DETERMINING THE VISCOPLASTIC PARAMETERS OF RUBBER-TOUGHENED PLASTICS

ANDRZEJ AMBROZIAK AND PAWEŁ KŁOSOWSKI

Department of Structural Mechanics and Bridges Structures,
Faculty of Civil and Environmental Engineering,
Gdansk University of Technology,
Narutowicza 11/12, 80-952 Gdansk, Poland
{ambrozan, klosow}@pg.gda.pl

(Received 10 June 2007)

Abstract: This paper describes the material parameter determination procedure for the elasto-viscoplastic Bodner-Partom model. A set of viscoplastic parameters is determined for rubber-toughened propylene-ethylene copolymer, and is used to numerical simulations of the material behaviour under different strain rate deformations. The evaluation of material parameters for Bodner-Partom constitutive equations is carried out using tensile tests.

Keywords: viscoplasticity, determination, Bodner-Partom, tensile test, propylene-ethylene copolymer

1. Introduction

Numerous unified constitutive models have been developed to describe the behaviour of elasto-viscoplastic materials (see *e.g.* Woznica [1]). The main advantage of unified models over the classical creep and plasticity models is their treatment of all behavioural aspects of inelastic deformation, including plastic flow under monotonic and cyclic loading, stress and creep relaxation, with a single inelastic equation (see *e.g.* Olszewski *et al.* [2]). At the same time, the practical engineering applications of the majority of unified models are limited by the difficulties in determining the large number of material parameters required.

The polymer investigated in the present study is a rubber-toughened propylene-ethylene copolymer containing about 17% by weight of fillers, mainly talc. This material belongs to the group of plastics used in many industries, *e.g.* Land Rovers' interior door trim panel. For detailed studies of plastics the reader is referred to the "Plastics" series published by Scientific-Technical Publishers, including such topics as production, processing and applications of plastics and covering more general aspects of the subject, *e.g.* the problems of structural plastics discussed by Żuchowska [3].

As the present authors have proposed the elasto-viscoplastic Bodner-Partom (B-P) [4] model to describe the behaviour of the propylene-ethylene copolymer, they have performed a detailed study in order to determine the material parameters of the B-P model. The process was based on laboratory test results taken from literature, *viz.* [5] and [6]. Notably, the authors of these papers have proposed elasto-plastic models for the propylene-ethylene copolymer, based on the Huber-von Mises and Drucker-Prager yield criteria and the cavitation model.

2. Bodner-Partom constitutive equations

The following additive decomposition of the total strain rate, $\dot{\boldsymbol{\varepsilon}}$, has been assumed:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^E + \dot{\boldsymbol{\varepsilon}}^I, \tag{1}$$

where $\dot{\boldsymbol{\varepsilon}}^E$ is the linear elastic strain rate and $\dot{\boldsymbol{\varepsilon}}^I$ is the inelastic strain rate. In the Bodner-Partom model, the inelastic strain rate, $\dot{\boldsymbol{\varepsilon}}^I$, is given by the following equation:

$$\dot{\boldsymbol{\varepsilon}}^I = \frac{3}{2}\,\dot{p}\,\frac{\boldsymbol{\sigma}'}{J(\boldsymbol{\sigma}')}, \tag{2}$$

where $\dot{p}$, $\boldsymbol{\sigma}'$ and $J(\boldsymbol{\sigma}')$ are the equivalent plastic strain rate, the stress tensor deviator and its second invariant, respectively. We have assumed the deformation to be inelastic from the very beginning of the process. The $J(\boldsymbol{\sigma}')$ invariant can be expressed directly in terms of the deviatoric part, $\boldsymbol{\sigma}'$, of the stress tensor, by the following formula:

$$J(\boldsymbol{\sigma}') = \sqrt{\frac{3}{2}(\boldsymbol{\sigma}' : \boldsymbol{\sigma}')}. \tag{3}$$

The equivalent plastic strain rate, $\dot{p}$, is defined by the following equation (*cf.* Bodner and Partom [4]):

$$\dot{p} = \frac{2}{\sqrt{3}}\,D_0\,\exp\left[-\frac{1}{2}\left(\frac{(R+D)}{J(\boldsymbol{\sigma}')}\right)^{2n}\frac{n+1}{n}\right], \tag{4}$$

where D is described as follows:

$$D = \boldsymbol{X} : \frac{\boldsymbol{\sigma}}{J(\boldsymbol{\sigma})} \tag{5}$$

and the D_0 and n constants are material parameters respectively representing the limit plastic strain rate and the strain rate sensitivity parameter. The isotropic, R, and kinematic, $\boldsymbol{X}$, hardening (with recovery effects neglected) is defined with the following expressions:

$$\dot{R} = m_1 \cdot (R_1 - R) \cdot \dot{W}^I, \tag{6}$$

$$\dot{\boldsymbol{X}} = m_2 \cdot \left(\frac{3}{2}\,D_1\,\frac{\boldsymbol{\sigma}}{J(\boldsymbol{\sigma})} - \boldsymbol{X}\right) \cdot \dot{W}^I, \tag{7}$$

with the following parameters:

m_1 – the hardening rate coefficient for isotropic hardening,
m_2 – the hardening rate coefficient for kinematic hardening,
R_1 – the limiting (maximum) value for isotropic hardening, and
D_1 – the limiting (maximum) value for kinematic hardening.

Additionally, isotropic hardening has an initial value, $R(t=0)=R_0$, which should also be treated as a material parameter. The plastic work rate, $\dot{W}^I$, used in Equations (6) and (7), is calculated from the following equation:

$$\dot{W}^I = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^I. \tag{8}$$

Nine parameters have to be determined in the described model: two being elastic constants (E, ν), the others – inelastic parameters (n, D_0, D_1, R_0, R_1, m_1, m_2).

3. Laboratory tests

The laboratory test results were taken from [5], where Dean and Read gave the stress-strain curves for the propylene-ethylene copolymer at 23°C, assuming various strain rates. They presented an explicit form of the $\sigma(\varepsilon^I)$ function for various strain rates, formulas follows:

$$\sigma(\varepsilon^I) = \left[\sigma_Y + (\sigma_f - \sigma_Y)\cdot\left(1 - e^{-(\varepsilon^I/\alpha)^\beta}\right)\right]\cdot(1 - q\cdot\varepsilon^I), \tag{9}$$

where the σ_Y, σ_f, α, β and q parameters of which are given in Table 1. The $\sigma(\varepsilon^I)$ functions obtained by Dean and Read for various strain rates are shown in Figure 1. Notably, parameter σ_Y is the yield stress value referring to the zero inelastic strain.

The $\sigma(\varepsilon^I)$ stress curves were obtained as functions of the inelastic strain, ε^I, assuming additivity of the elastic and inelastic strain components according to Equation (1). As the values of the elastic tensile modulus, $E = 2200$ MPa, and Poisson's

Table 1. Approximated parameters for the stress-inelastic strain curves of [5]

$\dot{\varepsilon}$ [s^{-1}]	σ_Y [MPa]	σ_f [MPa]	α [–]	β [–]	q [-]
0.00035	7.0	27.0	0.013	0.63	0.56
0.004	8.0	29.3	0.010	0.63	0.51
0.027	9.0	31.7	0.009	0.70	0.47
0.20	10.0	35.4	0.008	0.68	0.70
2.10	11.0	39.5	0.0075	0.70	0.76

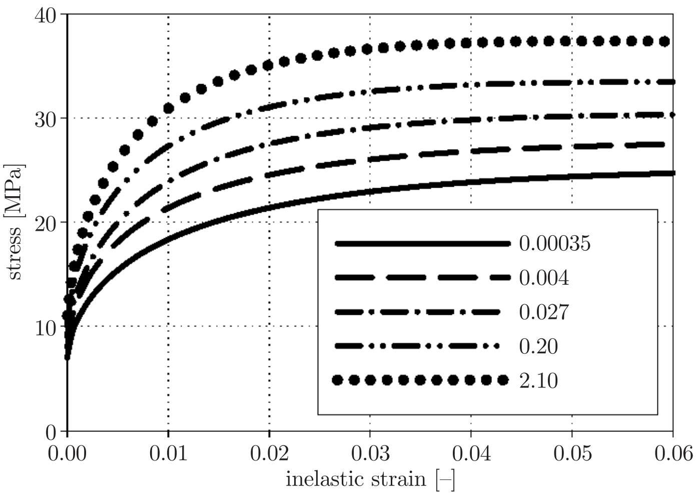

Figure 1. Stress-inelastic strain curves for various strain rates [s^{-1}]

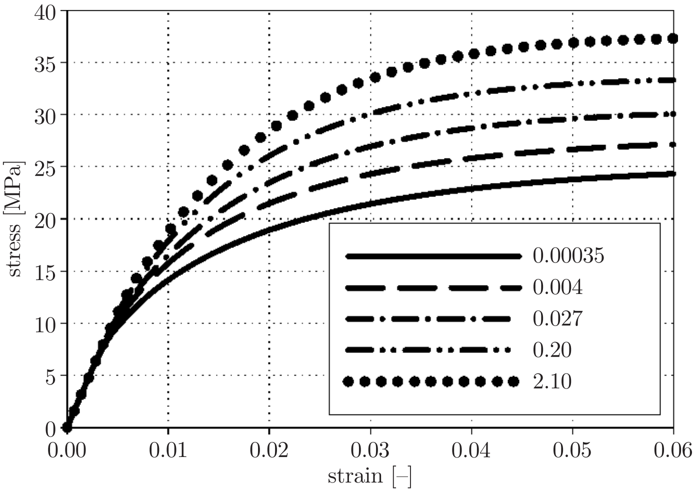

Figure 2. Uniaxial tension tests for various strain rates [s^{-1}]

ratio, $\nu = 0.37$ [–], were given (see [6] for details) it was possible to recalculate the total strain:

$$\varepsilon = \varepsilon^I + \frac{\sigma}{E}. \tag{10}$$

Finally, the stress-strain curves for various strain rates are shown in Figure 2.

4. Determining the viscoplastic material parameters

The viscoplastic material parameters in the strain range of $\varepsilon \in \langle 0, 0.06 \rangle$ were specified for specimens of propylene-ethylene copolymer at 23°C in the strain rate range of $\dot{\varepsilon} \in \langle 0.00035, 2.10 \rangle \, s^{-1}$. Viscoplastic material parameters were calibrated using the Marquardt-Levensberg variant of least square regression (see *e.g.* Marquardt [7] or Nash [8] for details).

The Bodner-Partom model parameters were determined on the basis of the uniaxial tension tests described in the preceding section. In the case of the uniaxial stress state, the inelastic strain rate could be rewritten as:

$$\dot{\varepsilon}^I = \frac{2}{\sqrt{3}} \, D_0 \cdot e^{-\frac{1}{2}\left(\frac{R+D}{\sigma}\right)^{2n} \cdot \frac{n+1}{n}} \cdot \operatorname{sign}\sigma, \tag{11}$$

where the hardening parts were described by the following equations:

$$R = R_1 \cdot \left[1 - e^{-m_1 \cdot W^I}\right] + R_0 \cdot e^{-m_1 \cdot W^I}, \tag{12}$$

$$D = D_1 \, \left[1 - e^{-m_2 \cdot W^I}\right]. \tag{13}$$

The $D_0 = 10^4 \, s^{-1}$ parameter (see [9] for details) was arbitrarily chosen in the beginning of the process, following which parameters n and R_0 were determined on the basis on the following equations (see [10] and [11]):

$$\frac{\sigma}{R+D} = \left[\ln\left(\frac{2 \, D_0 \cdot \operatorname{sign}\sigma}{\sqrt{3} \, \dot{\varepsilon}^I}\right) \cdot \frac{2n}{n+1}\right]^{-\frac{1}{2n}} = f\left(\dot{\varepsilon}^I\right), \tag{14}$$

a transformation of formula (11). When isotropic hardening has its initial value $R = R_0$, kinematic hardening is negligible; it was thus possible to formulate the following expression:

$$\sigma_Y = \frac{R_0}{\left[\frac{2n}{n+1} \cdot \ln\left(\frac{2}{\sqrt{3}}\frac{D_0}{\dot{\varepsilon}^I}\right)\right]^{\frac{1}{2n}}}, \tag{15}$$

where σ_Y was the initial value of the yield stress. As the yield stress, σ_Y, values for various strain rates were already determined (see Table 1), the $\sigma_Y(\dot{\varepsilon})$ function could be determined (see Figures 3 and 4).

Subsequently, the $\sigma\left(\varepsilon^I\right)$ function was established for all the tests. The form of this function used in the present investigation was suggested by Dean and Read [5] (see Equation (9)). Thus, the γ function of the work hardening rate was calculated:

$$\gamma = \frac{d\sigma(\varepsilon^I)}{d\varepsilon^I} \cdot \frac{1}{\sigma(\varepsilon^I)}, \tag{16}$$

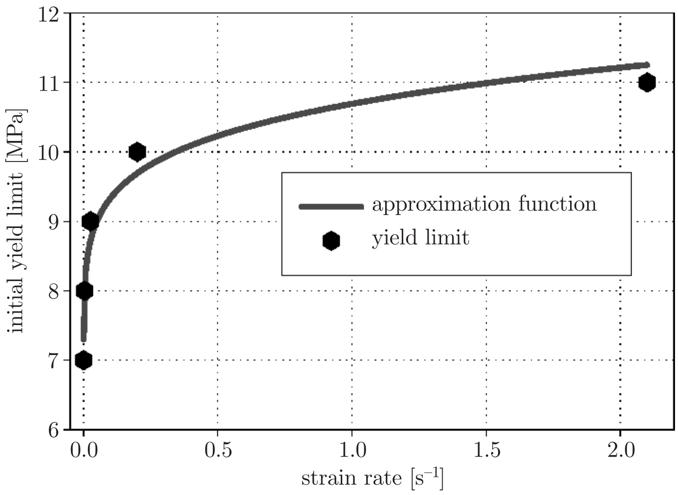

Figure 3. Yield stress as a function of the strain rate: linear scale

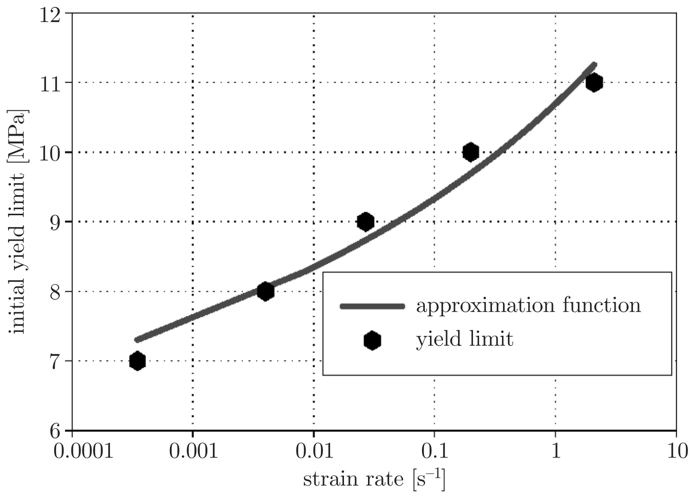

Figure 4. Yield stress as a function of the strain rate: decimal logarithmic scale

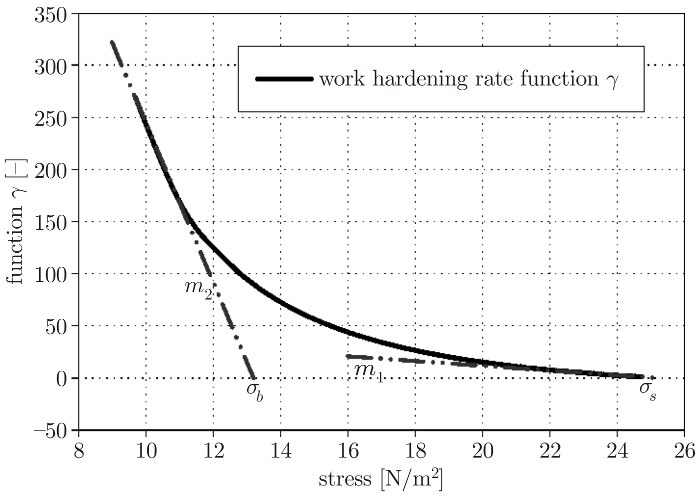

Figure 5. Evolution of function γ

where

$$\frac{d\sigma(\varepsilon^I)}{d\varepsilon^I} = -e^{-\left(\varepsilon^I/\varepsilon_{ps}\right)^\beta} \cdot \left(\frac{\beta\ (\sigma_Y - \sigma_f)(\varepsilon^I\ q - 1)(\varepsilon^I/\varepsilon_{ps})^\beta}{\varepsilon^I} + q\ (\sigma_Y - \sigma_f) \right) - q\ \sigma_f. \quad (17)$$

Establishing the form of the γ function made it possible to determine parameters m_1 and m_2 as slopes at the two extremes (see Figure 5).

For $\gamma = 0$, the σ_s and σ_b stress values were established. Finally, parameters D_1 and R_1 were calculated from the following equations:

$$D_1 = \frac{\sigma_b \cdot m_2}{f\left(\dot{\varepsilon}^I_{05}\right) \cdot (m_2 - m_1)} - \frac{\sigma_s \cdot m_1}{f\left(\dot{\varepsilon}^I_{5}\right) \cdot (m_2 - m_1)} - R_0, \quad (18)$$

$$R_1 = \frac{\sigma_s \cdot m_2}{f\left(\dot{\varepsilon}^I_{5}\right) \cdot (m_2 - m_1)} - \frac{\sigma_b \cdot m_2}{f\left(\dot{\varepsilon}^I_{05}\right) \cdot (m_2 - m_1)} + R_0. \quad (19)$$

The obtained Bodner-Partom model parameters are given in Table 2.

Table 2. Elasto-viscoplastic parameters for the propylene-ethylene copolymer at 23°C

E [MPa]	ν [–]	D_0 [s^{-1}]	n [–]	D_1 [MPa]	R_0 [MPa]	R_1 [MPa]	m_1 [MPa^{-1}]	m_2 [MPa^{-1}]
2200	0.37	$1 \cdot 10^4$	0.81	40.30	39.80	96.50	3.25	60.50

5. Validation of results

Numerical simulations for the propylene-ethylene copolymer at 23°C for various constant strain rates were compared with simulation results (see Figures 6–10).

The MSC.Marc system was used in the numerical calculations. As the standard MSC.Marc system does not support the Bodner-Partom material models, the authors introduced a user-defined UVSCPL subroutine [12] to apply the B-P model to the MSC.Marc system. The subroutine had been successfully implemented by Ambroziak for the Chaboche model with damage (see [13–15]) and for the Bodner-Partom constitutive equations (see [16, 17]).

Good agreement of stress distributions has been observed between the MSC.Marc calculations and the laboratory tests. The results of numerical simulation of uniaxial

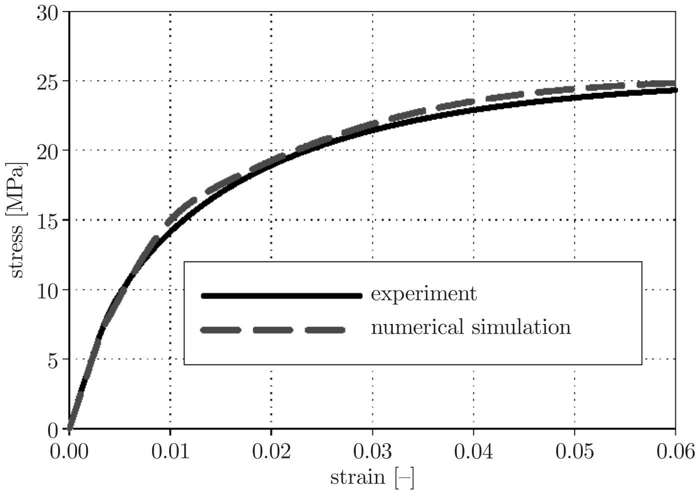

Figure 6. Results of a B-P simulation of a uniaxial tension test for $\dot{\varepsilon} = 0.00035\,\mathrm{s}^{-1}$

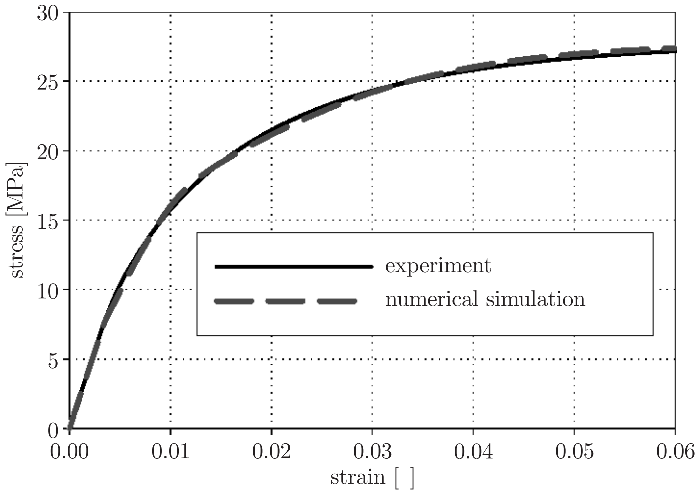

Figure 7. Results of a B-P simulation of a uniaxial tension test for $\dot{\varepsilon} = 0.004\,\mathrm{s}^{-1}$

tensile tests have confirmed the correctness of the ways in which the elasto-viscoplastic B-P model was introduced into an open commercial system and the viscoplastic material parameters were determined for the propylene-ethylene copolymer. The obtained results have encouraged the authors to continue their research on the basis of extended experimental data.

6. Concluding remarks

The presented elasto-viscoplastic Bodner-Partom constitutive equations appear to be useful in material modelling of the propylene-ethylene copolymer. A number of viscoplastic parameters can be identified in a straightforward way using the proposed constitutive relations.

Identification of the elasto-viscoplastic properties of the propylene-ethylene copolymer have been successfully determined merely on the basis of uniaxial tension

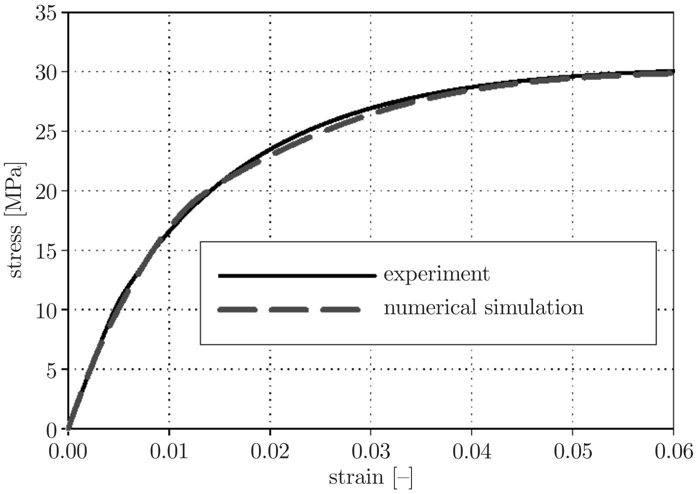

Figure 8. Results of a B-P simulation of a uniaxial tension test for $\dot{\varepsilon} = 0.027\mathrm{s}^{-1}$

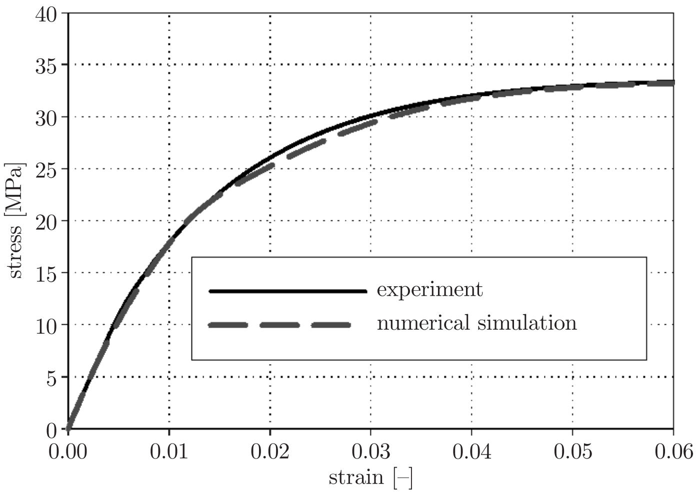

Figure 9. Results of a B-P simulation of a uniaxial tension test for $\dot{\varepsilon} = 0.20\mathrm{s}^{-1}$

tests for various strain rates. The elasto-viscoplastic Bodner-Partom material model can be used directly in the finite element analysis of structures made of the propylene-ethylene copolymer.

Acknowledgements

The calculations presented in the paper were performed at the Academic Computer Centre in Gdansk (TASK).

References

[1] Woznica K 2003 *Summer School "Simulation for Society"*, Gdansk (Kłosowski P, Ed.), pp. 77–120 (in Polish)
[2] Olszewski J, Sievert R, Meersmann J and Ziebs J 1990 *High Temperature Materials for Power Engineering* (Bachelet *et al.*, Eds), Kulwert Academic Publishers, pp. 1051–1060
[3] Żuchowska D 2000 *Constructional Polymers*, WNT, Warsaw (in Polish)
[4] Bodner S R and Partom Y 1975 *J. Appl. Mech.* **42** 385

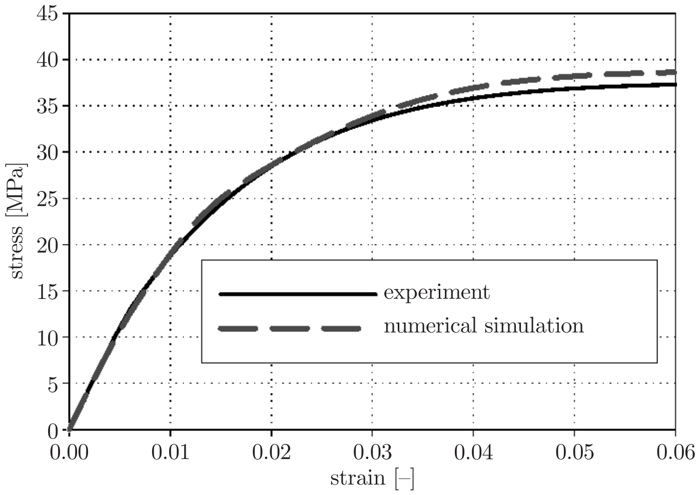

Figure 10. Results of a B-P simulation of a uniaxial tension test for $\dot{\varepsilon} = 2.10\mathrm{s}^{-1}$

[5] Dean G and Read B 2001 *Polymer Testing* **20** 677
[6] Dean G and Mera R 2004 *NPL Report DEPC-MPR 007*, National Phisical Labolatory Teddington, Middlesex-UK
[7] Marquardt D W 1963 *J. Soc. Ind. and Appl. Math.* **11** 431
[8] Nash J C 1979 *Compact Numerical Methods for Computers: Linear Algebra and Function Minimalization*, John Wiley & Sons Inc., New York
[9] Miller A K (Ed.) 1987 *Unified Constitutive Equations for Creep and Plasticity*, Elsevier Applied Science, London
[10] Chan K S, Bodner S R and Lindholm U S 1988 *J. Engng Materials and Technology* **110** 1
[11] Woznica K and Klosowski P 2000 *Arch. Appl. Mech.* **70** 561
[12] Users handbook: 2005 *MSC.MARC Volume D: User Subroutines and Special Routines, Version 2005*, MSC.Software Corporation
[13] Ambroziak A 2005 *TASK Quart.* **2** 167
[14] Ambroziak A 2005 *Shell Structures: Theory and Applications* (Pietraszkiewicz W and Szymczak C, Eds), Taylor & Francis Group, Balkema – Proceedings and Monographs in Engineering, Water and Earth Sciences, pp. 445–449
[15] Ambroziak A 2007 *Engineering Transactions* **1** 3
[16] Ambroziak A 2007 *Computer Assisted Mechanics and Engineering Sciences* **3** 405
[17] Ambroziak A 2007 *Engineering Transactions* **2** 115