DETERMINATION OF SELECTED PARAMETERS IN A 1D OPEN CHANNEL FLOW MODEL

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(Received 15 July 2007)

Abstract: Determination of the model's parameters is an important stage of mathematical models' application. In the case of a free-surface 1D unsteady flow model defined by the de Saint-Venant equations, one of the groups of parameters to be estimated is the set of parameters describing energy losses due to friction. The parameters can be estimated in different ways, but in most cases the task of their determination is an ill-posed problem. In such cases, optimization methods are the most common approach. In spite of numerous examples of such applications, these techniques are still not fully recognized, as there are several problems of different nature that require thorough analysis. Automatic optimization methods are discussed in the paper. The most important questions of choosing the objective function and the optimization algorithm are considered. Problems connected with data reliability and accessibility and their influence on the solution are discussed. The most common pitfalls of optimization applications are discussed. The analysis is supported with numerical examples.

Keywords: determination of parameters, optimization procedures, well-posed and ill-posed problems, numerical methods

1. Introduction

A condition of proper and effective application of mathematical models is proper estimation of its parameters. As the quality of such estimates obviously influences the accuracy of results obtained from the model's application, it is an important stage in the process of model construction. From the formal point of view, the task of parameter determination can be qualified as an *inverse* problem. Solving inverse problems is generally more difficult than solving *classical* (or *conventional*, *direct*) problems. As inverse problems must often be considered ill-posed, it is more difficult to achieve uniqueness and stability of their solutions, which can be very sensitive to minor changes of the model's input.

In the case of problems of identification, the parameters to be estimated can be diversified in character: from those which have a specified physical meaning and a clear physical interpretation to totally conceptual parameters, artificially introduced to the model by its user. Thus, various methods are applied to determination and their comprehensive and full classification would be difficult.

Several groups of parameters to be identified can be distinguished in the de Saint-Venant equations, the most popular mathematical description of 1D free-surface unsteady flow, which can be presented in the following form [1]:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial H}{\partial x} + gA(S_f - S_o) = 0, \tag{2}$$

where Q is flow discharge, H – water depth at the cross-section, A – the cross-section's area, S_f – hydraulic slope, S_0 – channel bottom slope, g – acceleration due to gravity, x – space and t – time.

The basic groups of parameters are connected with channel geometry and energy losses due to friction. If any additional factors influencing unsteady flow in the channel are included in the phenomenon's mathematical description, *e.g.* lateral inflow to the channel or the influence of wind, the list of parameters to be estimated will be extended to include values characterizing the analyzed factors. While the parameters describing channel geometry are usually measurable and estimation of their values is usually an experimental problem (or a question of approximation on the basis of measurements), the other group of parameters – those characterizing friction in the channel – are not measurable and require other methods to determine their values.

The parameters connected with friction losses in the channel are the coefficients appearing in the formula describing the friction term, or – more precisely – in the formula for hydraulic slope, S_f, which can be presented in its general form as follows [2, 3]:

$$S_f = \frac{rU|U|^{r_1-1}}{R^m}, \tag{3}$$

where R is hydraulic radius, U – average flow velocity and r, r_1 and m are coefficients, of which r is dependent on m, r_1 and the roughness of the channel. The r_1 parameter is connected with the type of flow in the channel and may assume the following values [3]:

- $r_1 = 1.00$ for laminar flow,
- $r_1 = 1.75$ for flow with mild turbulences,
- $r_1 = 2.00$ for fully developed turbulent flow

and values from the $\langle 1.75 - 2.00\rangle$ range for intermediate forms of turbulent flow.

As it is well known [4] that every maxi-scale flow of surface water is turbulent, it is most often assumed that $r_1 = 2.00$. Then formula (3) assumes the following form:

$$S_f = \frac{rU|U|}{R^m}, \tag{4}$$

which – by analogy to the commonly applied Manning formula – can be expressed as:

$$S_f = \frac{n^2U|U|}{R^m} \tag{5}$$

and $r = n^2$. If $m = 4/3$, n can be identified with Manning's roughness coefficient.

There are also empirical formulas, *e.g.* Pawlovsky's formula [5], representing the functional dependences between parameter m, hydraulic radius R and parameter n. However, it is usually assumed that the mentioned parameters are independent and the values of n and m (or n only, when the value of m is assumed, *e.g.* $m=4/3$) are searched in the process of determination. The number of estimated parameters is eventually reduced to a single coefficient n or a set of coefficients n when a channel of variable roughness or a network of channels is considered.

In the general case, the roughness coefficient is variable along the channel. It may also assume different values in a cross-section, due to diversified roughness of various sections of the wetted perimeter. In natural conditions, the coefficient may be a function of flow discharge, Q, or water stage, h [6]. Moreover, it usually depends on many other factors [7], which renders accurate estimation of its value extremely difficult or even impossible. Thus, the dependence of the roughness coefficient on flow discharge and water stage is sometimes neglected and for estimated the analyzed cross-section as a substitutive coefficient representative for the whole cross-section. In order to simplify the problem, the channel is often segmented into several sections of constant roughness or a uniform value of n is assumed for its whole length. Even if such assumptions are made, the problem of estimation of roughness coefficients is complicated. It becomes even more complex in the case of channel networks.

Determination of parameters describing friction in open channels is most often an ill-posed problem. There are only several, very simplified cases of free-surface flow for which proper formulation of the problem yielding a unique and stable solution is possible [8, 9]. In most often cases, application of numerical methods in solving the de Saint-Venant equations produces a set of algebraic equations in which the number of equations is not equal to the number of unknowns [10]. The problem so formulated does not have a unique solution [11] and other methods of parameter estimation must be considered.

2. Methods of determining friction parameters

The ways of determining parameters describing friction evolved with the technical progress in calculations and measurements. The first methods of roughness estimation were connected with observations of the channel – its shape, condition, cross-section dimensions, irregularities in cross-section shapes and the ground surface, the intensity and kind of vegetation, additional obstacles in the channel, *etc.* [7, 10]. The observations were sometimes supported with analogy to other channels of known roughness parameters. However, this kind of estimation suffers from relatively high subjectivity its accuracy depends on the researcher's experience. It is also possible to estimate the value of roughness coefficient on the basis of observations according to the procedure proposed by Cowan [7]. In this approach, the coefficient is calculated as a sum of components taking into account various roughness-influencing factors and the obtained sum is multiplied by a correcting coefficient, taking into account the channel winding. The values of components are estimated on the basis of tables.

The methods connected with observations continue to be applied nowadays. More accurate procedures of roughness estimation were developed on their basis, including detailed analysis of vegetation of the flood plains as well as the main channel [12, 13], and thus requiring a detailed description of the vegetation's structure or application of its substitutive structure. This requires relatively large volumes of reliable data.

A popular approach is to estimate roughness on the basis of tabular values. Chow [7] also presents a method of its determination on the basis of velocity profiles in channel cross-sections. Sometimes the formulas for steady flow are applied to unsteady cases, an approach proposed by Vervey, Baltzer and Lai [10]. However, the trial-and-error method persists as one of the most commonly used methods of estimating roughness. It requires observed flow and/or stage hydrographic data, which may render the estimation more difficult, but at the same time more adequate in the case of unsteady conditions in the channel. However, this method is subjective, as it requires visual comparison of simulated and observed values of flow and/or water stage. It can also be difficult to recognize the model's sensitivity to parameter changes, especially when the number of parameters is high.

All the methods presented above suffer from a relatively high degree of subjectivity, their accuracy is relatively low or they require large amounts of data (*e.g.* connected with the structure of vegetation), experience, or numerous calculations (*e.g.* the trial-and-error method). Moreover, most of the approaches presented above are oblivious to variations of parameters due to the type of flow in the channel. Values obtained for steady flow can be totally different from those describing unsteady conditions. It is therefore purposeful to search for other methods of solving this problem.

The ideas presented above can be replaced with a more formal approach, in which the problem of parameter estimation is formulated as an inverse problem, well- or ill-posed, depending on the analyzed case. As a result, the degree of subjectivity decreases, the effectiveness of calculations increases and in some cases it is possible to obtain a unique solution.

Unfortunately, cases for which well-posed problems can be formulated are quite rare, though very interesting. A properly posed system of equations satisfies three conditions: (i) there exists a solution, which is (ii) unique and (iii) stable [14]. One of such cases is a steady flow in a single channel of constant roughness, n, for which the flow discharge in the channel and water levels in upstream and downstream cross-sections are known. In such a case, a well-posed boundary problem can be formulated, the number of equations in the mathematical model is equal to the number of unknowns and it is possible to obtain a unique and stable solution [8, 9]. However, in most other cases, especially unsteady ones, the problem cannot be formulated as well-posed. In such situations automatic optimization procedures can be applied.

3. Theoretical background of automatic optimization

Automatic optimization is an approach in which a systematic iterative procedure is involved to search a set of parameter values for which a chosen function (called the optimization criterion) assumes its extreme value. As the optimization criterion, sometimes referred to as the "objective function", is usually formulated as a chosen error criterion, optimization is in such cases a problem of minimizing the objective function. The function chosen as an optimization criterion compares the simulated and observed values of flow and/or water stage for each set of parameters and the optimal values of parameters are found as a result, for which simulations are the closest to observations. A schema of the optimization problem is presented in Figure 1. The idea is similar to that of the trial-and-error method, but the whole process is automatic and parameter "improvement" is realized according to the chosen automatic procedure. Moreover, the optimal set of parameters is found not on the basis of visual comparison but on the basis of values of the objective criterion. Thus, the problems of subjectivity and large number of trial-and-error calculations are overcome.

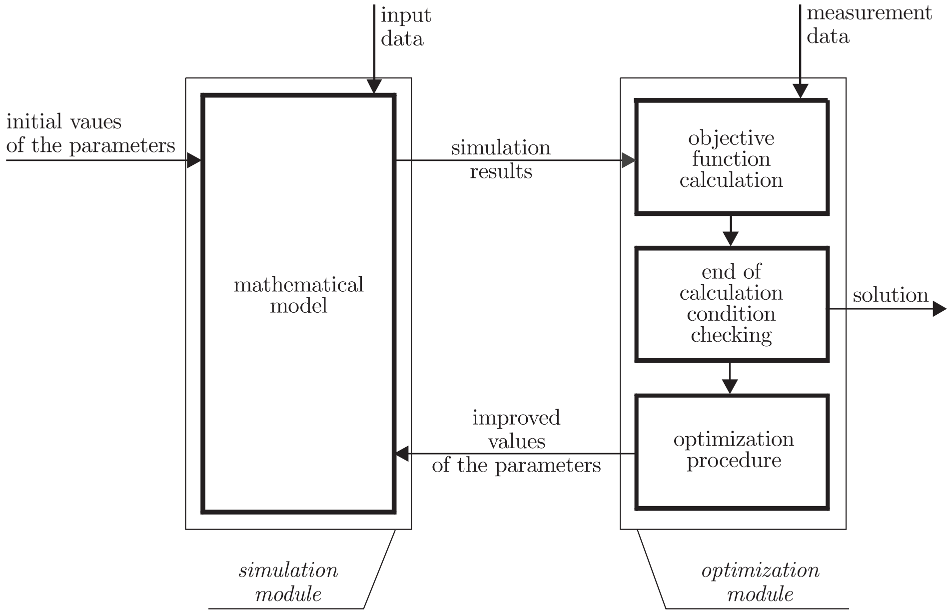

Figure 1. Schema of an automatic calibration (optimization) problem

When considering optimization procedures, an important aspect connected with data quality should be taken into account. It is important to remember that both observations and simulations suffer from different kinds of errors, *e.g.* measurement errors (including "gross errors" resulting from human mistakes, "random errors" resulting from a lack of precision, errors connected with irregular channel geometry, *etc.*), model errors (associated with imperfection of the phenomenon's mathematical description) and numerical errors (rounding errors, local truncation errors, *etc.*) [9, 10]. In the absence of such errors, automatic calibration would lead to exact, true values of the searched parameters. However, in practice, due to errors the results of calibration are "optimal" in the sense of minimizing the objective function and the "best" values of parameters will suffer from the influence of the errors involved. While it appears to be unavoidable in practical problems, the influence of the errors can be examined in theoretical cases based on synthetic data.

Automatic calibration is a very popular approach to solving calibration problems of diverse nature, not only those connected with open channel flow. Optimization procedures are widely applied in various ill-posed problems of many disciplines of science, in civil engineering, ground water flow, conceptual models, *etc.* In this range of applications, investigations in the field of open channel flows are relatively rare [10], however, many examples of such implementation can be found in the literature.

The first applications of free-surface flow model parameter optimization are usually attributed to Becker and Yeh [15, 16]. Later examples were presented in [6, 10, 17–22] and many other works. The various approaches differ in their choice of the optimization method, the objective function, flow conditions, error analysis, *etc.* After more than thirty years of experience in application of automatic calibration in many fields, there is a wide spectrum of analyzed types of objective functions and tested optimization procedures. One could presume that the problem is well-recognized now and easily solvable. However, despite the years of experience, the problem of parameter optimization is still not an easy one, especially for more complex cases (*e.g.* unsteady flow in a channel of variable roughness). This is due to various reasons, mostly connected with practical aspects of properly choosing the objective function and optimization procedures for particular cases and with limited accessibility and reliability of data, often determining the choice of optimization method and the calculations' accuracy. Some of the problems mentioned above are described and illustrated with numerical examples below.

4. Practical aspects of applying optimization procedures

As has been mentioned above, application of the optimization algorithm is connected with two main problems: (a) the choice of an error criterion and an objective function and (b) the choice of an optimization method, *i.e.* the algorithm of the objective function's minimization. Although there are many theoretical possibilities in this respect, as there is a wide range of objective functions and optimization methods, the problem is quite important and not trivial. The form of the optimization criterion and the type of the optimization procedure not only influence the duration and accuracy of calculations, but may produce incorrect parameter values if the choice of method has been improper. Another limitation is the accessibility of measurement data, key information in each optimization problem. The amount, kind and quality of data have a strong influence on the calculation results and often determine the choice of the objective function. Thus, a seemingly easy task of choosing the optimization method and error criterion appears to be a fundamental aspect of properly run calculations. As in many examples of automatic calibration applications, the most popular and simplest form of error criterion is chosen. The optimization method is chosen from a list of those most popular. Certain aspects of this choice will be discussed more thoroughly on the example of roughness parameter identification for the de Saint-Venant equations.

4.1. The choice of the objective function in friction parameter determination

As has been mentioned above, there are many possible forms of objective functions applicable in optimization problems for free-surface flow. The choice of the right formula is very important. The shape of the objective function for a particular case may facilitate and quicken the calculations or – on the contrary – complicate them, in extreme cases rendering them entirely inefficient and leading to false solutions. An objective function can have a single extreme or many local optima.

Moreover, it may "judge" the quality of the chosen parameters more or less severely and react with varying sensitivity to minor parameter changes. (It can even be insensitive to some of the parameters). The objective function can exhibit varying sensitivity to the bias of observation data, which can be examined if synthetically generated observation data are applied. Last but not least, it can yield good matching of one variable (*e.g.* water stages), while another (*e.g.* flow discharge) can be far from the "true" one in the "optimal" case. Thus the choice of objective function may prove to be essential, especially in more complex cases.

An ample survey of such functions can be found in [23]. An analysis of the above-mentioned aspects can also be found in [9, 10]. Nevertheless, the most popular optimization criterion continues to be the sum of squares of errors:

$$F=\sum_{i=1}^{N}\left(\bar{Y}_i-Y_i\right)^2, \tag{6}$$

where $\bar{Y}_i$ – an observed value, Y_i – a simulated (calculated) value, i – the temporal subscript (the index of the number of observations) and N – the total number of observations.

The following modifications of criterion (6) can be found in the literature (see [22, 24]):

$$F=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\bar{Y}_i-Y_i\right)^2}, \tag{7}$$

$$F=\sqrt{\frac{1}{N}\sum_{i=1}^{N}w\left(\bar{Y}_i-Y_i\right)^2}, \tag{8}$$

where w is a weight coefficient enabling better fitting of the time of occurrence and the maximum value of the observed variable, as well as many other forms of objective functions of various complexity. Authors have concluded that, in spite of the numerous possibilities of modification, there is often no justification for these forms and little advantage over the "traditional" criterion (5). Khatibi *et al.* have presented two other formulas for the error criterion (see [10]):

$$F=\sum_{i=1}^{N}\left(\frac{\bar{Y}_i-Y_i}{\bar{Y}_i}\right)^2, \tag{9}$$

or

$$F=\sum_{i=1}^{N}\left(\frac{\bar{Y}_i-Y_i}{Y_i}\right)^2. \tag{10}$$

Although analyzed in the literature on synthetic observation data, most of the above-mentioned formulas are very seldom applied in practice. The most popular formula seems to be (6) [10, 15, 16, 18, 25], rarely (9) or (10) [10].

When parameter optimization for a free-surface flow is analyzed, the choice of the error criterion is also connected with the physical aspects of the hydraulic problem. Unsteady flow in an open channel is known to be described by two types of unknowns (*e.g.* water stage and flow discharge, or water depth and flow velocity). Two functions describing the evolution of these variables are obtained as the result of the classical unsteady flow problems' solution. It is thus obvious that parameter estimation (in this case – of friction parameters) should be realized on the basis of information on measurements of both of the variables. In other words, in order to determine the friction parameters properly, one should have both water stage and flow discharge observations available. This involves two kinds of problems.

One of the problems is connected with the accessibility of data of both types of variables. Most often, it is relatively easy to observe water stages at selected cross-sections (usually only a few cross-sections along the channel). However, the practical possibilities of obtaining flow discharge values are much more limited; in many cases such measurements cannot be performed. Paradoxically, the sensitivity of water stages to the friction parameter is subject to relatively small changes. If water stage is the only variable observed, the optimization criterion usually assumes the following form:

$$F=\sum_{i=1}^{N}(\bar{h}_i-h_i)^2 \tag{11}$$

and „optimal" parameters values are obtained as a result of its application, but only in the sense of convergence of the calculated and observed water stage values. However, it is often relatively easy to find another set of parameter values (even by trial and error), for which the observed and calculated stage hydrographs are also very well matched, sometimes yielding visually identical result. As water stage observations are always subject to measurement errors, the optimization run may thus lead to false parameter values, especially when the number of observation samples is limited.

Variations of friction parameters have a stronger influence on flow discharge. The shape of the discharge hydrograph is much more sensitive to even minor changes of channel roughness. It complicates parameter estimation but can at the same time enhance its accuracy and reliability. However, as the possibilities of discharge measurements are much more limited than those of water stages, basing optimization on water surface elevation only is often the only possibility. This is a paradox of the practical side of optimization.

The other aspect of friction parameters' optimization in an unsteady flow is connected with the form of the objective function when both water stage and flow discharge observations are available, a highly favorable situation due to reasons given above. However, the use of the classical additive error criterion (6) for this case:

$$F=\sum_{i=1}^{N_h}\left(\bar{h}_i-h_i\right)^2+\sum_{i=1}^{N_Q}\left(\bar{Q}_i-Q_i\right)^2, \tag{12}$$

where N_h and N_Q are respectively the numbers of observations of water stage and flow discharge, may lead to incorrect parameter estimation. An interesting question concerns the notation of Equation (12), where it is formally improper for two kinds of variables of different physical meaning and units to be added. However, from a mathematical point of view, the values of h and Q can be treated as non-dimensional numbers, the physical interpretation of which is irrelevant for calculating the value of the objective function. From this perspective, the values are additive and the notation of Equation (12) can thus be considered proper and treated as an error criterion. However, applying this formula can have serious consequences for the calculations. In many practical cases, the absolute values of water stage and flow discharge are known to be quite different, often of different orders. As a result, the value of the objective function can be dominated by the errors of one variable, the other having no influence on the error criterion. Thus, when both water stage and flow discharge are observed, which is beneficial for the optimization process, the efficiency of applying formula (12) to a particular case should be analyzed or an alternative approach considered. One of the following possible solutions can be applied [9]:

- modification of Equation (12) with a weight coefficient, taking into account the variables' unequal influence on the objective function's value,
- applying a form of objective function based on relative errors (*e.g.* Equation (9) or (10)),
- scaling the constitutive values of the objective function, or
- constructing two error criterions and applying the so-called multi-criterion optimization.

The two first approaches appear to be the easiest to apply and thus particularly worth considering. Examples of their application will be presented below.

4.2. The choice of the optimization method

Another important practical aspect of proper application of automatic calibration is the choice of the optimization procedure, which is an algorithm determining the way in which the values of parameters will be corrected at next iterations and how quickly optimal values will be found. The choice of the optimization method is connected with the choice of the error criterion and the analyzed problem's specific features. One of the most important issues to be considered is whether the objective function used in the considered case has one or more local optima. This determines the method to be applied.

In general, optimization methods can be divided into two groups: (a) methods of local optimization and (b) methods of global optimization [16–18]. Methods of local optimization enable finding an optimum in the closest neighborhood of the starting point. If there are many local optima in an objective function, these methods will find the optimum closest to the point from which the searching has been started, which may "falsify" the solution. For objective functions of many optima, application of local optimization methods can lead to a local optimum being interpreted as the only one in the search domain. However, local optimization methods are much easier to apply than global optimization procedures and are often applied, even when more that two parameters are searched (when assessing the number of local optima of an objective function is difficult). The most popular local optimization methods are either non-gradient (the successive searching method, the Gauss-Seidel method, *etc.*) or gradient methods (*e.g.* the conjugate gradient method, the modified Newton-Raphson method and the influence coefficient method). Local optimization methods were applied by Becker and Yeh [15, 16], Wormleaton and Karmegam [19, 20], Wiggert *et al.* [17], Fread and Smith [6], Khatibi *et al.* [10], Morris and Anastasiadou-Partheniou [22] and many others (see [18]). Gradient methods are generally more effective and produce solution more quickly. However, they are really effective for single-minimum objective functions.

Global optimization algorithms are mostly random methods of finding global optima. However, random search is by no means chaotic; it means random procedures supporting directed processes. The most interesting group of such methods are evolution algorithms, in which an analogy to natural adaptation natural selection processes is applied. Evolution algorithms are particularly effective when applied to extremely complicated shapes of objective functions with many local optima [26–28], solving of which requires relatively high expenditure and multiple calculations. Obviously, such sophisticated methods are ineffective in simple single-optimum cases.

In unsteady flow problems, the choice of method must take into account the degree of complication of a particular case, the number of searched parameters and the chosen error criterion. The number of parameters is connected with assumptions made for the analyzed task (parameters m and n can be searched or n only, roughness can be constant or variable along the channel, *etc.*) and the channel or channel network's structure. Consequently, the various types of optimization problems in roughness estimation be quite diversified: from relatively easy tasks of single-parameter search and an objective function with a single optimum to complicated cases of multi-parameter global optimization. Thus, each case should be analyzed on its own merits.

Examples of optimization applications in several cases of open channel flow are presented in the following section. The above-mentioned practical aspects are discussed, the chosen methods' and objective functions' effectiveness is analyzed, and selected problematic aspects of the optimization techniques are presented. A more detailed analysis of these issues may be found in [9].

5. Numerical examples

5.1. Steady flow in a simple channel network: synthetic data

Our first example will be the "artificial" case of steady flow in a simple channel network consisting of three channels shown in Figure 2. The aim of our analysis is to compare the effectiveness of applying different objective functions and optimization methods and their mutual cooperation. The accuracy of the obtained solution will be evaluated.

The most popular approach to steady flow is assuming that m in friction term (5) is equal to 4/3, so that only roughness coefficients n are searched for in the determination process. Three rectangular channels of width $B = 40$m have been assumed in the exemplary network of Figure 2. The bottom slopes of channels 1 (nodes 1–10) and 2 (nodes 11–20) are equal to 0.0001 and 0.0004 for channel 3 (nodes 21–30). The distance between nodes, dx, is 2000 m. Assuming that "true" values of the roughness coefficient are constant and equal to 0.03 for each channel and that three conditions at boundaries are known, one can solve a properly-posed direct problem and obtain the "real" values of water stage and discharge at all nodes of the network. The calculated "real" values may be biased with noise representing measurement errors. In practice, it is usually assumed that such errors have normal distribution [10, 29] and may thus be easily generated in an artificial way. A more detailed description of observation of data generation for this case (and the other ones discussed below) may be found in [9]. In the presented case, the "real" data were biased with errors

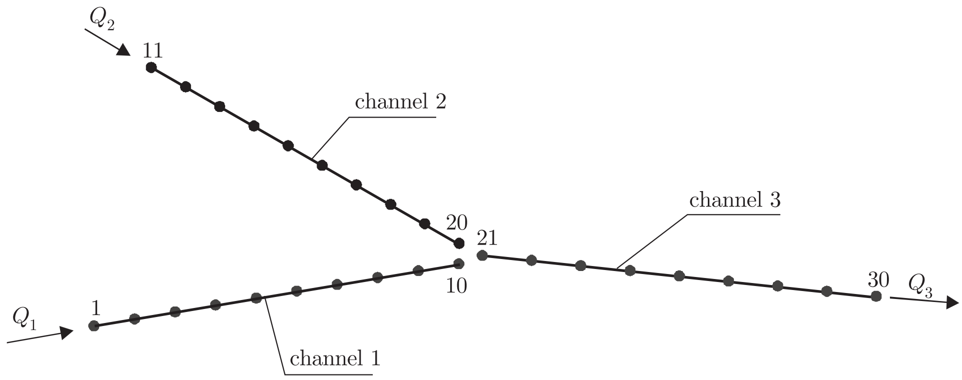

Figure 2. A schema of the channel network of example 5.1

of two different values: 10% and 5%. After such data preparation, the optimization procedure was developed assuming that roughness parameters are unknown.

Three values of n were searched in the analyzed case, with five objective functions chosen for analysis:

$$F_1 = \sum_{i=1}^{N_H} \left(\frac{\bar{H}_i - H_i}{\bar{H}_i} \right)^2 + \sum_{i=1}^{N_Q} \left(\frac{\bar{Q}_i - Q_i}{\bar{Q}_i} \right)^2, \tag{13}$$

$$F_2 = \sum_{i=1}^{N_H} \left(\frac{\bar{H}_i - H_i}{H_i} \right)^2 + \sum_{i=1}^{N_Q} \left(\frac{\bar{Q}_i - Q_i}{Q_i} \right)^2, \tag{14}$$

$$F_3 = (1-\lambda) \sum_{i=1}^{N_H} \left(\bar{H}_i - H_i \right)^2 + \lambda \sum_{i=1}^{N_Q} \left(\bar{Q}_i - Q_i \right)^2, \qquad \lambda = \frac{\sum_{i=1}^{N_H} \bar{H}_i}{\sum_{i=1}^{N_H} \bar{H}_i + \sum_{i=1}^{N_Q} |\bar{Q}|_i}, \tag{15}$$

$$F_4 = (1-\lambda) \sum_{i=1}^{N_H} \left(\bar{H}_i - H_i \right)^2 + \lambda \sum_{i=1}^{N_Q} \left(\bar{Q}_i - Q_i \right)^2, \qquad \lambda = \frac{\sum_{i=1}^{N_H} (\bar{H}_i)^2}{\sum_{i=1}^{N_H} (\bar{H}_i)^2 + \sum_{i=1}^{N_Q} (\bar{Q}_i)^2}, \tag{16}$$

$$F_5 = \sum_{i=1}^{N_H} \left(\bar{H}_i - H_i \right)^2 + \sum_{i=1}^{N_Q} \left(\bar{Q}_i - Q_i \right)^2. \tag{17}$$

It was also assumed that all the necessary values of water stage and flow rate could be measured, thus the number of discharge observations, N_Q, being equal to 3 and the number of nodes at which the water stage was observed, N_h, being 30.

The number of optima was checked for each objective function. The values of each objective function were calculated and analyzed in a search domain limited by the range of possible n values assumed as $\langle 0.01, 0.10 \rangle$. For the chosen channel, the value of n was assumed and the dependence of the objective function's value on the values of two other n coefficients was considered. The analyzed functions proved to have a single optimum for the considered case. Of course, such analysis is very complicated and ineffective in practical cases. However, even when performed roughly, it may help in properly choosing the optimization method or even indicate a potential optimum location, thus limiting the search domain. If such analysis is impossible, *e.g.* due to a large number of parameters, a local optimization method can be started multiple times from different points (to check if the same optimum is found) or a global search method can be used.

Three local optimization methods were chosen in the analyzed case: the Gauss-Seidel method (GS), the gradient method (GR) and the influence coefficient algorithm (ICA), each in conjunction with objective functions (13)–(17). The calculations' results were compared, the starting point for each case being $n_1 = n_2 = n_3 = 0.015$. The values of mean square errors were selected as the objective criteria of comparing the quality of optimization in each case:

$$\mathrm{SBQ} = \sqrt{\frac{\sum_{i=1}^{N_Q} \left(\bar{Q}_i - Q_i^*\right)^2}{N_Q - 1}}, \tag{18}$$

$$\mathrm{SBH} = \sqrt{\frac{\sum_{i=1}^{N_h} \left(\bar{H}_i - H_i^*\right)^2}{N_h - 1}}, \tag{19}$$

where Q_i^* and H_i^* were the values of discharge and water depth calculated for the set of parameters found as 'optimal'.

An example of results obtained for a 10% error in the 'observed data' is presented in Table 1,where the 'optimal' values of n found in each case, the number of objective function calls, SBH and SBQ are shown.

Table 1. The results of roughness coefficient determination with 10% error in observations

No. of function	n_1	n_2	n_3	SBQ	SBH	number of objective function calls	optimization method
1	0.02791	0.02717	0.03109	3.00860	0.12110	141	GS
2	0.02758	0.02684	0.03144	3.08001	0.12058	146	
3	0.02893	0.02816	0.03064	2.50463	0.12732	752	
4	0.02803	0.02725	0.03182	2.50470	0.12054	154	
5	0.03023	0.02947	0.02873	2.50462	0.16167	1090	
1	0.02808	0.02732	0.03125	2.67329	0.12113	61	GR
2	0.02771	0.02696	0.03157	2.79089	0.12057	37	
3	0.03189	0.03114	0.02600	2.52663	0.23783	2269	
4	0.02806	0.02728	0.03178	2.50474	0.12055	103	
5	0.03564	0.03490	0.01379	2.52250	0.63232	41	
1	0.02757	0.02684	0.03129	3.28338	0.12057	4	ICA
2	0.02751	0.02677	0.03154	3.06532	0.12067	5	
3	0.02801	0.02725	0.03182	2.50460	0.12054	6	
4	0.02802	0.02725	0.03181	2.50460	0.12054	5	
5	0.02802	0.02725	0.03187	2.50461	0.12054	6	

The following conclusions can be formulated after the first part of the experiment:

- the obtained values of roughness coefficients are sufficiently close to the 'real' ones. Their differences are mainly due to relatively large 'measurement errors', the form of the applied objective function being another reason;

- the calculations have proven limited efficiency of the Gauss-Seidel method, a non-gradient procedure. A significantly quick convergence of ICA has been observed;
- the form of the objective function significantly influences the quality and duration of calculations, especially with less effective methods (GS and GR). Generally, the best results are obtained for functions based on relative errors (F_1 and F_2). The effectiveness of the 'traditional' F_5 criterion is relatively poor; in some cases it can be increased by a modification (*e.g.* F_4).

Three objective functions, F_1, F_4 and F_5, and the ICA method were chosen for the experiment's second stage. Parameters were determined for 5 different samples of measurement data with 10% error: the results are presented in Table 2.

Table 2. The results of optimization of Manning roughness coefficients with the ICA method

Experiment number	objective function	n_1	n_2	n_3	SBQ	SBH	number of steps
1	F_1	0.02757	0.02684	0.03128	3.28338	0.12057	4
	F_4	0.02802	0.02725	0.03181	2.50460	0.12054	5
	F_5	0.02802	0.02725	0.03187	2.50461	0.12054	6
2	F_1	0.02260	0.02687	0.02929	3.35593	0.17058	4
	F_4	0.02264	0.02688	0.02944	3.35593	0.17026	4
	F_5	0.02264	0.02688	0.02944	3.35593	0.17020	4
3	F_1	0.03022	0.02887	0.02852	1.14294	0.12802	3
	F_4	0.03063	0.02924	0.02852	1.14249	0.12780	3
	F_5	0.03063	0.02924	0.02852	1.14249	0.12780	3
4	F_1	0.02904	0.03140	0.03331	7.60753	0.26549	4
	F_4	0.02683	0.03117	0.03343	7.31351	0.26445	5
	F_5	0.02683	0.03117	0.03343	7.31351	0.26445	10
5	F_1	0.03044	0.03057	0.02958	5.20078	0.12199	3
	F_4	0.03090	0.03129	0.02936	5.20078	0.12137	4
	F_5	0.03090	0.03129	0.02936	5.20078	0.12137	4

The mean values of 'optimal' n were calculated as follows, neglecting the results of sample 2 (with values of n_1 significantly different from the results obtained for other samples):

$$n_1 = 0.02917, \qquad n_2 = 0.02963, \qquad n_3 = 0.03077,$$

which can be considered as very good, taking into account the introduced magnitude of measurement error. The results were verified (see Table 3).

In summary of the first experiment, optimization methods have been proved to be an effective tool for identifying roughness parameters. The quality of identification depends on the quality of measurement data, the form of the objective function and the chosen optimization method. The form of the objective function is particularly important when both discharge and water stage are measured,.

Table 3. The results of verification of optimal Manning roughness coefficients with the ICA method

Sample number	SBQ	average SBQ	SBH	average SBH
first step of verification				
1	3.7777	9.254	0.1329	0.1748
3	1.4425		0.1588	
4	26.5946		0.2670	
5	5.2008		0.1403	
second step of verification				
6	3.8431	13.335	0.2354	0.2483
7	2.4508		0.1882	
8	43.9049		0.4561	
9	11.5460		0.1678	
10	4.9337		0.1942	

5.2. *Steady flow in a channel network: water stage measurements only*

Our second example is connected with steady flow in a more complicated channel network when observations of water stage at selected cross-sections are the only measurement data accessible, a much more common case than that presented in Subsection 5.1. Let us assume a channel network as shown in Figure 3, of characteristic features shown in Table 4. As was the case in Subsection 5.1 above, the measurement data were synthetically prepared on the basis of calculations for assumed 'real' roughness coefficients in the network.

Table 4. Characteristics of channels in the channel network shown in Figure 3

Channel number	1	2	3	4	5	6
Channel width, B [m]	10.0	10.0	8.0	6.0	10.0	10.0
Bottom slope, S_o [‰]	0.1	0.1	0.1	0.1	0.1	0.1
Channel length [km]	3.0	10.0	6.0	4.0	4.0	3.0
Number of upstream node	1	9	5	20	15	25
Number of downstream node	4	14	8	24	19	28
'Real' Manning roughness coefficient	0.022	0.020	0.020	0.015	0.018	0.018
Flow discharge [m^3/s]	4.561	2.511	2.050	0.394	1.656	2.904

The water surface elevations at boundary nodes were: $h_1 = 3.80$m, $h_{19} = 2.70$m and $h_{28} = 2.70$m.

Only water stage observations at channel junctions were assumed to be valid and the observed values were: $h_{\mathrm{I}} = 3.228$m for nodes 4, 5 and 9, $h_{\mathrm{II}} = 2.745$m for nodes 8, 15 and 20, and $h_{\mathrm{III}} = 2.740$m for nodes 14, 24 and 25.

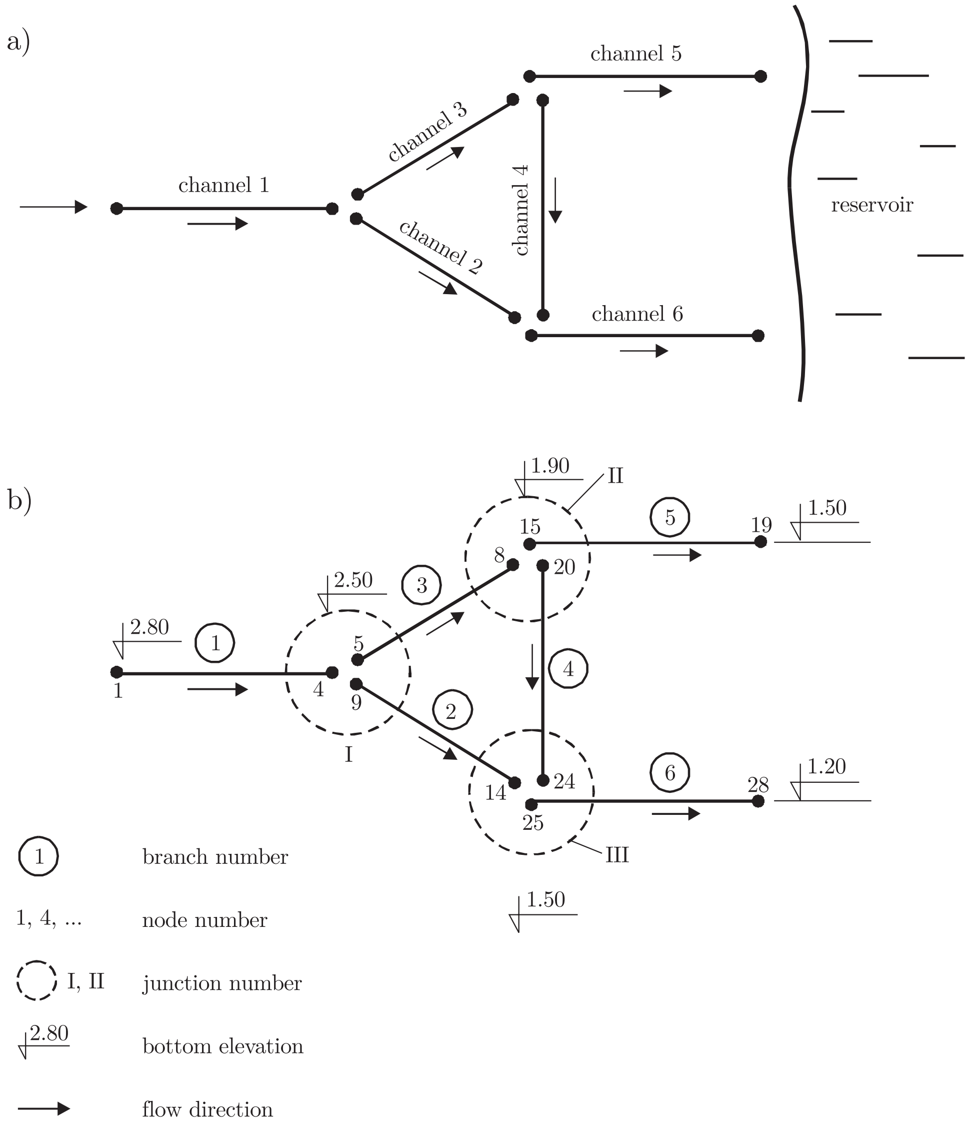

Figure 3. The channel network structure for experiment presented in Subsection 5.2

Six values of roughness parameters (one for each branch) were searched for in the $\langle 0.010; 0.035\rangle$ range using the ICA method and the error criterion most popular in such cases:

$$F = \sum_{i=\mathrm{I}}^{\mathrm{III}} \left(\bar{h}_i - h_i\right)^2. \tag{20}$$

The experiment was repeated for various initial values of Manning roughness coefficients and varying levels of measurement error. Exemplary results are presented in Tables 5 and 6.

A comparison of calculated values with the ‘real’ ones leads to important conclusions concerning the method of parameter estimation. First of all, different

Table 5. The results of optimization of Manning roughness coefficients for a channel network

Parameters	'Real' values of parameters	Values of parameters obtained in optimization			
		experiment 1	experiment 2	experiment 3	experiment 4
		$n_p = 0.012$	$n_p = 0.015$	$n_p = 0.020$	$n_p = 0.020$, measurement error 1.5%
n_1	0.022	0.01298	0.01637	0.02198	0.02151
n_2	0.020	0.01199	0.01500	0.02000	0.02000
n_3	0.020	0.01200	0.01500	0.02000	0.02000
n_4	0.015	0.01200	0.01500	0.02000	0.02000
n_5	0.018	0.01032	0.01270	0.01685	0.01480
n_6	0.018	0.01124	0.01390	0.01851	0.02650
value of objective function	$2 \cdot 10^{-7}$	$4.91 \cdot 10^{-7}$	$4.68 \cdot 10^{-7}$	$4.27 \cdot 10^{-7}$	$5.76 \cdot 10^{-6}$

n_p – the initial value of Manning roughness coefficient (starting point of the optimization procedure)

Table 6. 'Real' and calculated values of flow discharge in channels and water stage at channel junctions

Analyzed variable	'Real' value	Values for 'optimal' roughness coefficients			
		experiment 1	experiment 2	experiment 3	experiment 4
		$n_p = 0.012$	$n_p = 0.015$	$n_p = 0.020$	$n_p = 0.020$, measurement error 1.5%
Q_1 [m^3/s]	4.561	7.642	6.097	4.565	4.658
Q_2 [m^3/s]	2.511	4.214	3.359	2.514	2.560
Q_3 [m^3/s]	2.050	3.427	2.739	2.052	2.098
Q_4 [m^3/s]	0.394	0.493	0.390	0.291	−0.174
Q_5 [m^3/s]	1.656	2.934	2.349	1.761	2.273
Q_6 [m^3/s]	2.904	4.707	3.749	2.805	2.385
h_{I} [m]	3.228	3.228	3.228	3.228	3.239
h_{II} [m]	2.745	2.744	2.744	2.745	2.756
h_{III} [m]	2.740	2.739	2.739	2.739	2.758

sets of 'optimal' parameters were obtained in each experiment, depending on the chosen starting point. This is hardly surprising, as local optimization methods yield 'optimal' points closest to the starting point. At the same time, it suggests that there are many local optima, which renders optimization more difficult. An analysis of the obtained values of objective functions demonstrates them to be similar to each other and very small. Even more importantly, the values of water stage calculated at junctions were in each case very close to the 'observed' ones. As these were the only measurements accessible, optimization was formally successful in each case, as the roughness parameters for which calculations were coincident with observations were found. However, a practical problem persists, as the 'real' set and the Manning roughness coefficient are unknown. This means that there are many sets of n for which the same water stages are possible. Unfortunately, the values of discharge are different for each such set and as long as they cannot be measured the solution of the optimization problem is unknown. The problem will not be solved even if a more sophisticated optimization method is applied, as many sets of n values can give the same water stage values at junctions.

The experiment has proven the importance of practical aspects of optimization which can impose important limitations on solving the problem (especially accessibility of measurement data).

Finally, let us consider a real channel network case and the related problems of roughness estimation.

5.3. Unsteady flow in a channel network: Vistula Lagoon–Nogat–Jagielloński Channel–Elbląg–Lake Druzno

In this example, identification of roughness parameters for the Vistula Lagoon–Nogat–Jagielloński Channel–Elbląg–Lake Druzno hydrographic network is considered. The network's structure is presented in Figure 4. The system consists of:

- the Nogat – a regulated, cut-off arm of the Vistula, between the flood-gate in Michałowo and its outflow to the Vistula Lagoon (length 22.7km, width *ca.* 125m, maximum depth 2.5m),
- the Elbląg, a river connecting Lake Druzno with the Vistula Lagoon (length 17.7km, width at water level 35–64m, depth 2–4.25m) and
- the Jagielloński Channel, connecting the two (length 5.7km, width 36m, depth *ca.* 2.70m).

Unsteady flows of variable directions through the network are determined by changes of the water level in the Vistula Lagoon. Water-level fluctuations are mainly generated by wind and may cause flow towards Lake Druzno or in the opposite direction – during periods of low water level in the southern part of the Vistula Lagoon.

Lake Druzno is a relic of old broads of the Vistula. Its characteristic features are relatively small depth (*ca.* 1.20m on average) and great variation of the water surface, depending on the water level. The dependence between water surface area, F, and water stage, h, is as follows:

$$F(h)=\begin{cases}13.0\,\mathrm{km}^2 & \text{for } h\le 0\ ,\\ (13.0+200h^{2.2})\,\mathrm{km}^2 & 0<h\le 0.30\,\mathrm{m}\ ,\\ 27.0\,\mathrm{km}^2 & h>0.30\,\mathrm{m}\ .\end{cases}\tag{21}$$

In the considered case, water stage observations were valid for four cross-sections (see Figure 4): (A) Nowe Batorowo (water stages in the Vistula Lagoon, the same as in (B)), (D) Żukowo at Lake Druzno and (E) the city of Elbląg at River Elbląg.

For the purposes of numerical solution of an unsteady flow in the channel system, the network was digitized as shown in Figure 5. The hydrostatic state was assumed as the initial condition: for $t=0$ $h(x,t)=h_0=\mathrm{const}$ and $Q(x,t)=0$. The boundary conditions were specified as follows:

- the observed water stages $h_1(t)$ and $h_{28}(t)$ were imposed for nodes 1 and 28,

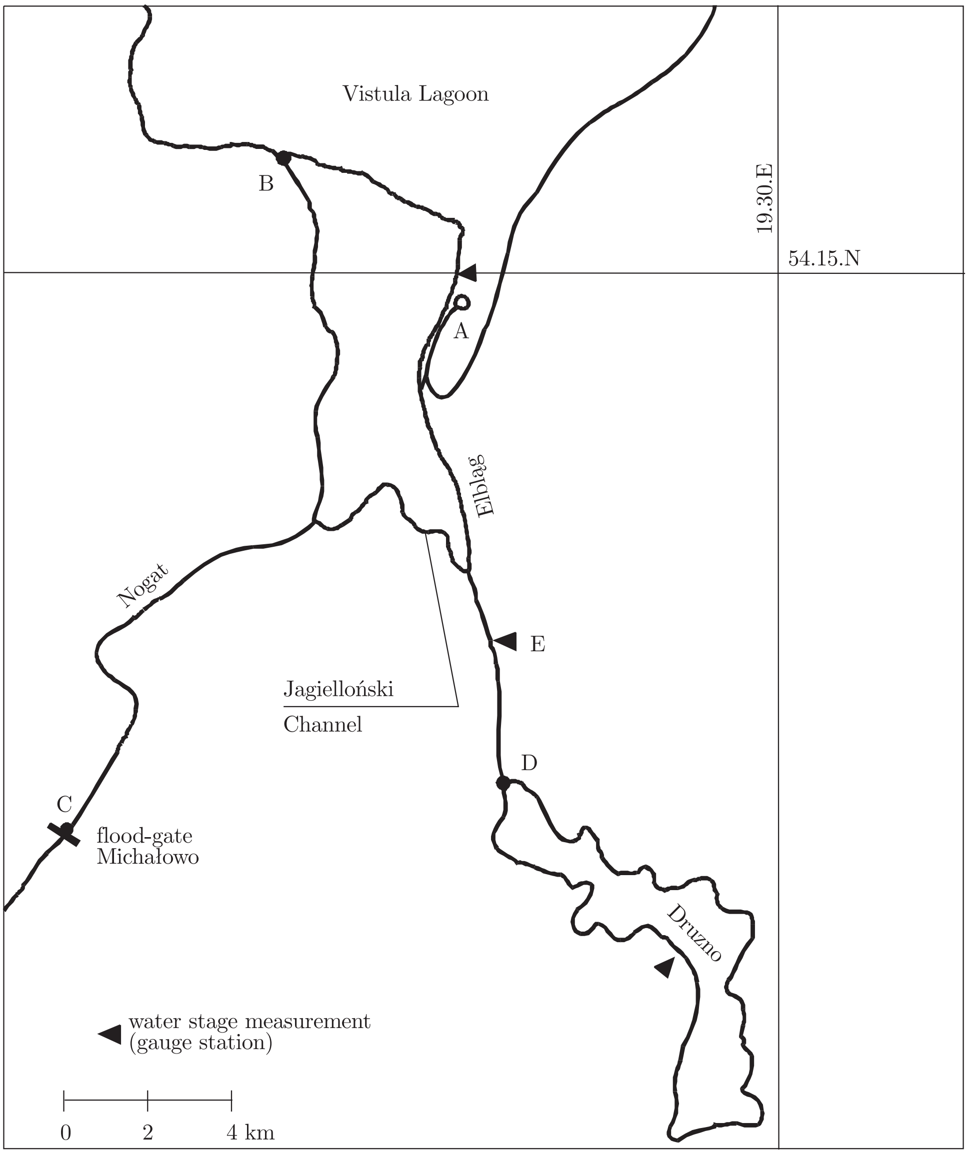

Figure 4. The system of channels connecting the Vistula Lagoon and Lake Druzno [30]

- the $Q = -2\,\mathrm{m}^3/\mathrm{s}$ condition at node 41 in Michałowo, being an approximate value of discharge through the flood-gate (the '$-$' sign representing the direction of flow towards the Vistula Lagoon),
- at node 20, either function $h_{20}(t)$ or function $Q_{20}(t)$ is unknown and thus the differential equation for lake retention was imposed as the boundary condition:

$$\frac{dh_{20}}{dt} = \frac{1}{F(h)}[Q_{20}(t) + P(t) + q(t)], \tag{22}$$

with the initial condition of $h_{20}(t=0) = h_0$, lateral inflow $q(t) = 8\,\mathrm{m}^3/\mathrm{s}$ and an assumed lack of rainfall, $P(t) = 0$.

In typical calculations of unsteady flow through the analyzed channel network, the value of the Manning roughness coefficient was usually assumed to be constant throughout the channel system and equal to 0.020 [31] or 0.022 [30]. Thus, the optimized values of n in the analyzed example were referred to those assumed in the literature.

The roughness coefficients were determined on the basis of water stage observations in Elbląg (node 15) and Żukowo (node 20). The traditional criterion (11) was assumed as the objective function. The trial-and-error method was used first to find the best value of n, assuming n to be constant throughout the system; the value of 0.017 was thus obtained. The next stage was automatic optimization of 5 values of n, assuming the roughness coefficient to be constant for each of the 5 channel sections shown in Figure 5. The local optimization method was applied. The search domain was limited by the values of n from the $\langle 0.01, 0.035\rangle$ range. The roughness parameters were determined and verified for another set of measurement data. Examples of sets of roughness coefficients obtained for various starting points and values of m are presented in Table 7.

It is interesting to compare the obtained values of objective functions with those achieved for values of n constant throughout the system:

- for $n = 0.022$ $f = 0.04985$,
- for $n = 0.017$ $f = 0.04552$.

The results of optimization (see Table 7) are quite similar for each set. The greatest differences can be found for branches 4 and 5, possibly due to the lack of gauge stations at these branches, which makes the calculation results less sensitive to the values of roughness coefficients in these channels. However, the obtained values of n are significantly different from the assumed constant roughness coefficients of $n = 0.022$ and 0.017; the values of objective functions suggest that the sets obtained through automatic calibration are better. An analysis of the calculation results for experiment 1, presented in Figure 6, leads to interesting conclusions.

Table 7. Optimal values of roughness coefficients for the network shown in Figure 5

Optimal values of parameters	$m = 4/3$			$m = 1.0$
	$n_p = 0.015$	$n_p = 0.020$	$n_p = 0.030$	$n_p = 0.020$
	1	2	3	4
n_1	0.03500	0.03500	0.03500	0.02654
n_2	0.01518	0.01522	0.01518	0.01436
n_3	0.01000	0.01000	0.01000	0.01000
n_4	0.01144	0.01167	0.01154	0.01409
n_5	0.01268	0.01716	0.02262	0.01829
value of objective function	0.03448	0.03460	0.03477	0.03648

The relatively small sensitivity of water stages to the changes of n was proved once again. However, the value of $n = 0.022$ proved not to be optimal; better values

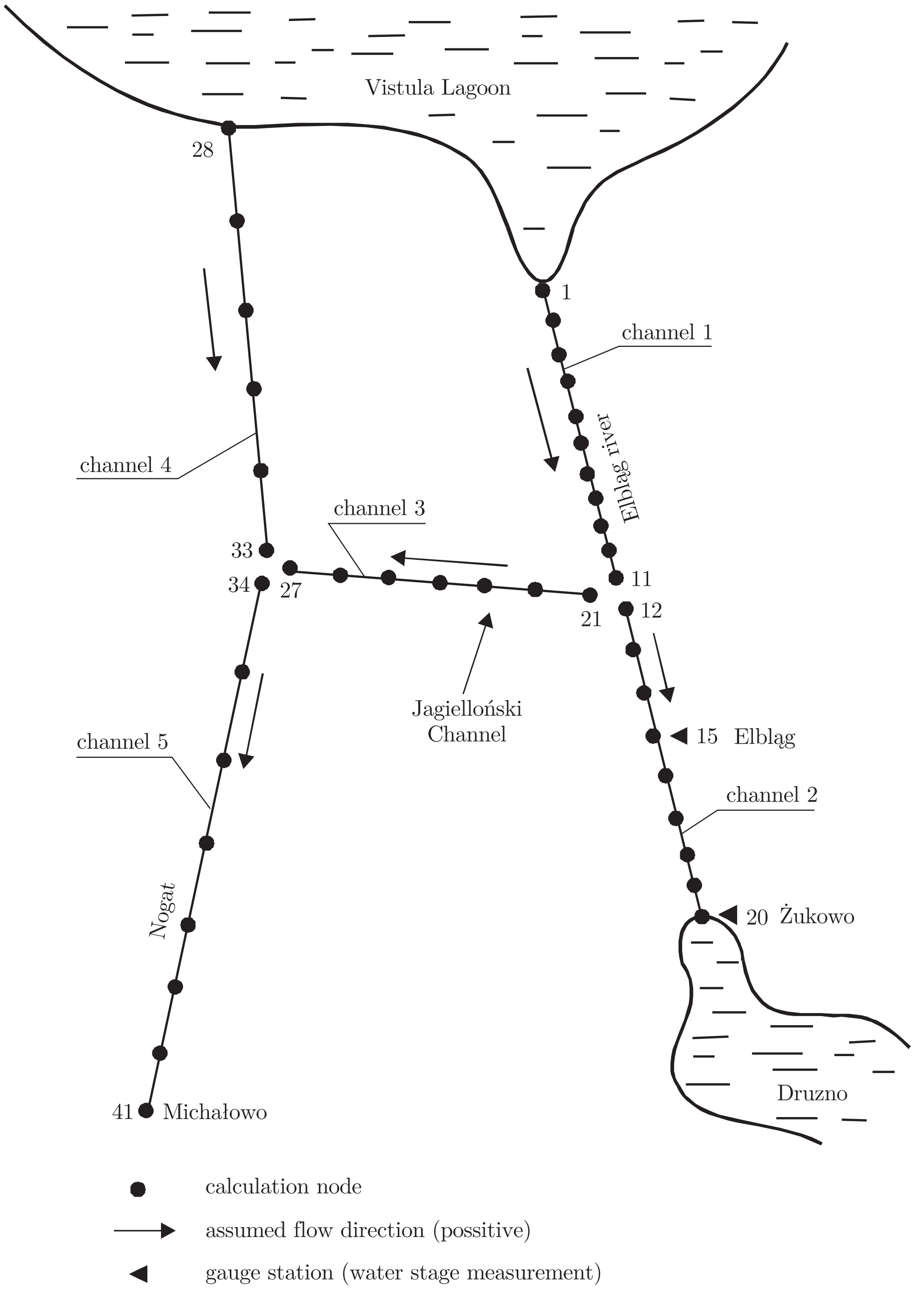

Figure 5. The schema of the digitized channel network of example presented in Subsection 5.3

of n could be found when optimization was applied (even the simple trial-and-error method). Another conclusion is connected with the number of gauge stations. Apparently, observations in Elbląg and Żukowo only are not enough to identify the 'true' values of n properly. Unfortunately, these data were the only ones valid for the analyzed channel system. Once again a practical aspect seems to be an obstacle to calculations. When discharge hydrographs are compared (Figures 7 and 8),

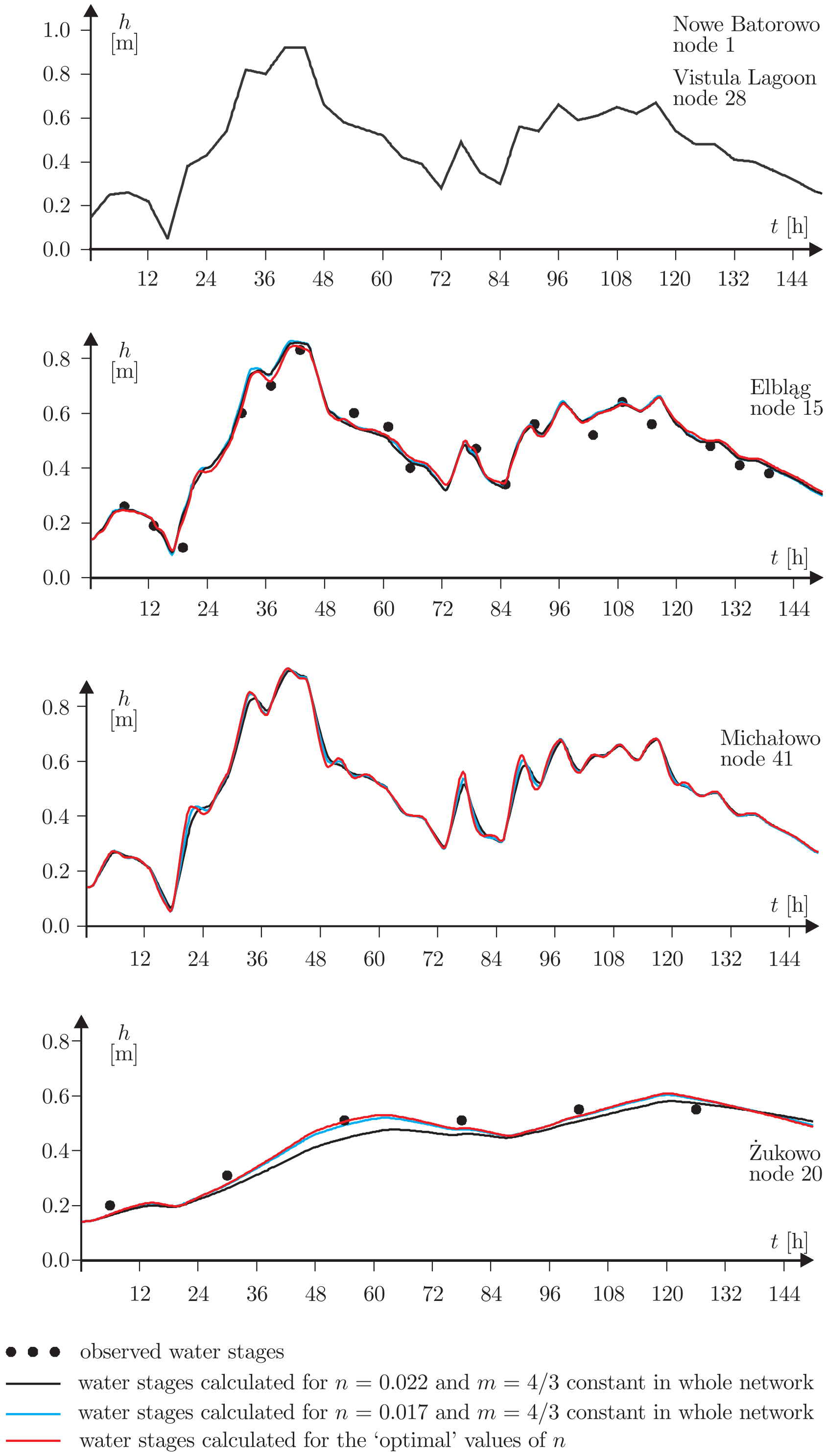

Figure 6. The roughness coefficients obtained for example presented in Subsection 5.3

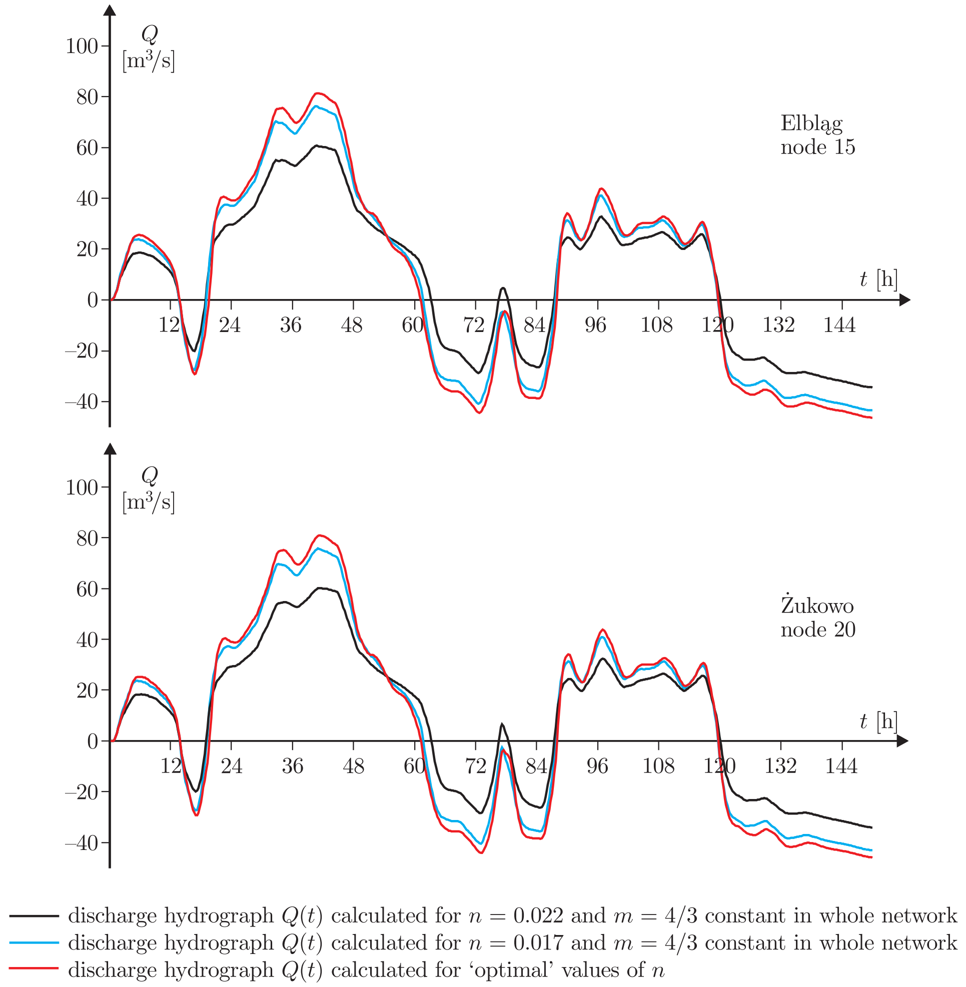

Figure 7. The roughness coefficients obtained for example presented in Subsection 5.3

their shapes are noticeably more sensitive to the changes of n. The differences are significant, up to $50\mathrm{m}^3/\mathrm{s}$ for node 28, but there are no observations of $Q(t)$, thus again (as in example presented in Subsection 5.3) the roughness parameters cannot be determined in a satisfactory manner.

6. Concluding remarks

The theoretical analysis and examples presented in the paper prove that the problem of roughness parameter identification is not trivial. Although it seems to be well recognized, there are still some problems potentially leading to 'false' solutions. Some of them are computational (*e.g.* the choice of the determination method and the objective function and optimization algorithm when the automatic calibration is applied) and controllable. However, unless carefully analyzed, they can lead to solutions which – although seemingly proper – may prove to be far from the 'true' ones.

There are also practical problems, connected with the physical aspects of unsteady flow, the sensitivity of discharges and water stages to parameter values,

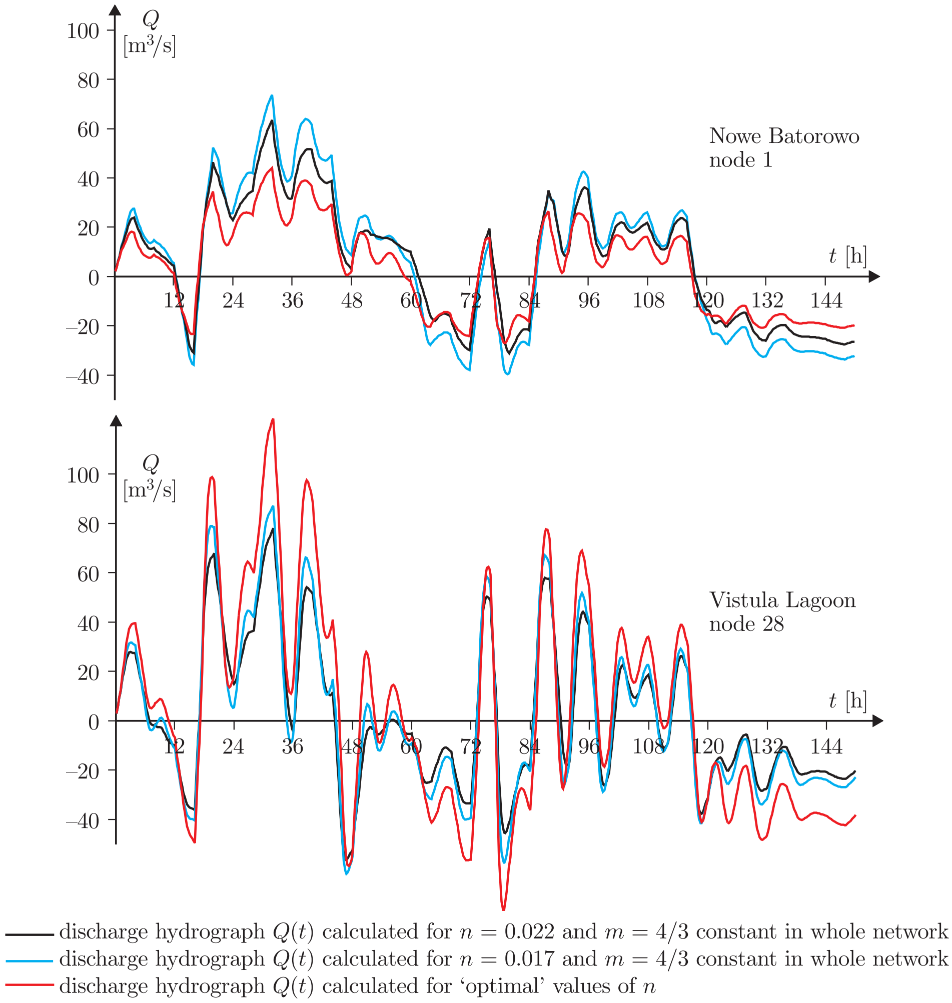

Figure 8. The roughness coefficients obtained for example presented in Subsection 5.3

accessibility of data, measurement problems, *etc.* Due to some of these problems (*e.g.* the lack of discharge observations), even a formally properly run optimization may not yield satisfactory solutions. Unfortunately, as such limitations are often impossible to overcome, it is important to realize the nature of such problems, their influence on the results and the potential consequences.

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