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MASS EXCHANGE MODEL IN FLOWS WITH CAVITATION

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Abstract: The paper presents a mathematical model of three-component flow of the liquid-liquid vapor-gas type including the mass exchange between the liquid and its vapor. The model is implemented in the Multi Flower 2D calculation package and used for computer simulations of flows with cavitation. An algorithm for model implementation, basic tests and an example of application are presented. The model's quality was assessed with a special test station equipped with a cavitation chamber allowing cavitation studies. The final section of the paper includes a qualitative comparison of observations made at the laboratory station with the results of numerical simulations, discusses the model's advanrages, disadvantages and possible further development.

Keywords: mechanics of liquids, cavitation, CFD

1. Introduction

Projecting processes occurring in flows with cavitation has long been the subject of numerous scientific studies and analyses. However, until the present day no single consistent mathematical model of that phenomenon has been developed that would allow performance of precise numerical simulations consistent with actual observations. Attempts are currently in progress to develop various mathematical models considering the influence of such parameters as pressure, flow velocity, temperature, density or viscosity [1, 2].

The present paper represents an attempt at developing a simple model – easily implemented and quick in operation – describing the mechanism of appearance and disappearance of liquid vapor in flows with cavitation. The mathematical description of the model is consistent with Euler's Finite Volumes Method.

2. Basic system of balance equations

The basic equations describing the dynamics of liquids are applicable to mass balance (Equation (1)), momentum balance (Equation (2)) and energy balance

(Equation (3)) [3]. For multi-component flows it is also necessary to determine the share of individual components (Equation (4)):

- mass balance: $$\frac{\partial}{\partial t}\rho+\nabla(\rho\vec{v})=0, \tag{1}$$
- momentum balance: $$\frac{\partial}{\partial t}(\rho\vec{v})+\nabla(\rho\vec{v}\otimes\vec{v})=\nabla\left(-p\overleftrightarrow{I}+\overleftrightarrow{t}^{c}\right)+\rho\vec{s}_b, \tag{2}$$
- energy balance: $$\frac{\partial}{\partial t}(\rho e)+\nabla(\rho e\vec{v})=\nabla\left(\left(-p\overleftrightarrow{I}+\overleftrightarrow{t}^{c}\right)\vec{v}+\vec{q}^{\,c}\right)+\rho s_e, \tag{3}$$
- component balance: $$\frac{\partial}{\partial t}(\rho Y_k)+\nabla(\rho Y_k\vec{v})=\nabla\left(\vec{J}_k^{c}\right)+\rho s_k, \quad k=1,\dots,n_s. \tag{4}$$

The meaning of the individual elements is as follows:

ρ – the mixture's density,
$\vec{v}$ – the mixture's average velocity,
$\overleftrightarrow{t}^{c}$ – the total stress tensor,
$\vec{s}_b$ – the source of mass forces,
e – total energy (sum of internal and kinetic energy),
p – the mixture's pressure,
$\overleftrightarrow{I}$ – the unit tensor,
$\vec{q}^{\,c}$ – the total heat flux,
s_e – the source of energy,
Y_k – the mass share of component k,
$\vec{J}_k^{c}$ – the total diffusion flux,
s_k – the mass source,
n_s – the number of mixture components.

The mass share of a component occurring in Equation (4) is determined according to the following formula:

$$Y_k=\frac{m_k}{\sum\limits_{i=1}^{n_s}m_i}, \tag{5}$$

where m_k denotes the unit mass of component k.

3. Types of flows with mass exchange

The component balance equation described by formula (4) includes a segment responsible for appearance or disappearance of individual mixture components (the mass source, s_k).

If mass exchange between components occurs in the flow, an appropriate equation should be added defining the character of s_k's changes dependent on the type of phenomenon. The mass of a component is subject to change in two cases only:

- when chemical reactions occur in the mixture – in majority of such situations both a change in the chemical composition of the mixture and the proportions between the individual components occur; or
- when a component changes its form, which occurs during phase changes of mixture components.

The mass exchange model presented in the paper refers to the latter case, *viz.* a phase change of the liquid-liquid vapor type. The model, after implementation in a numerical code, can be used to simulate flows with cavitation or evaporation and condensation [4].

4. Mathematical description of mass source: conditions of correctness

A correct description of the phenomenon of appearance and disappearance of mass requires satisfying two basic conditions:

- determining when the process of appearance or disappearance of a mass source is to start and in which areas of the flow should the equation describing phase transformation be included; and
- determining the degree of intensity of mass source appearance or disappearance depending on the actual parameters.

The existence of the first condition means that a stimulus must exist that will trigger the mass source equation. In the case of a liquid-liquid vapor phase transformation the stimulus may be the difference between static pressure at a given point of the flow and the saturation pressure for the current temperature. In the absence of formulas defining the relation between temperature and liquid saturation pressure it is possible to use data from the table. If static pressure at any point of the flow falls below the saturation pressure, the source of mass should be initiated; consequently, the quantity of liquid vapor will increase. It is particularly important that the mass exchange equation should operate also in the opposite direction, *i.e.* describe the loss of vapor mass in case of pressure increase above the saturation pressure.

The second condition is necessary to determine the quantity of vapor to be formed as a result of the first condition's operation. The difference between the static pressure in the flow and the saturation pressure for a given temperature can also be used as a process controlling parameter: – the greater the difference, the more intensive the mass source. Liquid vapor will be formed until that pressure exceeds the saturation pressure – at that moment the process should be reversed.

The process of vapor phase increment or decrement may also depend on other parameters of the flow (*e.g.* velocity) or additional functions taking into account the influence of other factors (*e.g.* air content).

5. The mass exchange model's assumptions

Before constructing the mathematical model, the following assumptions were made:

- the subject of the study is the three-component flow of the liquid, liquid vapor and gas type (*e.g.* water, steam and air). Any liquid with known relations between temperature and saturation pressure may serve as the liquid phase;
- mass exchange occurs between the liquid phase and the vapor phase only;
- the difference between static pressure and saturation pressure for a given temperature will be the basic parameter controlling the mass exchange equation;

- the mass share of liquid vapor in the area not covered by the evaporation and condensation phenomenon is nil;
- the walls limiting the model area are adiabatic;
- the influence of viscosity is disregarded;
- all phases are chemically inactive;
- Inter-phase surfaces are not assigned any physical properties such as mass, surface stress, *etc.*

6. The model of mass source in three-phase flow

Assuming a three-phase medium for the study will expand the system of mass, momentum and energy balance equations by adding further three equations:

- gas mass balance: $$\frac{\partial}{\partial t}(\rho Y_1)+\nabla(\rho Y_1\vec{v})=\nabla\left(\vec{J}_1^c\right)+\rho s_1, \tag{6}$$
- liquid vapor mass balance: $$\frac{\partial}{\partial t}(\rho Y_2)+\nabla(\rho Y_2\vec{v})=\nabla\left(\vec{J}_2^c\right)+\rho s_2, \tag{7}$$
- liquid mass balance: $$\frac{\partial}{\partial t}(\rho Y_3)+\nabla(\rho Y_3\vec{v})=\nabla\left(\vec{J}_3^c\right)+\rho s_3, \tag{8}$$

and as of this moment the sequence of defining the mixture's components shall be fixed and indices 1, 2, 3 shall respectively mean gas, liquid vapor and liquid.

If no mass exchange between components occurs in the flow within the entire calculation area, the following conditions are satisfied:

$$s_1=s_2=s_3=0, \tag{9}$$

$$Y_1\neq 0, \tag{10}$$

$$Y_2=0, \tag{11}$$

$$Y_3\neq 0, \tag{12}$$

$$0\leq Y_1,\,Y_2,\,Y_3\leq 1, \tag{13}$$

$$Y_1+Y_2+Y_3=1, \tag{14}$$

where:

s_1 – the source of gas mass,
s_2 – the source of liquid vapor mass,
s_3 – the source of liquid mass,
Y_1 – the mass share of gas,
Y_2 – the mass share of liquid vapor,
Y_3 – the mass share of liquid.

In the area of decreased pressure (when static pressure decreases below saturation pressure for a given temperature) the equation of mass exchange between the liquid and the liquid vapor starts operating according to the following relation:

$$s_2=-s_3. \tag{15}$$

Equation (15) must additionally be limited by the following conditions:

$$\begin{cases} s_2=0 \quad \Rightarrow \quad Y_2>(1-Y_1), \\ s_2=0 \quad \Rightarrow \quad Y_2<0. \end{cases} \tag{16}$$

The first condition means that the increase of liquid vapor cannot occur at the expense of the gas phase as the following equation must always be satisfied:

$$Y_2+Y_3=1-Y_1. \tag{17}$$

The second condition was assumed to avoid the appearance of negative values of liquid vapor mass share in the flow – according to formulas (13) and (14).

The major difficulty in mass exchange modeling is defining the relation between flow parameters and the mass source intensity. This applies in particular to three-component flows of the liquid, liquid vapor and gas type. Generally, in the studies on cavitation projections (that is the standard of the presented mass exchange model) the intensity of vapor source is dependent on the difference between static pressure and saturation pressure for a given temperature (the so-called cavitation discriminant is a linear function of that pressure difference).

The following liquid vapor phase mass source model is proposed in this paper:

$$s_2=\alpha_{(w,z)}C^{\max}(p-p_{\mathrm{sat}})f(\vec{v})f(Y_1), \tag{18}$$

where:

$\alpha_{(w,z)}$ – the liquid vapor intensity increase/decrease coefficient,
$C^{\max}$ – the model constant,
p – static pressure of the liquid,
p_{sat} – saturation pressure for a given temperature,
$f(\bar{v})$ – a function relating mass source intensity to local velocity,
$f(Y_1)$ – a function relating mass source intensity to the presence of gas solved in the liquid.

7. Standardization of the mass exchange intensity coefficient

Model constant $C^{\max}$ is one of the components of formula (18). It allows determining the maximum content of the vapor phase that can occur under the defined flow conditions.

The $C^{\max}$ constant was included for the purpose of standardization of the vapor phase increase or decrease coefficient, $\alpha_{(w,z)}$. According to this approach, coefficient $\alpha_{(w,z)}$ will assume values within the range from zero to one:

$$0\le\alpha_{(w,z)}\le 1. \tag{19}$$

8. The velocity influence function

The presence of a clearly visible cloud of cavitation bubbles in areas of high velocities is frequently observed in cases of cavitation . For example, this appears in liquid jets zones occurring in systems with significant narrowing [5, 6]. Therefore it was decided to add in this model an appropriate function allowing for increased mass exchange intensity in areas of the highest velocities. At the current stage of research the following formula has been proposed:

$$f(\vec{v}) = \frac{v^2}{2}. \tag{20}$$

9. Limiting the influence of the gas phase

Determining the influence of the content of gas in the liquid on the intensity of the vapor source represents another problem. The literature only states that the influence is most clearly visible at low gas contents because of the possibility of appearance of the so-called "nuclei", *i.e.* small gas bubbles. The energy necessary for the vapor to penetrate the liquid-gas inter-phase surface is much lower in that case than the energy necessary to create a new, purely vaporous bubble. At lower gas contents that influence is significantly weaker. Generally, the higher the mass content of gas dissolved in the liquid, the easier and more numerous gas bubbles will form under decreasing pressure; that influence will increase only to the maximum gas content that can be dissolved in the liquid. An additional limiting formula can be applied to consider this phenomenon in the mass exchange process, as follows:

$$f(Y_1) = \begin{cases} \sqrt{Y_1^{\mathrm{limit}} Y_1} & \Rightarrow \quad Y_1 \le Y_1^{\mathrm{limit}} \\ Y_1^{\mathrm{limit}} & \Rightarrow \quad Y_1 > Y_1^{\mathrm{limit}} \end{cases} \tag{21}$$

The limiting function defines the maximum level of gas content which can occur in the liquid under specific thermodynamic conditions. At higher gas contents large gas bubbles will appear in the liquid and – according to Equation (21) – the limiting function will assume a constant value.

In flows of the water, steam and air type the maximum unit quantity of air dissolved in water will amount – at the pressure of 1013hPa and temperature 288K – to *ca.* 0.02 of volume or *ca.* 0.00025 of mass. The graph of the limiting function for this case is presented in Figure 1.

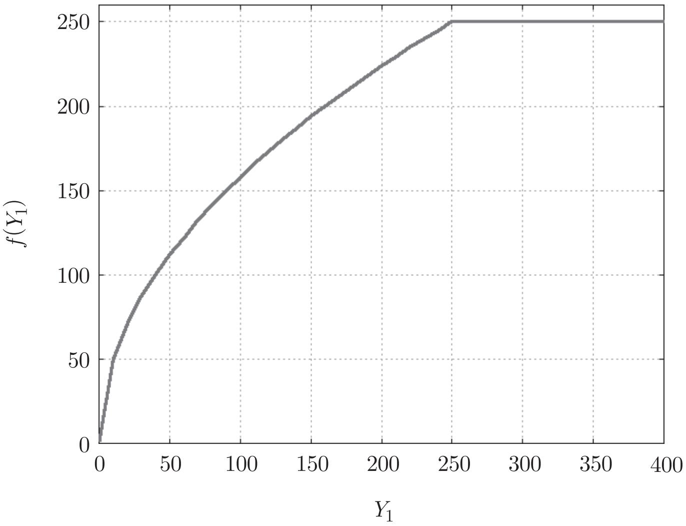

Figure 1. Graph of the function limiting the gas phase influence

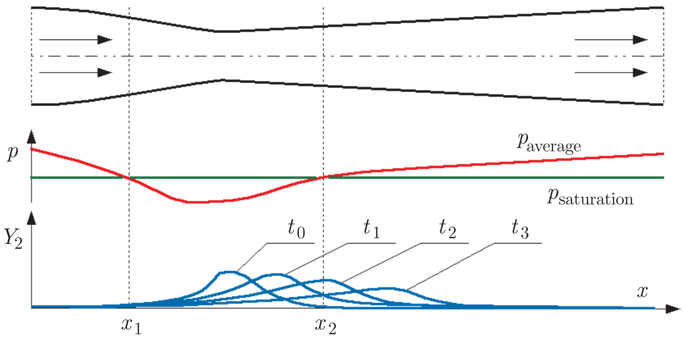

Figure 2. Appearance and disappearance of the volatile phase during cavitation in the area of geometric narrowing of a convergent-divergent nozzle

10. The oscillation equation

A typical flow of liquid through a convergent-divergent nozzle considering the phenomenon of cavitation is shown in Figure 2.

In the geometric narrowing of the nozzle (after exceeding the x_1 diameter) a decrease of static pressure below the saturation pressure for a given temperature occurs and, as a result, air and vapor bubbles start forming, represented in the graph by the increase in the quantity of liquid vapor; the mass quantity of gas does not change, only its volume share increases as a consequence of decompression. The period of increase in the number of cavitation bubbles continues until the maximum value of diameter x_2 is achieved, following which they slow disappear during further progress down the flow.

However, this process is not constant in time: practical observations and experimental tests have shown that in real flows the intensity of air-vapor bubble cloud's appearance and disappearance is irregular, which over time results in a change of the maximum vapor phase content in the cavitation area, as well as of the length and shape of the cavitation cloud down the flow past the narrowing (see curves for $Y = Y(t)$, for $t_0 < t_1 < t_2 < t_3$ in Figure 3).

Increase coefficient α_w is the parameter responsible for the increase of liquid vapor in the presented mathematical model. Consequently, by changing its value in the iteration process we will obtain the maximum value of the vapor phase changing over time. The maximum length of the cavitation cloud forming in the flow depends mainly on the disappearance coefficient, α_z – the higher its value the shorter the cavitation cloud. When describing the "oscillation equation" we should remember that the signs of coefficients α_w and α_z cannot change, as that would cause a change in the sign of the mass source and the final effect would be opposite to the intended one: α_w would decrease the quantity of the volatile phase and α_z would increase it. As expected, a change in the value of α_z will cause a change in the cavitation cloud's length and the character of the function describing the change of α_w will influence its shape.

At the current stage of our studies it was assumed that the oscillation equation should possess the features of a periodic function, *e.g.*:

$$\alpha_{(w,z)}(t) = \bar{\alpha}_{(w,z)}\,(1 + A\sin(ft)), \tag{22}$$

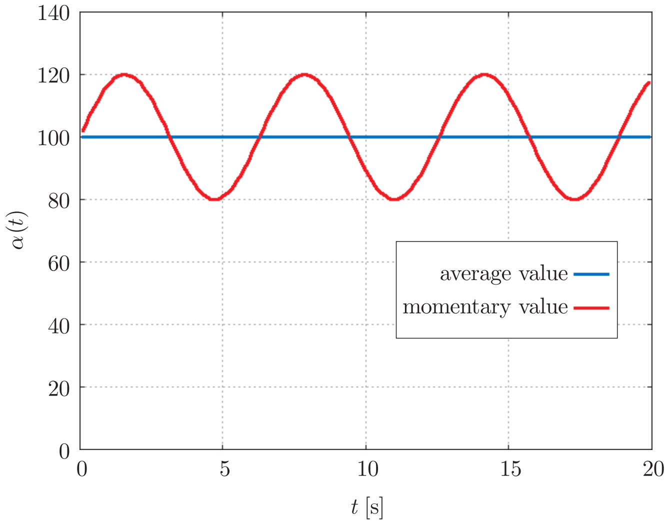

Figure 3. Changes of increase or decrease coefficient

where:

$\alpha_{(w,z)}(t)$ – the time-dependent appearance or disappearance coefficient,
$\bar{\alpha}_{(w,z)}$ – the average value of the appearance or disappearance coefficient,
A – oscillation amplitude
f – oscillation frequency,
t – time.

A graph of the function described by formula (22) is presented in Figure 3.

The value of coefficients α_w and α_z is determined from Equation (22), while the values of A and f are new independent variables defined in the initial conditions of a problem.

In Equation (22), the value of amplitude is limited as follows:

$$0 \leq A \leq 1, \tag{23}$$

which means that the maximum deviation from the average value cannot exceed the average value of $\bar{\alpha}_w$ or $\bar{\alpha}_z$, as that would cause a periodic change of the constant's sign and reversing its role in Equation (18).

Nowadays constants A and f are selected experimentally; in the future it will be recommended to find a formula relating these values to the parameters characterizing a given flow (energy, velocity, pressure, *etc.*). Such studies have not been commenced yet.

Although the "oscillation equation" presented above is based on the sinus function. there are no obstacles to apply any other periodic function in the mass exchange model. Studies on the influence of the function type on the results of calculations [7] have shown that application of other periodic functions has a minor influence on the results' quality and we have thus decided to apply the simplest function.

11. The source segment

Considering the mass exchange equation and the "oscillation equation", the source segments of Equations (6)–(8) assume the following form for the analyzed problem:

$$\begin{cases} s_1 = 0, \\ s_2 = \bar{\alpha}_{(w,z)} C^{\max}(1 + A\sin(ft))(p - p_{\mathrm{sat}})\dfrac{v^2}{2} f(Y_1), \\ s_3 = -\bar{\alpha}_{(w,z)} C^{\max}(1 + A\sin(ft))(p - p_{\mathrm{sat}})\dfrac{v^2}{2} f(Y_1). \end{cases} \tag{24}$$

12. The calculation algorithm

In order to determine new values of mass shares of individual components for a flow with mass exchange the following operations should be performed in relation to a single i-calculation cell:

- determine static pressure p_i,
- determine temperature T_i,
- check whether the value of temperature is within the range of use of physical tables for a given medium,
- determine the value of saturation pressure, p_{sat}, on the basis of current temperature,
- determine the difference, Δp_i, between static pressure and saturation pressure,
- determine the unit mass shares of individual components,
- depending on the value of the pressure difference and the mass shares of particular phases, initiate the appropriate procedure to calculate mass source $s_{2,i}$.

With information available on all the necessary parameters, we can define the conditions for appearance or disappearance of mass. The logical procedure for the n-time step initiating the equation of mass exchange between components in any cell consists of two parts.

First, a procedure is defined applicable in the case of a pressure decrease below the saturation pressure for a given temperature.

If $\Delta p_i \le 0$ then $s_{2,i} > 0$ and $s_{3,i} < 0$ and $Y_{2,i} \uparrow$ and $y_{3,i} \downarrow$
unless $Y_{2,i} \ge 1 - Y_{1,i}$, then $s_{2,i} = 0$ and $Y_{2,i} = 1 - Y_{1,i}$.

In this case, the mass source of the second component (liquid vapor) has a non-zero positive value, which causes an increase in the mass share of that component in the call. The increase occurs at the expense of the third component (liquid) through condition (15) and is limited by the first of conditions (16), which means that the process of evaporation can be continued at maximum to "exhaustion" of the source material, *i.e.* the third component.

Second, conditions are defined for disappearance of liquid vapor under pressure increased above the saturation pressure for a given temperature.

If $\Delta p_i > 0$ then $s_{2,i} < 0$ and $s_{3,i} > 0$ and $Y_{2,i} \downarrow$ and $Y_{3,i} \uparrow$
unless $Y_{2,i} \le 0$, then $s_{2,i} = 0$ and $Y_{2,i} = 0$.

In this case, the source of mass of the second component (liquid vapor) has a negative value, which reduces the mass share of that component in the cell with the accompanying increment of the third component (liquid) according to formula (15). That decrement is limited by the second of conditions (16), which means that the mass source is to stop if the second component disappears entirely.

A sample pressure change path and the corresponding distribution of mass shares are presented in Figure 4. Characteristically, the figure may simultaneously analyzed as a line of coordinate function (*e.g.* decrease of pressure along the flow in a geometrical narrowing) or as a function of time (*e.g.* local decrease of pressure as a result of non-stationary flow).

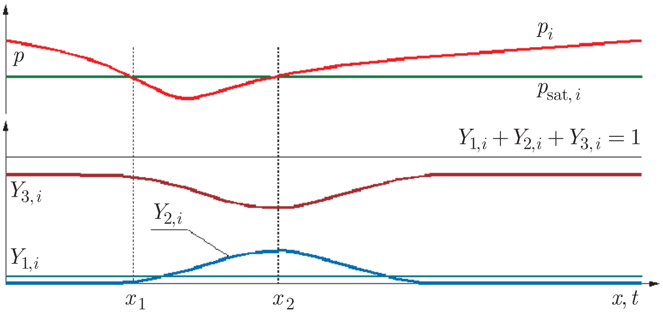

Figure 4. Visualization of the mass exchange algorithm's implications

The calculation algorithm presented above was implemented in the Multi Flower 2D package, which enabled numerical simulations of any flow problems involving mass exchange.

13. Testing the model

The mass exchange model was tested on the example of water flowing through a convergent-divergent duct. The test phase allowed us to determine the influence of the α_w and α_z coefficients on the quantity of vapor and its average distribution along the duct (Figure 5).

Computer simulations have corroborated the assumed importance of coefficients α_w and α_z: the former determines the maximum content of the vapor phase, the latter – the length of the cavitation cloud.

The conducted tests have also demonstrated a linear relation between the increment coefficient, α_w, and the maximum content of the vapor phase (at constant distribution of velocities over time), as shown in Figure 6.

A more detailed description of a test stage conducted on a simplified version of the mass exchange model can be found in [4] and [7]. Also paper [8], in which the presented mass exchange model was used to design a cavitation chamber at a laboratory station, may also be considered as belonging to the testing stage.

14. Laboratory and simulation tests

In 2005, a station was constructed for visualization and testing the cavitation phenomenon, which enabled further development of the mass exchange model. The

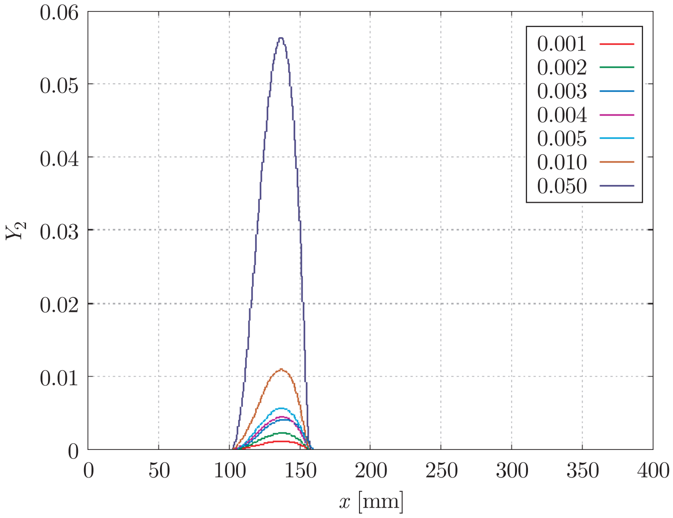

Figure 5. Content of the vapor phase for various values of coefficients α_w and α_z, averaged after sections

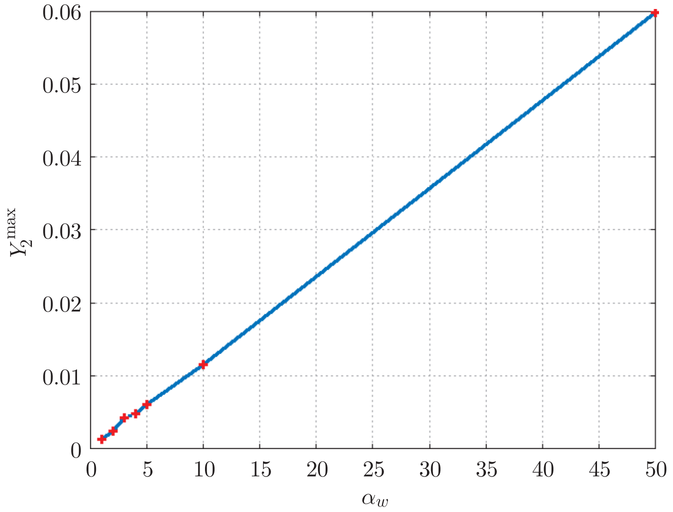

Figure 6. The influence of α_w coefficient on the maximum content of water vapor in the flow

station allows visual observation of the cavitation phenomenon occurring in the exciter, as well as vibro-acoustic analysis of cavitation noise [5, 6, 9]. The results obtained in the laboratory station have served as qualitative comparative material for numeric simulations carried out in the Multi Flower 2D code with the mass exchange model implemented. The station's cavitation chamber consists of a transparent pipe 1 meter in length and 40mm in internal diameter in which a cavitation exciter in the form of a narrowing has been placed 0.4m from the inlet. The diameter of the narrowing is 3mm.

The tests were carried for two narrowing setups: convergent-divergent and convergent-flat. The working pressure was altered within the range from 1 to 5 at-

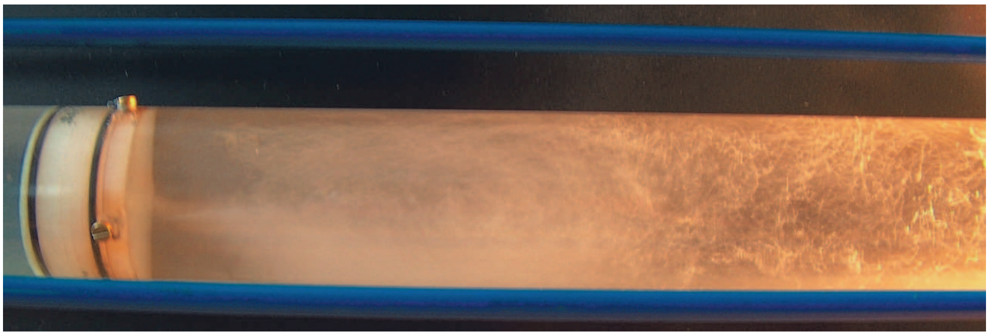

Figure 7. Structure of the cavitation cloud for the convergent-divergent exciter and working pressure of 4at

mospheres and the pressure at the chamber's outlet was constant and equaled atmospheric pressure. The chamber was supplied with water from water mains.

A typical structure of the flow in the cavitation chamber after the narrowing is shown in Figure 7. A characteristic feature is the appearance of a bifurcation leading to the formation of a recirculation area ended with a zone of rapid flow velocity decrease. The phenomenon is known from literature and has been studied by the present author [3, 10–12].

In order to perform simulation tests, the flow system was modeled in the Multi Flower 2D package. The principles of design for the computer model were identical with those assumed during earlier tests and described in [4].

The distribution of water vapor's mass share is shown in Figure 8. It is qualitatively similar to the structures observed at the test station. The main difference is found in the area of the liquid jet flowing from the narrowing: in the simulation the cavitation appears in the jet's "shade" rather than its center. This is related to the low-pressure zone visible immediately past the narrowing (Figure 9 – the upper edge of the narrowing), which could mean that the influence of velocity is much greater in relation to the influence of pressure than assumed so far. It indicates the necessity for further work on the formula considering the influence of velocity on the progress of the mass exchange process.

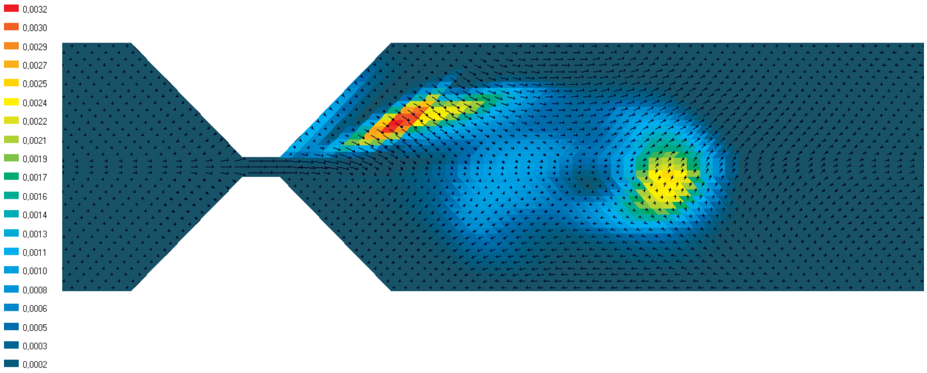

Figure 8. Distribution of water vapor's mass share in the recirculation area, Y_2 (with velocity vectors shown): $\alpha_w = 0.5$, $\alpha_z = 0.1$, iteration 8045

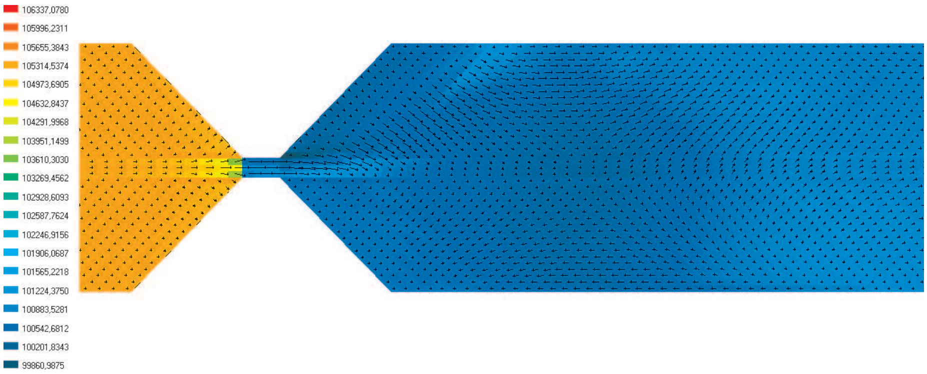

Figure 9. Distribution of static pressure in the recirculation area, p [Pa] (with velocity vectors shown): $\alpha_w = 0.5$, $\alpha_z = 0.1$, iteration 8045

The area of operation of the mass exchange equation, *i.e.* the area in which the source of liquid vapor assumes non-zero values, is shown in Figure 10. The intensity distribution of that source is also shown in Figure 11.

The structure of the cavitation cloud presented in Figures 8–11 is highly unstable and subject to frequent changes in the iteration process (although the bifurcation character of the flow remains unchanged at all times); the phenomenon is intensified after the oscillation equation has been switched on. Similar changes in the shape and intensity of the cavitation cloud have been observed during laboratory tests, although a precise confirmation of their similarities and differences requires

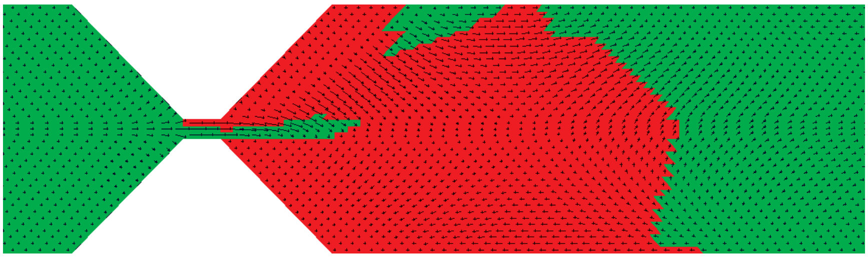

Figure 10. Area of operation of the mass exchange equation (with velocity vectors shown): $\alpha_w = 0.5$, $\alpha_z = 0.1$, iteration 8045

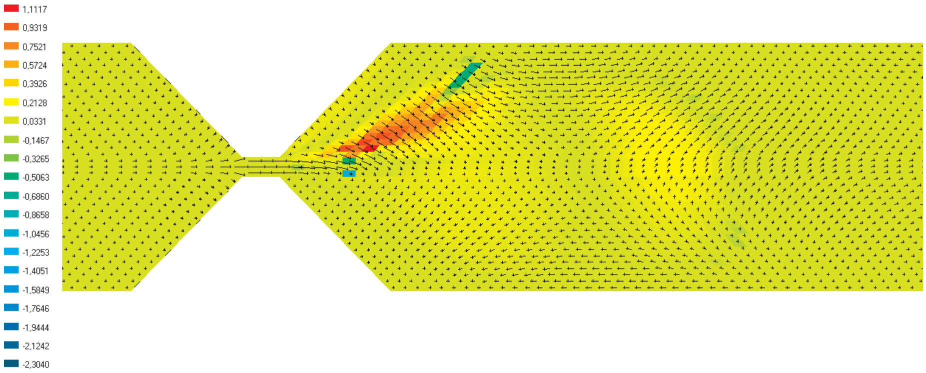

Figure 11. Intensity of water vapor's source (with velocity vectors shown): $\alpha_w = 0.5$, $\alpha_z = 0.1$, iteration 8045

conducting additional studies based on precise measurements of the vapor phase's quantity and distribution.

All the presented results of computer simulations were obtained with the same values of the α_w and α_z coefficients, with the oscillation equation switched off. The value of the model constant, $C^{\max}$, was 100.

Comparing the structure of the cavitation cloud observed at the test station with the results of computer simulations, one more difference is noticeable: bubbles of varying diameter and status of grouping are visible throughout the volume of the cavitation chamber, while the numerical simulations have shown compact and concentrated zones of bubble groupings instead. A partial solution for this problem could be decreasing the disappearance constant: the cavitation cloud will then have more "blurred" edges and a certain quantity of vapor phase will be present in the entire area of recirculation (bifurcation). An example is presented in Figure 12.

The sample's status after switching on the oscillation equation is shown in Figures 13 and 14; changes in the character of the cavitation cloud are much more

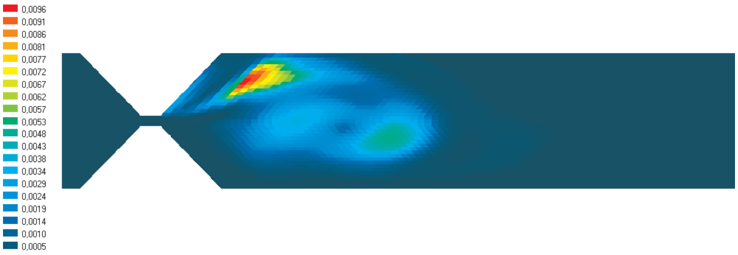

Figure 12. Distribution of water vapor's mass share in the recirculation area, Y_2: $\alpha_w = 0.5$, $\alpha_z = 0.0001$, iteration 11 000

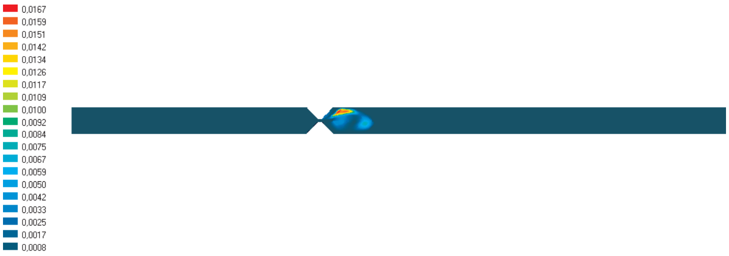

Figure 13. Distribution of water vapor's mass share in the recirculation area, Y_2: $\alpha_w = 0.5$, $\alpha_z = 0.1$, $A = 0.2$, $f = 2.0$, iteration 9100

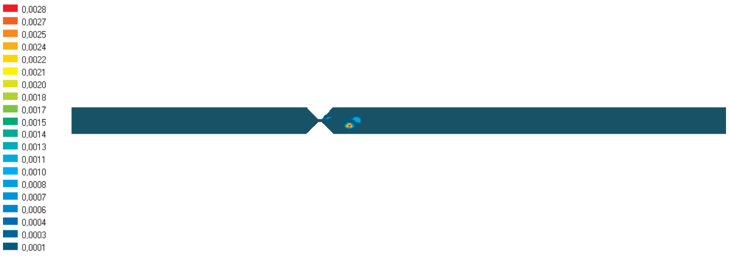

Figure 14. Distribution of water vapor's mass share in the recirculation area, Y_2: $\alpha_w = 0.5$, $\alpha_z = 0.1$, $A = 0.2$, $f = 2.0$, iteration 9200

dynamic in that case, causing much its faster movement and greater changes in its structure and intensity. Switching on the oscillation equation causes moments of total disappearance of the cavitation cloud – a similar phenomenon has been observed at the laboratory station.

15. Conclusion

The conducted laboratory tests and computer simulations have led us to the following conclusions concerning a nozzle and a duct with a narrowing:

- numerical methods enable correct modeling of the flow's structure, in particular the type of bifurcations frequently present in such cases;
- the mass exchange model presented in the paper yields results similar in quality to the results known from laboratory observations;
- the increase coefficient present in the model determines the maximum content of the vapor phase;
- the disappearance coefficient present in the model is responsible for the length of the cavitation cloud;
- qualitative differences in the area of jet formation after the narrowing suggest an excessively simplified velocity influence function – further studies should be carried out in this area;
- future work should aim at obtaining precise experimental data allowing scaling the model's coefficients, particularly the $C^{\max}$ constant and parameters of the oscillation equation.

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