}\right)}, \tag{22}$$

$$\dot{q}=\eta\,H(F)\,\langle\boldsymbol{n}:\boldsymbol{n}^*\rangle\dot{p},\quad \dot{\xi}=\frac{2}{3}(1-\eta)\,H(F)\,\langle\boldsymbol{n}:\boldsymbol{n}^*\rangle\,p\,\dot{\mathbf{n}}, \tag{23}$$

$$\dot{\mathbf{n}}=\frac{3}{2}\frac{\boldsymbol{S}'-\mathbf{X}'}{J(\boldsymbol{S}'-\boldsymbol{X}')},\quad \boldsymbol{n}^*=\sqrt{\frac{2}{3}\frac{\left(\dot{\boldsymbol{E}}^I-\boldsymbol{\xi}\right)}{I\left(\dot{\boldsymbol{E}}^I-\boldsymbol{\xi}\right)}}. \tag{24}$$

Following paper [23], non-linear kinematic hardening was divided into two parts (see Equation (7)):

$$\begin{aligned}\dot{\boldsymbol{X}}_{(1)}&=\frac{2}{3}a_1\,\dot{\boldsymbol{E}}^I-c_1\,\Phi(p)\,\boldsymbol{X}_{(1)}\dot{p}-\beta_1\left(J_2\left(\boldsymbol{X}_{(1)}\right)\right)^{r_1-1}\boldsymbol{X}_{(1)}\\ \dot{\boldsymbol{X}}_{(2)}&=\frac{2}{3}a_2\,\dot{\boldsymbol{E}}^I-c_2\,\Phi(p)\,\boldsymbol{X}_{(2)}\dot{p}-\beta_2\left(J_2\left(\boldsymbol{X}_{(2)}\right)\right)^{r_2-1}\boldsymbol{X}_{(2)}\end{aligned} \tag{25}$$

where a_1, a_2, c_1, c_2, β_1, β_2, r_1 and r_2 were material constants. The function $\Phi(p)$ was given by the following equation:

$$\Phi(p) = \phi_S + (1-\phi_S)\exp(-b\,p). \tag{26}$$

For the Chaboche model presented above the following material constants for S316LN stainless steel at 600°C can found (see [29] or [48]): $\alpha = 2\cdot 10^6$, $n = 24$, $k = 10$, $K_0 = 116$, $c_1 = 45$, $c_2 = 1300$, $\phi_S = 0.5$, $\alpha_K = 2$, $a_1 = 3600.0$, $a_2 = 87\,750.0$, $b = 12$, $\alpha_R = 0.0$, $\beta_1 = 0.5\cdot 10^{-14}$, $\beta_2 = 0.9\cdot 10^{-11}$, $\gamma = 0.2\cdot 10^{-6}$, $\mu = 19$, $r_1 = r_2 = 4$, $m = 2$, $\eta = 0.6$, $Q_{\max} = 455$, $Q_0 = 30$, $Q_R^* = 200$.

Notably, Rive *et al.* [49] specified all the above 23 parameters as functions of temperature, in the temperature range from 0°C to 600°C.

3.2. Elements of continuum damage mechanics

The scalar isotropic damage concept can be considered as belonging to the group of continuum damage mechanics proposed by Kachanov [34]. The foundations of continuum damage modelling should be based on homogenization of micromechanical models (see *e.g.*: Gurson [50], Marigo [51], Krajcinovic and Lemaitre [52], Böhm [53]). The micromechanical category defines the damage internal variable by averaging the microscopic defects that characterize the state of internal deterioration. Numerous models have been developed to describe isotropic damage (see *e.g.*: Lemaitre [54], [55], Simo and Ju [56], Krajcinovic and Lemaitre [52], Ju [57], Mazars and Pijaudier-Cabot [58], Lemaitre and Chaboche [59] Krajcinovic [60], Vree *et al.* [61]).

A model of ductile fracture based on continuous damage mechanics and its applications to metal formation, particularly to deep-drawing of sheets, was presented by Lemaitre [62]. Used the Ramberg-Osgood hardening law with damage the concept of parameter identification was presented in detail. A model for combined elastoplasticity and damage was developed by Hesebeck [63]. The model was based on the maximum dissipation principle and implemented a strong coupling between plasticity and damage. Hambli [64] presented numerical results obtained by a finite element analysis for the metal sheet blanking process. These results were compared with the experimental results in order to verify the Gurson and Lemaitre damage models describing the initiation and propagation of cracks during the process's evolution. A strain-based thermodynamic framework was proposed for modelling the continuum damage behaviour of viscoelastic materials by Abdel-Tawab and Weitsman [65]. In their paper damage was represented by the internal state variable in the form of a symmetric second-rank tensor. Their approach accounted for time-dependent damage as well the changes in material symmetry due to the damage.

The double scalar damage variables, characterizing the state of isotropic damage, were described by Tang *et al.* in [66]. The damage influence tensor relating to the double scalar damage variables of the damaged material was thus formulated on the basis of the hypothesis of stress equivalence for this model. Additionally, the damage influence tensor and the specific damage energy release rate were obtained using the experimental results of 2024T3 pre-strained aluminium alloy specimens under uniaxial tension tests. Zako and Uetsuji [67] showed the simulation damage behaviour of FRP through finite element analysis, with the use of an anisotropic damage model based on damage mechanics. The proposed procedure was performed twice in the case of FEM: in aided damage simulation and in the finite element analysis of microscopic damage propagation in woven fabric composite materials. The authors gave the mechanical properties of boron-aluminium lamina in centrally notched specimens. A series of deformation experiments was conducted by Yaguchi *et al.* [46] on IN738LC nickel-base polycrystalline super alloy at 850˚C. A kinematic hardening rule was proposed within a viscoplasticity framework. A new internal variable of the back stress dynamic recovery property was incorporated into the back stress evolutionary equation. An evolutionary law of the proposed internal variable was assumed to be a function of the back stress static recovery property.

Optimal algorithm results for the shape optimization of mechanical engineering structures used in continuum damage mechanics were presented by Lemaitre [68]. Cormey and Welemane [69] investigated macroscopic modelling of the brittle damage unilateral effect, due to the opening-closure of micro-cracks. These authors examined precisely two formulations proposed by Chaboche [70], and Halm and Dragon [71] and demonstrated that they exhibited major inconsistencies. Different results for the calculation of fatigue strength of components made of ductile materials under complex cyclic load were presented by Roos *et al.* [72]. The shear stress intensity hypothesis and the critical plane approach were considered as typical representatives of stress theories. Viscoplastic material models of creep-and-fatigue combination were applied. A calculation of multiaxial creep and fatigue tests with a modified material model by Chaboche and Nouailhas [73] was presented.

3.3. Variants of Chaboche model equations with damage

Generally, a scalar variable D $(0 \leq D \leq 1)$, describing isotropic damage is introduced into the constitutive equations using the strain equivalence principle, see *e.g.*: Chaboche [74], Lemaitre [55].

In the first presented variant of the Chaboche model with damage, the viscoplastic strain rate, $\dot{\boldsymbol{E}}^I$, is based on the creep plasticity isotropic hardening theory with damage evolution, as given by the following extension of the viscoplastic constitutive theory by Chaboche and Rousselier [75] to the creep plasticity isotropic hardening model with damage evolution (see *e.g.*: Dune and Hayhurst [76], Skrzypek and Ganczewski [77]):

$$\dot{\boldsymbol{E}}^I = \frac{3}{2}\dot{p}\frac{\mathbf{S}'}{J(\boldsymbol{S}')}, \tag{27}$$

where $\dot{p}$ can be written in the following form:

$$\dot{p} = \left\langle \frac{\frac{J(\boldsymbol{S}')}{(1-D)} - R - k}{K} \right\rangle^n. \tag{28}$$

To determine the evolution of isotropic hardening, R, and the isotropic damage variable, D, the following equations are employed,

$$R = Q_1\,p + Q_2\,[1 - \exp(-b\,p)], \tag{29}$$

$$\dot{D} = \left(\frac{Y}{S}\right)^s \dot{p}. \tag{30}$$

The elastic energy release rate is described as

$$Y = \frac{1}{2E(1-D)^2}\left[\frac{2}{3}(1+\nu)\left(J(\boldsymbol{S}')\right)^2 + 3\,(1-2\nu)\left(\frac{\mathrm{tr}(\boldsymbol{S})}{3}\right)^2\right], \tag{31}$$

where E is the Young modulus, ν is Poisson's ratio, and k, K, n, Q_1, Q_2, b are material parameters. S and s are damage constants.

When the accumulated equivalent strain exceeds its limit value, ε_D, damage develops as per Equation (30), namely:

$$\begin{aligned} &\dot{D} = 0 \qquad p < \varepsilon_D \\ &\dot{D} > 0 \qquad p \geq \varepsilon_D \end{aligned} \tag{32}$$

The following physical parameters for SM490 steel at the 450°C can be found in [78]: $E = 178\,400.0$MPa, $\nu = 0.3$, yield stress 223.4MPa, ultimate strength 450.8MPa, elongation 32%, $K = 181.0$MPas, $n = 30.0$, $k = 90.0$MPa, $Q_1 = 650.0$MPa, $Q_2 = 140.0$MPa, $b = 37.0$, $S = 0.2$MPa, $s = 0.85$, and the critical value of the damage variable $D_{\mathrm{cr}} = 0.45$. For "ductile" analysis the threshold strain for damage initiation is $\varepsilon_D = 0.155$. For the "quasi-brittle" analysis the threshold strain of $\varepsilon_D = 10^{-10}$ is assumed.

In the next variant of the Chaboche equations with damage, the inelastic strain rate, $\dot{\boldsymbol{E}}^I$, is specified by Equation (1). In this variant the $\dot{p}$rate $\dot{p}$ is defined by the following equation (see *e.g.*: Lemaitre [68]):

$$\dot{p} = \left\langle \frac{\frac{J(\boldsymbol{S}'-\mathbf{X}')}{1-D} - R - k}{K} \right\rangle^n, \tag{33}$$

where damage evolution, D, is expressed by Equation (30). Identification and validation of material parameters for nickel-based super alloys was performed by Amar and Dufailly (see [79]). The authors gave the following parameters, dependent on temperature, $(0^\circ\mathrm{C} \leq T \leq 627^\circ\mathrm{C})$:
$E(T) = 30\,000\,[\mathrm{MPa}]\left(1-\exp\left(1.45\cdot 10^{-3}\left[{}^\circ\mathrm{C}^{-1}\right]\cdot T\right)\right) + 206\,000\,[\mathrm{MPa}], \quad \nu = 0.3\,[-]$,
$k(T) = 70\,[\mathrm{MPa}]\left\{1-\exp\left(3.10\cdot 10^{-3}\left[{}^\circ\mathrm{C}^{-1}\right]\cdot T\right)\right\} + 920.5\,[\mathrm{MPa}], \quad n = 2.4$,
$K(T) = 1/\left(1.80\cdot 10^{-6}\exp\left(-1.51\cdot 10^{-2}\left[{}^\circ\mathrm{C}^{-1}\right]\cdot T\right)\right)^{2.4}, \quad a = 8\cdot 10^4\,[\mathrm{MPa}]$,
$c = 200\,[-], \quad b = 15\,[-], \quad R_1(T) = 6.6\,[\mathrm{MPa}]\left\{1-\exp\left(5.2\cdot 10^{-3}\left[{}^\circ\mathrm{C}^{-1}\right]\cdot T\right)\right\}$,
damage parameters $s = 3.0\,[-]$ and $S(T) = -8.80\cdot 10^{-3}\left[\mathrm{MPa}^\circ\mathrm{C}^{-1}\right]\cdot T + 10\,[\mathrm{MPa}]$.

4. Final remarks and conclusions

The present paper collects and describes several variants of the Chaboche model and their practical engineering application. Selected material parameters are also included. It follows from our review that the elasto-viscoplatic equations of the Chaboche model are still being developed and modified. The model has some disadvantages; the presented variant of the Chaboche model, based on the assumption of additivity of elastic and inelastic strains, is applicable only in the small strain range. At the same time, some constitutive models are based on multiplicative decomposition of elastic and inelastic strains. To offer an example, Brocks and Lin [80] extended the Chaboche viscoplastic law assuming the multiplicative decomposition of the strain into elastic and inelastic parts.

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