THE TRUSS OVERLOAD ANALYSIS UNDER CORROSIVE DEGRADATION

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Abstract: A contribution to analytical and numerical tools is presented that permits deterministic evaluation of structures' behavior under multiparameter and/or cyclic mechanical, thermal and chemical loads. Particular structure elements undergo plastic and corrosive degradation and dissipate energy, which consists of irreversible contributions, like the work of inelastic strains. The construction and its units' lifetime are estimated according to the dissipated energy criterion. Modeling and numerical implementation of degradation effects are discussed, including cyclic plasticity generated by mechanical and thermal loads, stress corrosion, electrochemical corrosion and low-cyclic corrosion.

Keywords: cycle loading, corrosion degradation, numerical symulator, FEM

1. Introduction

Elastic ranges for which working structures are designed are often exceeded under operational conditions. The structures' elements undergo material degradation of various forms, the most dangerous of which are plasticity, creep, thermal fatigue and corrosion. The dominant effects resulting from interaction between a structure and an aggressive environment are: (i) *stress corrosion*, a function of extensional stresses and the corrosive power of the environment, (ii) *electrochemical gas-corrosion*, mainly due to high temperatures and the presence of products of combustion, and (iii) *low-cyclic corrosion*, an effect of the interaction between electrochemical corrosion and varying thermo-mechanical loads. Experimental data concerning for modeling corrosion process are obtained with testing machines for axial extension of cylindrical samples [1–4]. Therefore, the current research activity has been restricted to rod structures discretized with one-dimensional truss finite elements. Structures' degradation is evaluated on the basis of the energy criterion, *viz.* the value of energy dissipated in the material during loading cycles [5]. The equilibrium path, energy dissipation and degradation of the structure's elements' cross-sections are analyzed in numerical simulations.

2. Modeling of corrosion degradation

It has been assumed that corrosion acts on the surface of a structure's elements thus diminishing the rod's cross-section. The following three forms of corrosion have recently been analyzed (see [5]):

- stress corrosion,

$$\dot{d}_S = C_S |\sigma - \sigma_{\mathrm{gr}}|^n \, e^{(T-T_0)/B}, \tag{1}$$

- electrochemical and gas corrosion,

$$\dot{d}_H = C_H \, (T/T_0)^{\kappa} \, |\nabla T|^m, \quad \text{and} \tag{2}$$

- low-cyclic corrosion,

$$\dot{d}_L = C_L \, N^{\mu} (\Delta\varepsilon)^b \, e^{(T-T_0)/B}. \tag{3}$$

In the foregoing formulas, d denotes the loss in an element's thickness [mm], T – temperature, N – the number of cycles, σ – stress, σ_{gr} – the stress limit, below which stress corrosion does not occur, $\Delta\varepsilon$ – the strain range, while C_S, n, B, C_H, κ, m, C_L, μ, b are model constants to be calibrated in one-dimensional experiments. After every load increment the loss in the element's thickness is calculated as the sum $d = d_S + d_H + d_L$ and the cross-section area is updated.

3. The energetic criterion of structures' degradation

The basis of the energetic criterion is the elementary energy irreversibly dispersed (dissipated) in the material during variable loading up to the element's destruction. The energy dissipated by element j during cyclic load changes is described by the following formula:

$$U_j = \frac{1}{V_j} \sum_{i=1}^{k \cdot N} \Delta W_{ij}, \tag{4}$$

where W_{ij} means the increase of stress work on plastic deformations of element j corresponding to increase of load i, N is the number of load changes (load cycles), k – the number of load increases in a cycle, and V – the element's volume.

The destruction condition of j element may be represented as follows:

$$U_{\mathrm{kr}} - U_j = 0. \tag{5}$$

The critical energy, U_{kr}, was assumed to be equivalent to the threshold energy of deformation in the case of static stretching [6]. According to the specified energetic criterion (5), the cycle during which the elementary energy dissipated by any element of the structure exceeds the limit value determined from the Broniewski formula [7] was assumed to represent the moment when damage occurs:

$$U_{\mathrm{kr}} = 0.0025 \, (3R_m + R_e) \, A_{10}, \tag{6}$$

where R_m is the tensile strength, R_e – the yield stress, A_{10} – percentage unit elongation after tensile failure of the specimen.

4. Numerical applications

The models of stress corrosion, electrochemical-gas corrosion and low-cyclic corrosion described by Equations (1)–(3) were implemented in the D-KRAT numerical code developed by the author [5] on the basis of a Mini-Mod library containing Finite Element Method solver procedures developed by Chróścielewski and Branicki [8]. The modeled phenomena were nonlinear and the solution of a non-linear system of equations was obtained in an incremental-iteration process. For non-linear loading paths tracing, the numerical technique developed by Chróścielewski [9] was applied. Additionally, thanks to subprograms developed to visualize the calculation results [5], it was possible to follow the equilibrium track (Figure 1b), structure deformation (Figure 1a), the variation of control displacement q in time t (Figure 1c), degradation of rods' cross-section area (Figure 1d) and energy dissipation (Figure 1e).

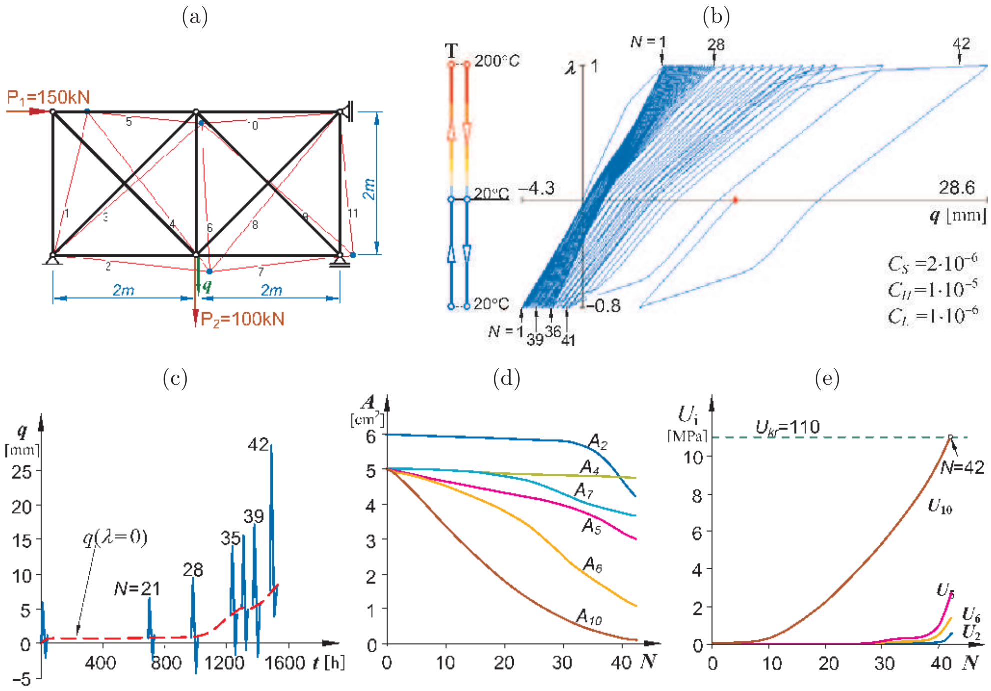

Figure 1. (a) The structure's geometry, (b) equilibrium path, $q = f(\lambda)$, (c) variation of control displacement in time, $q = f(t)$, (d) changes in rods' cross-section area in subsequent load cycles, and (e) strain energy density dissipation for selected rods in subsequent load cycles

5. Results of the truss overload simulation

The plane truss shown in Figure 1a was considered in the simulation process. Cross-section areas of rods were assumed to equal $A_{2,7,10,11} = 4\text{cm}^2$, $A_5 = 5.2\text{cm}^2$, and 5cm^2 others. The following material properties were assumed: Young's modulus $E = 2.1 \cdot 10^5$ MPa, strain linear hardening modulus $E_T = 10^4$ MPa, yield stress $R_e = 230$ MPa, tensile strength $R_m = 400$ MPa, $A_{10} = 13.1\%$. The corrosion model constants were $C_S = 2 \cdot 10^{-5}$, $C_H = 4.6 \cdot 10^{-6}$, $C_L = 3.58 \cdot 10^{-6}$, $n = 1$, $\sigma_{\text{gr}} = 150$ MPa, $B = 170$, $\kappa = 1$, $\vartheta = 1$, $\mu = 1.6$ and $b = 1$.

Therefore, the maximum velocity of each corrosion type's values were $\dot{d}_S = 10^{-3}$ mm/h, $\dot{d}_H = 0.4$ mm/year and $\dot{d}_L = 1.5 \cdot 10^{-4}$ mm/cycle. The strucure was loaded cyclically by forces $P_1 = \lambda \cdot 95$ kN and $P_2 = \lambda \cdot 45$ kN. The load control parameter λ was

Table 1. Number of cycles with overload

Series' number	Number of cycles with overload						
I	—	—	—	—	—	—	—
II	1	50	100	—	—	—	—
III	—	50	100	150	—	—	—
IV	—	—	100	150	200	—	—
V	—	—	—	150	200	250	—
VI	—	—	—	—	200	250	300

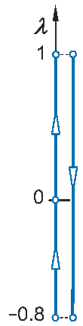

Figure 2. The sequence of variation of load parameter λ

modified with an increment of $\Delta\lambda = 0.01$ according to the scheme shown in Figure 2. A single cycle time was equal to 360 hours and was increased in proportion to $\Delta\lambda$. Acording to the set of cycles (see Table 1), the structure was overloaded in 15% of the forces' positive operating range as well as in the negative range.

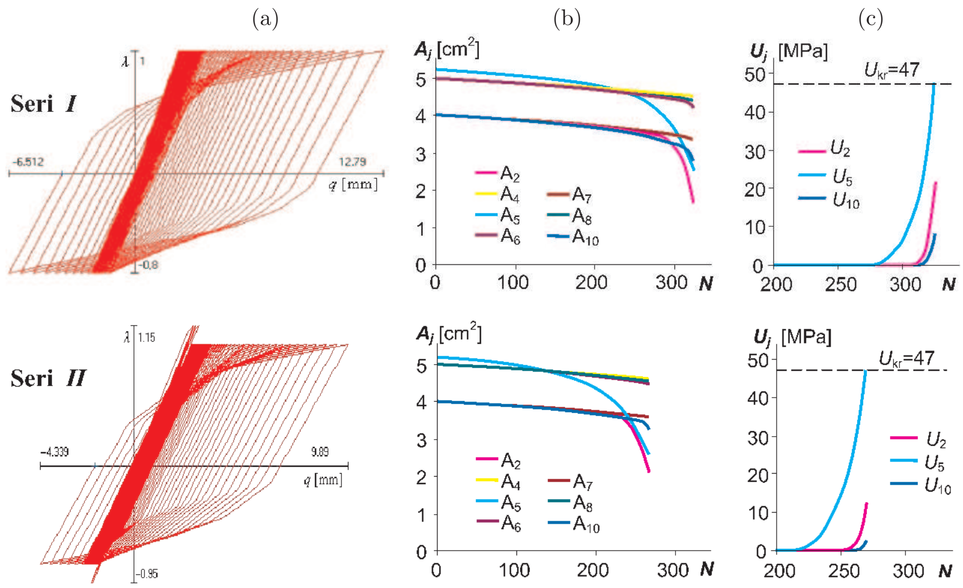

Figure 3. Series I and II: (a) equilibrium path $q = f(\lambda)$, (b) degradation of cross section area A_j and (c) dissipation of energy U_j (continued on the next page)

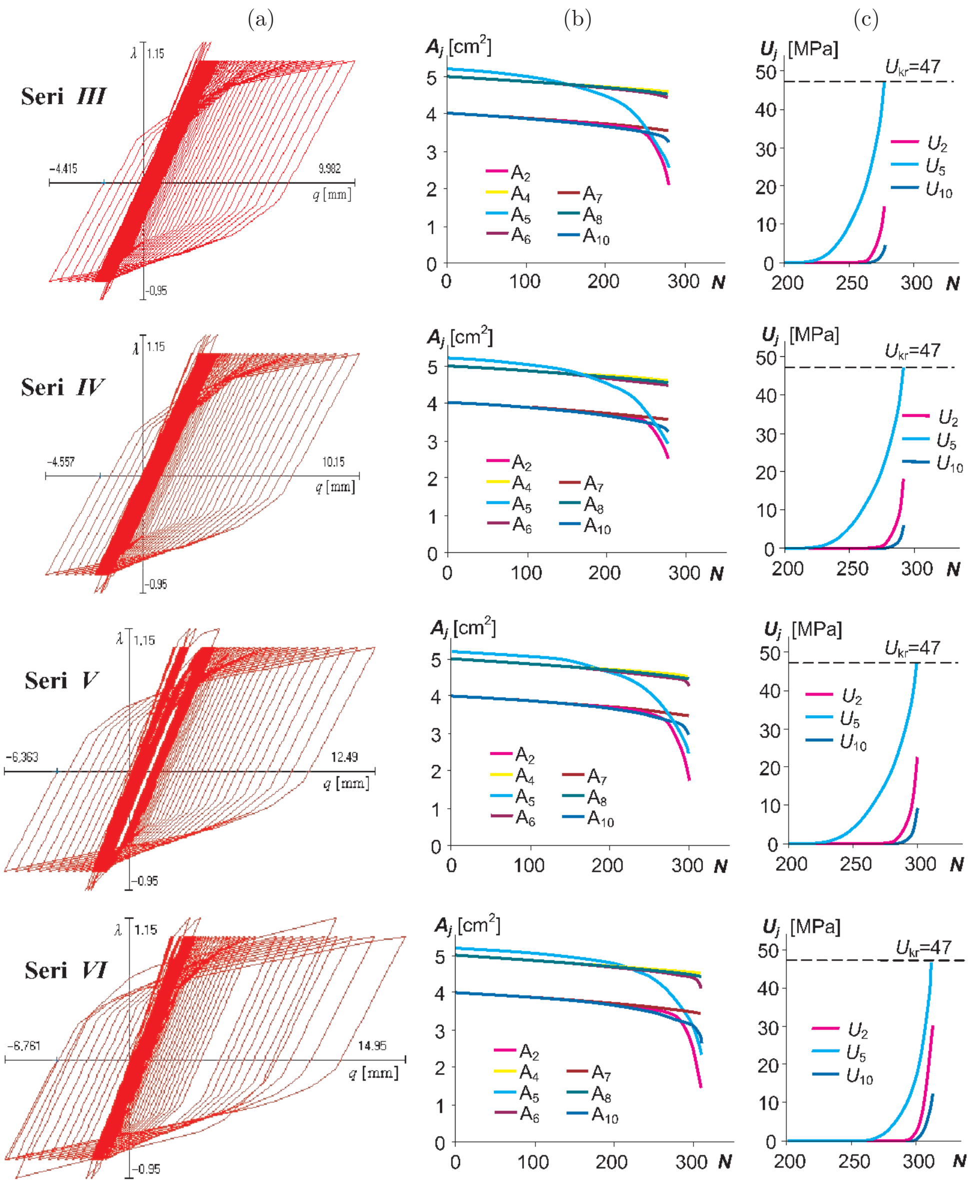

Figure 3 – continued. Series III, IV, V and VI: (a) equilibrium path $q=f(\lambda)$, (b) degradation of cross-section area A_j and (c) dissipation of energy U_j

The relation between control displacement q and load parameter λ (Figure 1b), the variation of control displacement q in time t (Figures 3a), the changes in rods' cross-section area, A_j (Figures 3b) and the increments of dissipated energy U_j (Figures 3c), were observed in subsequent load series.

6. Discussion

In order to determine the influence of a structure's overload on its behavior during cyclic loading, the results of all calculation series were correlated in two graphs

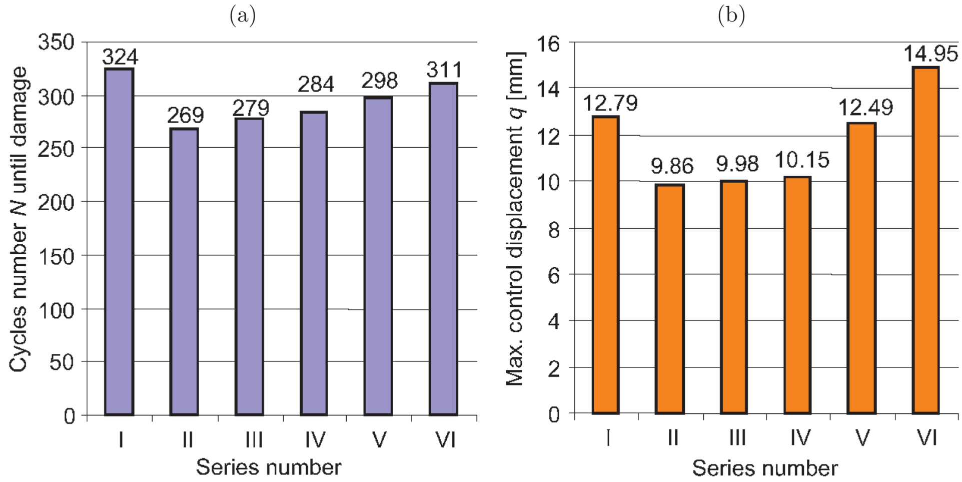

Figure 4. Distributions of (a) number of cycles, N, until damage and (b) maximum control displacement q in the following loading series

(Figure 4). The graph of Figure 4a presents the number of cycles's distribution until the moment of damage appearance in the following loading series. The graph of Figure 4b shows the greatest value of control displacement distribution in the following loading series.

It follows from an analysis of Figure 4 that, with respect to the energetic criterion, the structure is more susceptible to overload in the first working cycles. In series II, where overload first occurred, the rod no. 5 was damaged in 269 cycles, whereas in the series VI, the damage of this rod occurred already after 311 cycles.

However, considering the displacement criterion (*i.e.* the minimum displacement q), the displacements in series II, III and IV were lower than in series I, no overload occurred. In these series, the overload moment (instant) did not considerably influence control displacement, which differred very little in these three cases. Displacements in series V and I were nearly equal, although there was an overload condition in one case. The overload in series VI generated considerably greater displacements, compared with the unloaded structure. Furthermore, when the maximum values of control displacement stabilized in series II, III and IV, the significant increase of that displacement occured in series V and VI. One should suppose that overloads appearing at times greater than for series VI, may lead to excessive structural strains.

Comparing the graphs shown in Figure 4, one may conclude that the structure is the least susceptible to the oveloading method specified for series IV. With respect to energy, structures can be subjected to the load of 284 cycles (Figure 4a) with simultaneous maintenance of the low displacement level of $q = 10.15$ mm (Figure 4b).

7. Summary

The exemplary calculations carried out and analysis of the results have shown that the tools of numerical mechanics of structure enable numerical models to be developed capable of simulating the process of structures' degradation under cyclic loads. The calculation results supply information on the behavior of the structure as a whole at the same time offering information on the degradation of its constituent elements.

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