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DYNAMIC CHARACTERISTICS OF COARSE-GRAINED TRICKLING FILTERS

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Abstract: A structural method of dimensioning trickling filters is proposed. Reactors applied in sanitary engineering are usually designed on the basis of formally simple technical instructions, but laboratory experiments have shown that the flow through a trickling filter can be described with the plug-flow model. With an additional function describing reaction intensity, the object's parameters can be calculated and its operation simulated for various technical conditions. If variability of the reaction rate is taken into account, numerical integration of the governing equations will be necessary.

Keywords: hydraulic design, waste disposal, trickling filters

Notation

c – concentration of dissolved matter
c_k – conservative tracer concentration
c_0 – initial concentration
c_w – degradable tracer concentration
d – mean size of filling elements
D_G – coefficient of hydrodynamic dispersion
D_M – coefficient of molecular diffusion
D_Z – trickling filter's diameter
F_T – cross-section area
H_Z – depth
i – index
I – number of components
I_H – hydraulic slope
k – reaction rate
k_F – hydraulic conductivity of porous medium
K_L – pipe-flow dispersivity
L – BOD of waste water
L_V – organic load

M_P – total mass
M_R – mass removed from the reactor
n – porosity
N – number of trickling filters
p – pressure
Pe – Peclet number
q – unit water consumption
q_F – hydraulic load
Q – discharge
Q_S – discharge for unit hydraulic slope
r – reactor's efficiency
r_{ef} – effective efficiency
t – time
t_M – modal time
t_P – plug-flow detention time
t_S – detention time
T – temperature
$\boldsymbol{u}$ – velocity vector
u_F – Darcy flux
V – total volume of trickling filter
V_A – gaseous fraction volume
V_F – filling volume
V_W – suspended water volume
V_P – porous space volume

1. Introduction

The term "reactor" denotes a technical object serving practical realization of chemical, physical and/or biological reactions or processes. As the intensity of these transformations depends on the development of the interfacial surface, the best efficiency can be obtained in fluid media (liquid or gaseous). Therefore, *a hydraulic reactor* should be considered as the basic system, although there are also solid-medium reactors (*e.g.* charcoal retorts) or intermediate ones (*e.g.* iron blast furnaces).

Generally, two regimes of hydraulic reactors' operation may be distinguished: *fluid-flow*, where reactants and products move continuously through the reactor, and *periodic*, in which the device works in cycles.

The interior of each reactor can be either *free*, *i.e.* void of significant obstacles or elements, or *filled*, *i.e.* containing a considerable number of technical elements, which cannot be neglected in the reactor's functional characteristics.

The following classification of filled reactors concerns the size of their internal elements. From this point of view, it is convenient to distinguish:

- *fine-grained reactors*, wherein the motion of fluid can be described by the classical Darcy law [1],
- *medium-grained reactors*, the internal motion of which is subject to the non-linear theory of filtration, and
- *coarse-grained reactors*, with the characteristic size of their filling elements so large that the flow through the reactor cannot be described in terms of filtration.

The filling of reactors, especially those of the third group above, may be either the reactor's technical equipment (*e.g.* cooling or heating coils [2]) or crushed medium – *active* (*e.g.* an ion exchanger) or *passive* (*e.g.* stone or plastic).

It should also be borne in mind, that the pore space of a reactor's packing can be filled with liquid to varying degree, which presents three possibilities:

- *a liquid reactor*, filled with liquid only,
- *a gaseous reactor*, where the active stream is a gaseous mixture,
- *a "dripping" reactor*, the pore space of which is partly occupied by a liquid medium.

This paper is a presentation of dynamic characteristics of dripping, fluid-flow and coarse-grained reactors applied in sanitary engineering. Trickling filters have been selected as the most typical example of this category of technical devices.

2. Functional description of reactor's operation

The general dynamic characteristics of a reactor, being a fluid-flow system, is given by the following variables: *velocity* $\boldsymbol{u}$, *pressure* p, *density* ϱ, *temperature* T and *concentration of each component* c_i (index $i=1,\ldots,I$).

These variables can be determined with various levels of accuracy. The most general method, based on the fundamental equations of mass, momentum and energy conservation [3, 4], is of no assistance in technical practice. These equations are so difficult formally, that their solution is expensive and time-consuming. Moreover, the information contained in this general solution is too complex and too rich for technical purposes. Therefore, simplified methods are very popular in engineering practice, sometimes going as far as algebraic technical guidelines. This attitude is very attractive, but methods of reactor design based on compact instructions have a very important disadvantage. As a matter of fact, these instructions are statistical in character, as they describe proper functioning of a number of similar objects, and they cannot simulate the structure of the considered processes.

Moreover, consistency of the chosen method is ever important. Let us consider a classical recommendation considering trickling filters as an example [5]. For a high rate reactor, one should assume the following hydraulic loading:

$$q_F = Q/F_T = 10\text{–}40\,\mathrm{m^3/m^2d} \tag{1}$$

and organic loading:

$$L_V = Qc/V = Qc/(F_T H_Z) = 0.50\text{–}2.50\,\mathrm{kgBOD_5/m^3d}. \tag{2}$$

Dividing these expressions side by side, one obtains:

$$L_V/q_F = c/H_Z = 0.05\text{–}0.06\,\mathrm{kgBOD_5/m^4d}. \tag{3}$$

Assuming that the $\mathrm{BOD_5}$ of the mechanically treated waste water equals $c = 0.2\,\mathrm{kgBOD_5/m^3}$ [6], we can state that the filter's depth should be equal to:

$$H_Z = 3.3\text{–}4.0\,\mathrm{m}, \tag{4}$$

in disagreement with the value of $H_Z = 0.9\text{–}2.4\,\mathrm{m}$ recommended in [5].

Thus, in spite of their attractive formal simplicity, oversimplified methods should be applied very carefully. Structural methods are recommendable instead: they are more difficult but much more precise.

Practical experience appears to prefer dynamic characteristics of reactors containing (see *e.g.* [7, 8]):

- the terminal concentration of a conservative tracer, introduced into the reactor's inflow as an impulse, and
- functions describing the kinetics of each considered reaction or transformation.

The $c_k(t)$ function offers information about the detention time for each sub-element of the mass entering the system as a whole at the same moment of time (see Figure 1a). This function can be determined with varying accuracy (from high precision to rough approximation, see [7]), theoretically or using experimental tracer methods [9–12].

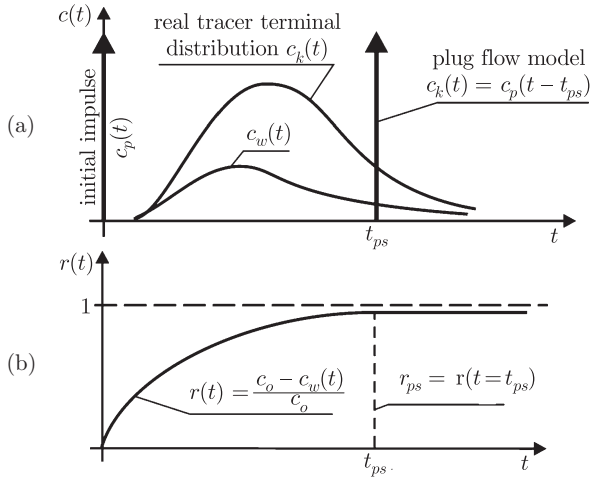

Figure 1. Functional characteristics of a reactor

The other element of reactor characteristics expresses the reaction rates. The following reactor efficiency can be used for the especially important processes of water and waste water purification (Figure 1b):

$$r(t) = \frac{c_o - c_w}{c_o}. \tag{5}$$

The function can be found theoretically for some simple cases, often under the assumption that the interesting process can be treated as a first-order reaction [13]. When the considered phenomenon is more complicated, the $r(t)$ function should be determined experimentally.

The $r(t)$ expression enables us to recalculate the terminal concentration of a conservative tracer, $c_k(t)$, into the concentration of a degradable component, $c_w(t)$:

$$c_w(t) = c_k(t)[1 - r(t)]. \tag{6}$$

The total mass of the component introduced as an impulse into the reactor equals:

$$M_P = \int_0^\infty c_k(t) Q \, dt, \tag{7}$$

whereas the mass removed from the reactor is equal to:

$$M_R = \int_0^\infty [c_k(t) - c_w(t)] Q \, dt. \tag{8}$$

The effective efficiency of the reactor can be expressed by the following evident expression:

$$r_{ef} = M_R / M_P. \tag{9}$$

This important parameter of the reactor's operation is usually given in advance and the engineer should design it so as to fulfill this requirement.

3. Dynamic characteristics of coarse-grained reactors

The velocity field of the considered category of reactors is so complex that the $c_k(t)$ function should be determined empirically in this case. In order to investigate this important kind of water or sewage treatment devices, two laboratory stands have been set up (see Figure 2 for their general schema) [14, 15].

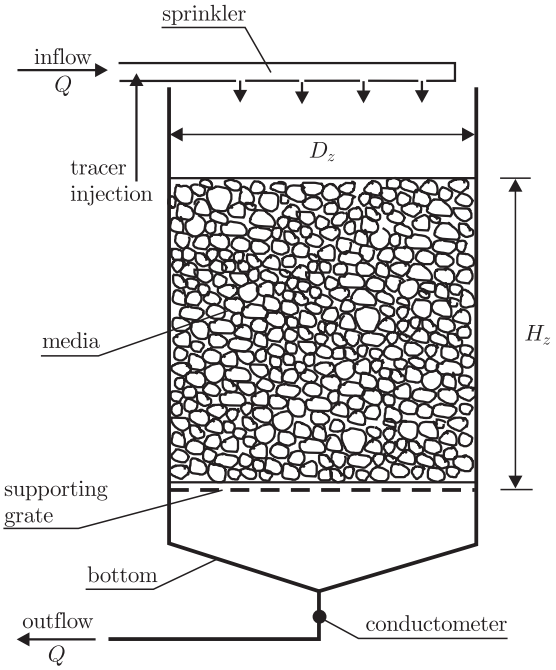

Figure 2. Schema of the experimental stands

One of them (of $D_{ZK} = 0.375\,\mathrm{m}$) was filled with KERAMSIT ceramic sinter ($d_K = 20\,\mathrm{mm}$, $n = 47\%$) to three different depths ($H_{ZK1} = 0.3\,\mathrm{m}$, $H_{ZK2} = 0.4\,\mathrm{m}$, $H_{ZK3} = 0.6\,\mathrm{m}$). The other (of $D_{ZH} = 0.66\,\mathrm{m}$) contained openwork plastic rings called HUFO ($d_H = 130\,\mathrm{mm}$, $n = 97\%$, $H_{ZH} = 0.41\,\mathrm{m}$).

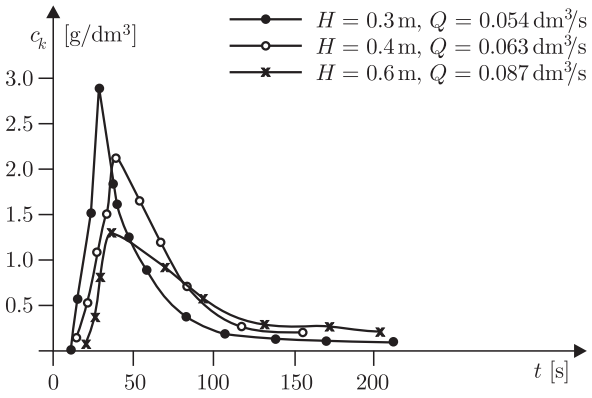

Figure 3. Terminal tracer concentration for KERAMSIT TF

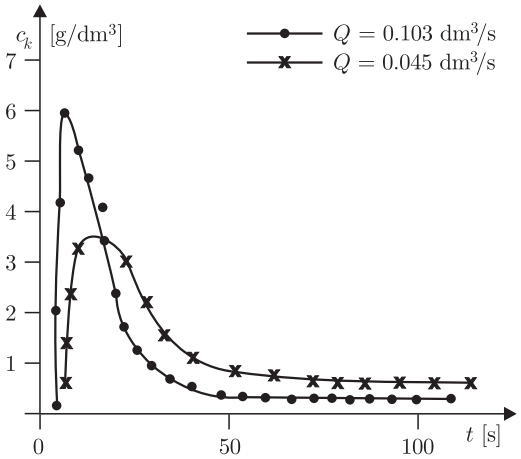

Figure 4. Terminal tracer concentration for HUFO TF

Both laboratory filters were fed dripping water. A tracer (water solution of NaCl, $c_0 = 100\,\mathrm{g/dm^3}$) was introduced into the supply conduit in so short an instant of time that its initial concentration could be treated as an impulse (see Figure 1). The tracer's terminal concentration of was measured with a conductometer (see Figure 2). Selected examples of the measured $c_k(t)$ function are shown in Figure 3 (for the KERAMSIT medium) and Figure 4 (for the HUFO medium).

The shapes of the obtained lines suggest that mass dispersion in the investigated objects is relatively low. Their ascending and descending parts are rather steep. The fields below the diagrams (proportional to the total mass of the tracer) are narrow and the characteristic "tails" behind the curves (due to adhesion and "dead zones") are moderate.

The conclusion concerning low intensity of dispersion is consistent with the theoretical evaluation of the dispersion coefficient. For the typical hydraulic load of a trickling filter (about $q_F = 10\,\mathrm{m/d}$) and mean filling elements' size of $d = 0.10\,\mathrm{m}$, the Peclet number is equal to:

$$\mathrm{Pe} = \frac{Q_F \cdot d}{D_M} = 10\,000, \tag{10}$$

given molecular diffusivity $D_M = 10\text{m}^2/\text{d}$, so the coefficient of hydrodynamic dispersion equals [16]:

$$D_G = 1.4\text{Pe}D_M = 1.4\text{m}^2/\text{d}. \tag{11}$$

The corresponding value for a pipe flow according to the Taylor formula [1] would be much higher for similar parameters:

$$K_L = \frac{q_F^2 d^2}{192 D_M}. \tag{12}$$

Another important conclusion from these measurements is good compatibility of the modal detention time, t_M (*i.e.* the time after which the maximal concentration appears in the outflow), and the mean detention time, t_S:

$$t_S = V_W/Q. \tag{13}$$

This conformity is presented in Figure 5.

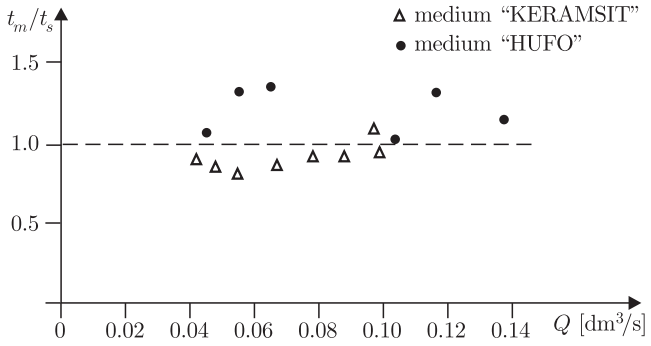

Figure 5. Comparison of modal and mean detention time

The above conclusions enable us to assume that *the flow through a dripping reactor can be described by the plug-flow model* (Figure 1a).

Generally speaking, this statement is not a completely new conclusion, as similar opinions have been formulated in earlier papers (*e.g.* [2]). However, these investigations were performed for liquid or gaseous reactors (completely filled with the working fluid), where the total reactor volume V was the sum of the filling volume, V_F, and the fluid volume, V_W (equal to the pore space volume, V_P):

$$V = V_F + V_W = (1-n)V + nV. \tag{14}$$

However, we have a different situation in the considered case, as the pore space, V_P, is occupied by two fractions – liquid V_W and gaseous V_A:

$$V = V_F + V_A + V_W. \tag{15}$$

This means, that we deal with a special case of the plug flow, when the mean detention time, t_S, should be calculated for the volume of liquid "suspended" in the trickling filter.

4. Determination of the suspended liquid volume

The experimental points in Figure 5 were obtained for measured values t_M (*e.g.* Figures 3 and 4) and calculated values t_S (Equation (13)), where volume V_W

was also measured. In order to make it possible, the laboratory stand was equipped with two valves, an initial and a terminal one. At the final stage of each experiment, both valves were closed at the same moment, so that the water suspended in the system could not leave and remained at the reactor's bottom. Measurement of its volume V_W posed no problem.

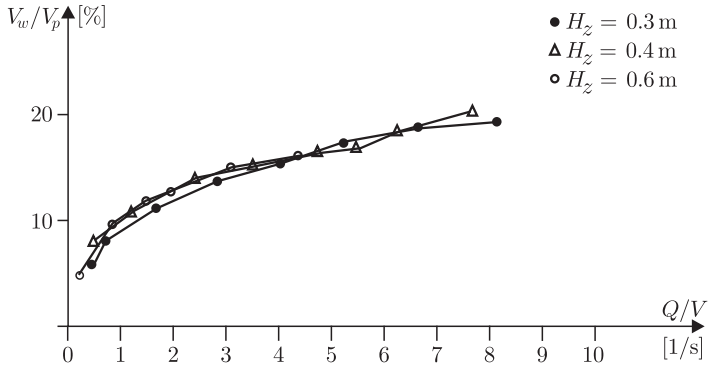

Figure 6. Volume of suspended water (KERAMSIT medium)

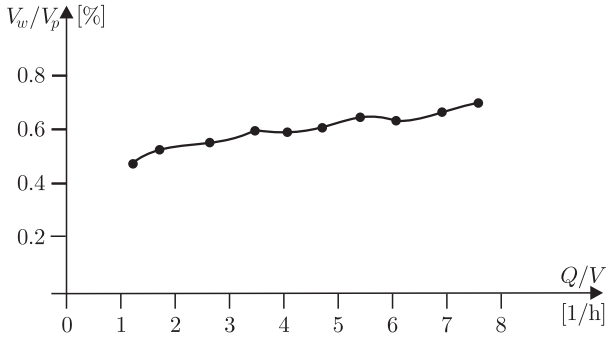

Figure 7. Volume of suspended water (HUFO medium)

This experimental procedure can be applied only for existing devices. Hence, it would appear that the $V_W(Q)$ function should be an element of technical characteristics of every kind of reactor fittings, determined in advance and provided by the manufacturer. The lines in Figures 6 and 7 show two examples of the equivalent expression, $V_W/V_P = f(Q/V)$, determined for the two investigated media.

However, a theoretical method of determining volume V_W would be desirable. In order to derive such a relation, let us note that the value under consideration is limited by the following terminal levels:

$$V_W = \begin{cases} 0 \text{ (and } V_A = V_P = nV) & \text{when } Q = 0, \\ V_P \text{ (and } V_A = 0) & \text{when } Q = Q_S. \end{cases} \tag{16}$$

Discharge Q_S appears when gravitational flow persists, but the whole pore space of the filter is filled with liquid and its free surface is identical with the upper surface of the filling, what means that the hydraulic slope

$$I_H = 1. \tag{17}$$

For systems which can be described by the classical Darcy law, we have:

$$u_F = q_F = k_F, \tag{18}$$

$$Q_S = q_F F = k_F \frac{\pi D_z^2}{4}, \tag{19}$$

in this case.

Unfortunately, determination of hydraulic conductivity was impossible for two investigated cases, as they were not subject to the filtration laws. Therefore, in order to obtain at least approximated results, an experiment was carried out for the fine-grained medium (gravel, $n = 72\%$, $k_F = 0.0117\,\mathrm{m/s}$, $D_Z = 0.195\,\mathrm{m}$, $H_Z = 0.86\,\mathrm{m}$ – Figure 2). The value of $Q_S = 0.35\,\mathrm{dm^3/s}$ was obtained from Equation (19) (when $V_P = 18.5\,\mathrm{dm^3}$). Empirically determined values of $V_W < V_P$ obtained for 12 different discharges $Q < Q_S$ are shown in Figure 8.

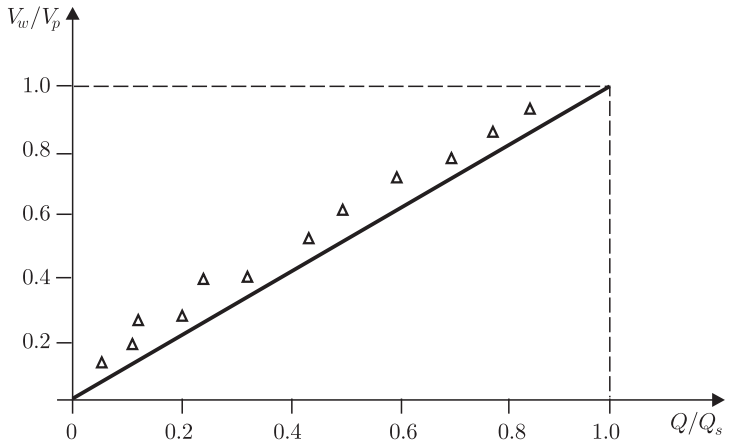

Figure 8. Volume of suspended water (gravel medium)

The set of experimental points yields a convex line, but its curvature is so moderate that for the practical purposes it can be approximated to the straight line given by the following formula:

$$V_W/V_P = Q/Q_S. \tag{20}$$

This theoretical result (supported with experimental arguments) would be the most convenient way of determining V_W. Otherwise, one should be satisfied with proper empirical information (*e.g.* Figures 6 and 7).

5. Determination of the reaction rate

The final element necessary in reactor design is an $r(t)$ curve. As has been mentioned above, this line can be described experimentally or but theoretically, provided that the course of the considered reaction is regular enough [13].

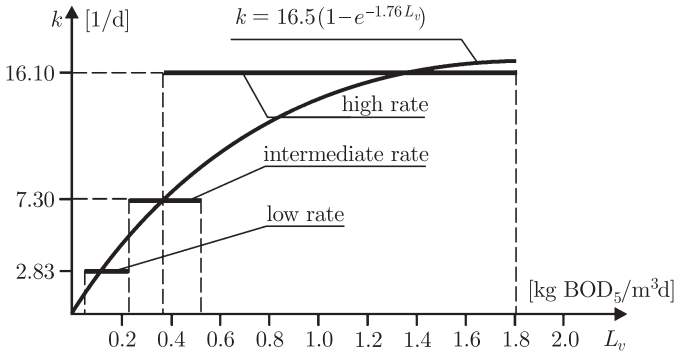

Figure 9. Reaction rate constant, k, versus organic loading, L_V

The first-order reaction model is often applied in technical practice. According to this model, we can write:

$$r(t) = 1 - e^{-kt}, \tag{21}$$

which means that the $r(t)$ curve is univocally described by the reaction rate, k. An analysis of the experimental data (*e.g.* [9]) suggests that the k coefficient depends in this case on the organic load of the trickling filter (Figure 9).

6. Dimensioning of trickling filters: algebraic and numerical models

On the basis of the above considerations, the following method of dimensioning trickling filters can be proposed:

Preliminary decisions: determination of waste water discharge, Q, the initial concentration of harmful component (*e.g.* BOD), c_0, the required terminal concentration of this component, c_W, and the reactor's loading (low, intensive, high rate, *etc.*).

Designing procedure:

1. choice of reaction rate (*e.g.* Figure 9),
2. calculation of the necessary reduction of organic matter, r_0 (Equation (5)),
3. determination of the computational detention time, t_S (*e.g.* Equation (21) for $r = r_0$),
4. calculation of the necessary volume of suspended waste water, V_W (Equation (13)),
5. calculation of the necessary porous space, V_P (*e.g.* Figures 6 and 7),
6. calculation of the necessary volume of filling:

$$V = V_P/n, \tag{22}$$

7. choice of filter depth, H_Z (*e.g.* technical instructions [5]),
8. calculations of the filter's cross-section area:

$$F_T = V/H_Z, \tag{23}$$

9. choice of the number of reactions, N, and calculation of each unit's diameter:

$$D_Z = \sqrt{4F_T/N\pi}. \tag{24}$$

Let us consider a housing estate for $M = 250$ inhabitants as an example. For a unit water consumption of $q = 0.2$ m^3/Md, the total sewage discharge equals $Q = 50\,\mathrm{m}^3/\mathrm{d}$. Let us assume that the initial BOD_5 equals $c_0 = 250\,\mathrm{g/M}$ [6] and that the necessary reduction of organic matter equals $r_0 = 90\%$. Let us assume that the trickling filter will be filled with rock medium ($n = 60\%$) and operate as a high rate reactor ($k = 16.11/\mathrm{d} = 0.671/\mathrm{h}$).

According to the procedure described above, we obtain: $t_S = 3.4\,\mathrm{h}$, $V_W = 7.0$ m^3, $V_P = 12.25\,\mathrm{m}^3$ (for $V_W/V_P = 0.57$) and $V = 20.4\,\mathrm{m}^3$. Assuming that $H_Z = 2.0\,\mathrm{m}$, we have $F_T = 10.2\,\mathrm{m}^2$, so we can design $N = 3$ units of diameter $D_Z = 2.1\,\mathrm{m}$ each.

Arguably, formally equivalent results could be obtained with the traditional, simplified method. However, as has been mentioned above, we would then be unable to discuss the reactor's functioning in various situations, analysis of which can be performed using the method proposed in this paper.

For instance, let us determine the influence of twenty-four hours' variation in waste-water discharge. Let us assume that the mean discharge, $Q = 50\,\mathrm{m}^3/\mathrm{d}$, varies from $Q_M = 25\,\mathrm{m}^3/\mathrm{d}$ to $Q_D = 75\,\mathrm{m}^3/\mathrm{d}$. We respectively obtain $t_{SM} = 6.72\,\mathrm{h}$ and $t_{SD} = 2.23\,\mathrm{h}$ for these two parameters, resulting in different expected levels of BOD biodegradation: $r_M = 98.8\%$ and $r_D = 77.6\%$.

These calculations have been performed by means of the algebraic model. Its formal simplicity was achieved by assuming the reaction rate to be constant. However, analysis of empirical data (see Figure 9) has shown that this parameter varies and depends on the trickling filter's organic load:

$$L_v = \frac{LQ}{V} = \frac{L}{t_P}. \tag{25}$$

The data presented in Figure 9 were approximated with the empirical formula (obtained by means of the least square method):

$$k(L) = 16.5(1 - e^{-1.76L/t_P}). \tag{26}$$

Substituting this expression into the equation of dissolved matter transfer (for the plug-flow model, demonstrated above to be acceptable for this case), we obtained [17]:

$$\frac{DL}{Dt} = -1.65(1 - e^{-1.76L/t_P})L = F, \tag{27}$$

with a simple and obvious initial condition:

$$t = 0 - L = L_0. \tag{28}$$

Equation (27) was solved numerically by means of the Runge-Kutta method [18] according to the following relation:

$$L_{j+1} = L_j + \frac{1}{6}(K_1 + 4K_2 + K_3), \tag{29}$$

where:

$$K_1 = F(L_j)\Delta t, \quad K_2 = F(L_j + \frac{1}{2}K_1)\Delta t, \quad K_3 = F(L_j + K_1 - 2K_2)\Delta t. \tag{30}$$

Equation (27) was integrated from $t = 0$ up to $t = t_P$ (for $\Delta t = t_P/100$). This period of time is long enough from the practical point of view, because in the face of

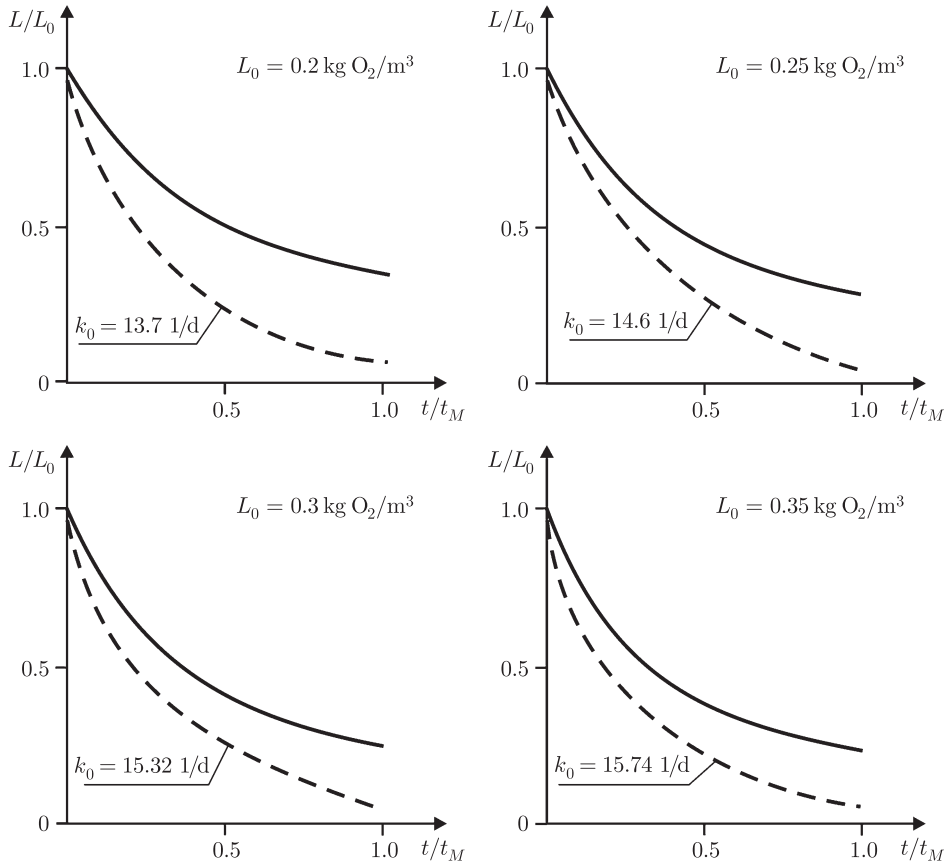

Figure 10. Numerical solutions of Equation (27) (solid line – numerical solution of Equation (27), dashed line – analytical formula, Equation (6))

Equations (13) and (25) we have $t_P \gg t_S$ (as $V \gg V_W$). Consequently the diagram shown in Figure 10, which presents an example of numerical results, can be used to determine the level of reduction of pollutants' concentration for real waste-water detention times, t_S.

The obtained diagram shows that the actual course of the considered process of sewage treatment (approximated by Equation (27)) is less intensive than that resulting from the commonly accepted assumption that $k = k_0 = \text{const}$, when we have

$$\frac{DL}{Dt} = -k_0 L, \tag{31}$$

instead of Equation (27), whence:

$$L(t) = L_0 e^{-k_0 t}. \tag{32}$$

This is a consequence of the simplifying assumption that the reaction rate is the same throughout the process, while in fact (see Figure 9) the coefficient decreases together with the drop of BOD, thus reducing the reaction's effectiveness. This conclusion is very important for dimensioning trickling filters, demonstrating the importance of analyzing the structure of the considered process. The method proposed above offers this possibility.

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