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# ON ELECTROMECHANICAL PHENOMENA IN THIN DIELECTRIC FILMS

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**Abstract:** A model of electro-magneto-thermo-mechanics for electroconducting polarized non-ferromagnetic medium is proposed which takes into account the local *mass* displacement in addition to the local electric charge displacement. The corresponding key set of equations is written. Using the isothermal approximation, the model is applied to describe the interface inhomogeneity of a stressed state, the polarization and coupled electric charge in thin dielectric films. An anomalous dependence of the electric capacity on the thickness of a thin dielectric film, observed experimentally by Mead, is also studied and is shown to be well captured by the present approach.

**Keywords:** coupled electro-magneto-thermo-mechanical processes, non-local materials, local mass displacement, interfacial phenomena, thin dielectric films

## 1. Introduction

It is well known that the linearized classical theory of piezoelectricity does not take into account the interaction between the mechanical and electromagnetic fields in isotropic materials [1, 2]. However, some experimental investigations have demonstrated that piezoelectric phenomena can also be observed in centrosymmetric crystals [2]. In [3, 4] the anomalous dependence of the capacitance of a thin dielectric film on its thickness has been reported. This effect cannot be described by the classical Voight theory. The classical piezoelectric theory predicts the electric potential linear distribution across the film, and that the polarization vector is constant within a thin film, which contradicts the experimental results of Mead [3, 4]. Such disagreements

between the classical theory and the experiment have stimulated development of new models.

One of the first attempts to account for interactions between the mechanical and electromagnetic fields in isotropic materials has been made in [5] where the parameters space which describes the state of a dielectric has been extended to contain the tensors of induced charges and electromagnetic stresses in addition to usual parameters, such as temperature, entropy, strain tensor and mechanical stresses. In [6] a gradient model of piezoelectrics has been proposed on the basis of the Toupin linear theory [7, 8], in which model it is assumed that the state of a dielectric depends on the polarization gradient. The model modified in this way describes correctly an anomalous dependence of the capacitance of a thin dielectric layer on its thickness [9, 10], observed experimentally by Mead [3, 4]. It also predicts the nonlinear distribution of the electric potential and polarization in thin dielectric films [9, 10].

Unlike gradient models in which the functional dependence of the internal energy on the polarization gradient is *postulate*, a *nonlocal* model, proposed in [11, 12], is constructed taking into account the the local mass displacement process (in addition to the local displacement of the electric charge). The physical reason for the local mass displacement is the reordering of the molecular structure of a solid. Such reordering can be caused by a change of the locations of atoms at the interface due to the surface formation or due to the polarization of the whole solid, or due to the displacement of the neighboring subsystems of a heterogeneous solid in case of an accelerating motion.

The local mass displacement leads to an extension of the state parameter space and nonlocal constitutive equations, to a redefinition of the stress tensor and additional bulk ponderomotive forces [11, 12]. In addition to the conventional conjugated parameters (such as strain and stress tensors, temperature and entropy, the electrical field intensity vector and the vector of polarization), two additional pairs of parameters are introduced:

(i) the specific density of an induced mass $\rho_m$ and the reduced potential $\mu'_\pi = \mu_\pi - \mu$;
(ii) the specific mass displacement $\boldsymbol{\pi}_m$ and the reduced potential gradient $\nabla\mu'_\pi$.

Here $\mu_\pi$ is the energy measure of the influence of the mass displacement on the internal energy and $\mu$ is the chemical potential. $\boldsymbol{\pi}_m = \boldsymbol{\Pi}_m/\rho$, where $\boldsymbol{\Pi}_m$ is the vector of the local mass displacement, $\rho$ is the mass density, $\rho_m = \rho_{m\pi}/\rho$, where $\rho_{m\pi} = -\nabla\cdot\boldsymbol{\Pi}_m$ is the induced mass density.

The purpose of this paper is to analyze the above-mentioned mathematical model. The model equations are presented and discussed in detail in Section 2. Using the linear approximation, we will show that our approach brings a correct description of the surface polarization of a solid body and the inhomogeneity of a stress-strained state (Section 3.1), and describes the Mead anomaly properly (Section 3.2). A brief summary of our work is given in Section 4.

## 2. The model

At first model equations are introduced and discussed, then the linear approximation is considered which will be applied in Section 3 to study coupled electromagnetic fields in a thin dielectric film.

### 2.1. A basic set of equations

A complete set of equations of a model of electro-magneto-thermo-mechanics of an electroconducting non-ferromagnetic isotropic polarized medium with local mass and charge displacements includes [11, 12]:

– the momentum equation:

$$\rho\frac{d\boldsymbol{v}}{dt}=\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\boldsymbol{F}_e+\rho\boldsymbol{F}_*; \tag{1}$$

– the mass and entropy balance equations:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v})=0, \tag{2}$$

$$\rho T\frac{ds}{dt}=-\nabla\cdot\boldsymbol{J}_s+T\sigma_s+\rho\Re; \tag{3}$$

– the Maxwell's equations:

$$\nabla\cdot\boldsymbol{B}=0,\qquad \nabla\cdot\boldsymbol{D}=\rho_e,\qquad \nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t},\qquad \nabla\times\boldsymbol{H}=\boldsymbol{J}_{ef}; \tag{4}$$

– the conservation law of the induced mass and electric charges:

$$\frac{\partial\rho_{m\pi}}{\partial t}+\nabla\cdot\boldsymbol{J}_{ms}=0, \tag{5}$$

$$\frac{\partial\rho_{e\pi}}{\partial t}+\nabla\cdot\boldsymbol{J}_{es}=0; \tag{6}$$

– the state equations which in the linear approximation takes the form:

$$s=s_0-\left[a_T^s(T-T_0)+\rho_0^{-1}a_{eT}e+a_{\rho T}\rho_m\right], \tag{7}$$

$$\hat{\boldsymbol{\sigma}}_*=2a_2^\sigma\hat{\boldsymbol{e}}+\left[a_1^\sigma e+a_{eT}(T-T_0)+a_{e\rho}\rho_m\right]\hat{\boldsymbol{I}}, \tag{8}$$

$$\mu'_\pi=\mu'_{\pi0}+a_\rho^\mu\rho_m+\rho_0^{-1}a_{e\rho}e+a_{\rho T}(T-T_0), \tag{9}$$

$$\boldsymbol{p}=-a_E^p\boldsymbol{E}_*-a_{E\mu}\nabla\mu'_\pi, \tag{10}$$

$$\boldsymbol{\pi}_m=a_\mu^\pi\nabla\mu'_\pi+a_{E\mu}\boldsymbol{E}_*; \tag{11}$$

– the relations between the fluxes and thermodynamical forces (kinetic relations):

$$\boldsymbol{J}_{e*}=\sigma_e\boldsymbol{E}_*+\sigma_e\eta\nabla T,\qquad \boldsymbol{J}_q=-\lambda\nabla T+\pi_t\boldsymbol{J}_{e*}; \tag{12}$$

– an expression which relates the strain tensor $\hat{\boldsymbol{e}}$ with the displacement vector $\boldsymbol{u}$ (the so-called strain-displacement relation):

$$\hat{\boldsymbol{e}}=\left[\nabla\boldsymbol{u}+(\nabla\boldsymbol{u})^T\right]/2. \tag{13}$$

Here

$$\hat{\boldsymbol{\sigma}}_*=\hat{\boldsymbol{\sigma}}-\rho(\boldsymbol{E}_*\cdot\boldsymbol{p}-\rho_m\mu'_\pi-\boldsymbol{\pi}_m\cdot\nabla\mu'_\pi)\hat{\boldsymbol{I}}, \tag{14}$$

$$\boldsymbol{F}_*=\boldsymbol{F}+\rho_m\nabla\mu'_\pi-\boldsymbol{\pi}_m\cdot\nabla\nabla\mu'_\pi, \tag{15}$$

$$\boldsymbol{F}_e=\rho_e\boldsymbol{E}_*+\left(\boldsymbol{J}_{e*}+\frac{\partial(\rho\boldsymbol{p})}{\partial t}\right)\times\boldsymbol{B}+\rho(\nabla\boldsymbol{E}_*)\cdot\boldsymbol{p}, \tag{16}$$

$$\sigma_s = \boldsymbol{J}_{e*} \cdot \frac{\boldsymbol{E}_*}{T} - \boldsymbol{J}_q \cdot \frac{\nabla T}{T^2}, \tag{17}$$

$$\boldsymbol{E}_* = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}, \qquad \boldsymbol{J}_{e*} = \boldsymbol{J}_e - \rho_e \boldsymbol{v}, \tag{18}$$

where $\hat{\boldsymbol{\sigma}}$ is the Cauchy stress tensor, $\boldsymbol{v}$ is the mass center velocity, $s$ is the specific entropy and $T$ is the absolute temperature. $\boldsymbol{J}_s$ is the entropy flux density and $\boldsymbol{J}_q = T\boldsymbol{J}_s$ is the heat flux density. $\sigma_s$ is the entropy source strength, $\Re$ denotes the distributed thermal sources. $\boldsymbol{F}$ is the mass force vector and $\boldsymbol{F}_e$ is the ponderomotive force. $\boldsymbol{E}$, $\boldsymbol{H}$ are electric and magnetic fields, and $\boldsymbol{D}$, $\boldsymbol{B}$ are electric and magnetic inductions. For the non-ferromagnetic medium, $\boldsymbol{B} = \mu_0 \boldsymbol{H}$, $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$, where $\boldsymbol{P}$ denotes the local electric charge displacement (polarization vector), $\boldsymbol{p} = \boldsymbol{P}/\rho$ is the specific polarization vector, and $\varepsilon_0$, $\mu_0$ are the electric permittivity and the magnetic permeability of vacuum (electric and magnetic constants). $\rho_e$, $\rho_{e\pi}$ are the density values of free and induced electric charges. $\boldsymbol{J}_{ef} = \boldsymbol{J}_e + \boldsymbol{J}_{ed} + \boldsymbol{J}_{es}$ is the total electric current density, where $\boldsymbol{J}_e$ is the electric current density (convection and conduction currents), $\boldsymbol{J}_{ed} = \varepsilon_0(\partial \boldsymbol{E}/\partial t)$, and $\boldsymbol{J}_{es} = \partial \boldsymbol{P}/\partial t$ is the current density caused by the ordering of a charged system (polarization current). $\boldsymbol{J}_{ms} = \partial \boldsymbol{\Pi}_m/\partial t$ is the vector of a mass flux related to the local mass displacement. $e \equiv \hat{\boldsymbol{e}} : \hat{\boldsymbol{I}}$ is the first invariant of the strain tensor, where $\hat{\boldsymbol{I}}$ is the unit tensor. $\rho_0$ is the mass density in the reference state, and $a_1^\sigma$, $a_2^\sigma$, $a_T^s$, $a_\rho^\mu$, $a_E^p$, $a_\mu^\pi$, $a_{eT}$, $a_{\rho T}$, $a_{e\rho}$, $a_{E\mu}$, $\sigma_e$, $\eta$, $\lambda$, and $\pi_t$ are material parameters. $s_0$ and $\mu'_{\pi 0}$ is the entropy and the reduced potential $\mu'_\pi$ in the reference state, respectively. $t$ is the time, and $d\ldots/dt = \partial\ldots/\partial t + \boldsymbol{v} \cdot \nabla\ldots$ is the substantive derivative. Finally, the upper index $T$ denotes a transposed tensor.

It is noted that according to the state Equations (7)–(11) the polarization fields and the displacement of mass are coupled: the electric polarization is caused not only by the electric field but also by the gradient of $\mu'_\pi$. In the surface region of a thin film the value of $|\nabla \mu'_\pi|$ can be sufficiently large to induce an essential surface polarization. This can be important in studies of the electro-magnetic emission caused by the formation of a new surface within the body, or an electro-magnetic response of the body to an external dynamic influence on its surface [11, 12].

Therefore the above-mentioned equations supplied by the appropriate boundary conditions constitute a nonlocal theory of electro-magneto-thermo-elasticity of a polarized medium and take into account the local displacements of mass and electric charges. Such a theory is in fact an extension of the classical theory of piezoelectricity, in which the mass displacement process is taken into account. This theory conforms with electro-mechanical interactions in centrosymmetric materials including isotropic materials [11–13].

### *2.2. The key set of equations*

If the displacement vector $\boldsymbol{u}$, temperature $T$, magnetic induction $\boldsymbol{B}$, electric field $\boldsymbol{E}$ and the function $\tilde{\mu}'_\pi = \mu'_\pi - \mu'_{\pi 0}$ are selected as key functions, then the key set of linear equations takes the form:

$$\rho_0 \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \left( a_1^\sigma + a_2^\sigma - \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu} \right) \nabla(\nabla \cdot \boldsymbol{u}) + a_2^\sigma \Delta \boldsymbol{u} + \left( a_{eT} - \frac{a_{e\rho} a_{\rho T}}{a_\rho^\mu} \right) \nabla T + \\ + \frac{a_{e\rho}}{a_\rho^\mu} \nabla \tilde{\mu}'_\pi + \rho_0 \boldsymbol{F}, \tag{19}$$

$$\left(a_T^s-\frac{a_{\rho T}^2}{a_\rho^\mu}\right)\frac{\partial T}{\partial t}=(\lambda-\pi_t\sigma_e\eta)\,\Delta T+T_0\left(a_{eT}-\frac{a_{e\rho}a_{\rho T}}{a_\rho^\mu}\right)\frac{\partial(\nabla\cdot\boldsymbol{u})}{\partial t}+ \\ +\rho_0 T_0\frac{a_{\rho T}}{a_\rho^\mu}\frac{\partial\tilde{\mu}_\pi'}{\partial t}-\sigma_e\pi_t\nabla\cdot\boldsymbol{E}+\rho_0\Re, \tag{20}$$

$$\nabla\cdot\boldsymbol{B}=0,\qquad \nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t},\qquad \nabla\cdot[(\varepsilon_0-\rho_0 a_E^p)\,\boldsymbol{E}-\rho_0 a_{E\mu}\nabla\tilde{\mu}_\pi']=\rho_e, \tag{21}$$

$$\nabla\times\boldsymbol{B}=\mu_0\sigma_e\,(\boldsymbol{E}+\eta\nabla T)+\mu_0\,(\varepsilon_0-\rho_0 a_E^p)\frac{\partial\boldsymbol{E}}{\partial t}-\rho_0\mu_0 a_{E\mu}\frac{\partial\nabla\tilde{\mu}_\pi'}{\partial t}, \tag{22}$$

$$\Delta\tilde{\mu}_\pi'+\frac{1}{a_\mu^\pi a_\rho^\mu}\tilde{\mu}_\pi'=\frac{1}{a_\mu^\pi a_\rho^\mu}\left[\frac{a_{e\rho}}{\rho_0}\nabla\cdot\boldsymbol{u}+a_{\rho T}\,(T-T_0)\right]-\frac{a_{E\mu}}{a_\mu^\pi}\nabla\cdot\boldsymbol{E}. \tag{23}$$

It is noted that if the function $\tilde{\mu}_\pi'$ is excluded from the set of Equations (19)–(23), a set of linearized integro-differential (nonlocal) equations is obtained which takes into account the interaction between the mechanical, thermal, and electromagnetic processes.

## 3. Example

As an example, the above-mentioned equations are applied to study the coupled electromechanical fields in thin dielectric films. For the sake of simplicity the isothermal approximation is used and our considerations are restricted to the processes of deformation, polarization, and mass displacement.

### *3.1. Interface inhomogeneity of mechanical stresses, polarization and electric charges*

An infinite elastic polarized layer of an ideal dielectric is considered. At time $t=0$ the layer is cut from an infinite medium in such a way that at time $t>0$ it is in contact with a medium which behaves as a vacuum with regards to its electromagnetic properties. It is assumed that the layer is $2l$ in thickness and it is bounded by planes $x=\pm l$ which are free of stresses. The potential $\mu_\pi'$ at $x=\pm l$ is zero.

In this case, the basic functions $\boldsymbol{u}=(u,0,0),\boldsymbol{E}=(E,0,0)$ and $\tilde{\mu}_\pi'$ are the of space coordinate $x$ and time $t$ functions only.

Consequently, in the absence of a body force the set of Equations (19)–(23) for the layer $-l\le x\le l$ can be written as follows:

$$\rho_0\frac{\partial^2 u}{\partial t^2}=\left(a_1^\sigma+2a_2^\sigma-\frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\frac{\partial^2 u}{\partial x^2}+\frac{a_{e\rho}}{a_\rho^\mu}\frac{\partial\tilde{\mu}_\pi'}{\partial x}, \tag{24}$$

$$\mu_0\,(\varepsilon_0-\rho_0 a_E^p)\frac{\partial^2\varphi}{\partial t\partial x}+\rho_0\mu_0 a_{E\mu}\frac{\partial^2\tilde{\mu}_\pi'}{\partial t\partial x}=0, \tag{25}$$

$$\frac{\partial^2\tilde{\mu}_\pi'}{\partial x^2}+\frac{1}{a_\mu^\pi a_\rho^\mu}\tilde{\mu}_\pi'=\frac{1}{a_\mu^\pi a_\rho^\mu}\frac{a_{e\rho}}{\rho_0}\frac{\partial u}{\partial x}+\frac{a_{E\mu}}{a_\mu^\pi}\frac{\partial^2\varphi}{\partial x^2}, \tag{26}$$

where $\varphi$ is the electric potential and $E=-\partial\varphi/\partial x$.

The set of equations for the potential $\varphi_v$ of the electric field in vacuum ($x<-l$, $x>l$) is:

$$\frac{\partial^2\varphi_v}{\partial x^2}-\varepsilon_0\mu_0\frac{\partial^2\varphi_v}{\partial t^2}=0. \tag{27}$$

Note that the non-zero component $E_v$ of the electric field in vacuum $\boldsymbol{E}_v=(E_v,0,0)$ and the potential are related by $E_v=-\partial\varphi_v/\partial x$.

Taking into account the continuity condition for the electric potential, the boundary and radiation conditions become:

$$\left(a_1^\sigma + 2a_2^\sigma - \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\frac{\partial u}{\partial x} + \frac{a_{e\rho}}{a_\rho^\mu}\tilde{\mu}'_\pi = 0, \tag{28}$$

$$\tilde{\mu'}_\pi = -\mu'_{\pi 0}, \qquad \varphi = \varphi_v \quad \text{if} \quad x = \pm l, \tag{29}$$

$$\lim_{x\to\pm\infty}\left(\frac{\partial\varphi_v}{\partial x} \pm \sqrt{\varepsilon_0\mu_0}\frac{\partial\varphi_v}{\partial t}\right) = 0. \tag{30}$$

We suppose that all the unknown functions are zero at $t = 0$.

By neglecting the inertial forces, the solution of the boundary problem becomes:

$$\sigma_{yy} = \sigma_{zz} \equiv \sigma(x,t) = -\mu'_{\pi 0}\frac{a_{e\rho}}{a_\rho^\mu}\left[1 - \frac{a_1^\sigma}{a_1^\sigma + 2a_2^\sigma - \dfrac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}}\left(1 - \frac{a_{e\rho}^2}{\rho_0 a_1^\sigma a_\rho^\mu}\right)\right]\frac{\cosh(\lambda x)}{\cosh(\lambda l)}\theta(t), \tag{31}$$

$$\tilde{\mu}'_\pi(x,t) = -\mu'_{\pi 0}\frac{\cosh(\lambda x)}{\cosh(\lambda l)}\theta(t), \quad E(x,t) = -\lambda\mu'_{\pi 0}\frac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\frac{\sinh(\lambda x)}{\cosh(\lambda l)}\theta(t), \tag{32}$$

$$\varphi(x,t) = \mu'_{\pi 0}\frac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\frac{\cosh(\lambda x)}{\cosh(\lambda l)}\theta(t), \quad p(x,t) = \lambda\mu'_{\pi 0}\frac{\varepsilon_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\frac{\sinh(\lambda x)}{\cosh(\lambda l)}\theta(t) \tag{33}$$

for $-l \le x \le l$, and

$$\varphi_v(x,t) = \begin{cases} \mu'_{\pi 0}\rho_0 a_{E\mu}(\varepsilon_0 - \rho_0 a_E^p)^{-1}\,\theta\big(t + \sqrt{\varepsilon_0\mu_0}(x+l)\big), & \text{for } x < -l, \\ \mu'_{\pi 0}\rho_0 a_{E\mu}(\varepsilon_0 - \rho_0 a_E^p)^{-1}\,\theta\big(t - \sqrt{\varepsilon_0\mu_0}(x-l)\big), & \text{for } x > l, \end{cases} \tag{34}$$

where $\theta(x)$ is the Heaviside step function, $\sigma_{yy}$ and $\sigma_{zz}$ are the non-zero components of the stress tensor, and

$$\lambda^2 = -\frac{1}{a_\rho^\mu a_\mu^\pi}\left[1 + \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\cdot\frac{1}{a_1^\sigma + 2a_2^\sigma - a_{e\rho}^2/(\rho_0 a_\rho^\mu)}\right]\times\left[1 - \frac{\rho_0 a_{E\mu}^2}{a_\mu^\pi\varepsilon_0(1+\chi) + \rho_0 a_{E\mu}^2}\right] > 0. \tag{35}$$

It is noted that $a_\rho^\mu a_\mu^\pi < 0$ [13] and the quantity $1/\lambda$ has a length dimension and describes the characteristic distances of our problem.

For the density of the bound surface charge $\sigma_{se}(\pm l) = \rho_0 p(\pm l)$ for $t > 0$:

$$\sigma_{se}(\pm l) = \mu'_{\pi 0}\lambda a_{E\mu}\frac{\varepsilon_0\rho_0}{\varepsilon_0 - \rho_0 a_E^p}\tanh(\lambda l). \tag{36}$$

An analysis of the solution shows that the distribution of the stresses $\sigma_{yy}$, $\sigma_{zz}$, the reduced energy measure $\tilde{\mu}'_\pi$ and functions $E$, $\varphi$, and $p$ exhibit inhomogeneities close to the surface. Figure 1 displays the distribution of the normalized stress $\sigma/\sigma^*$, the electric potential $\varphi/\varphi^*$, and the electric polarization $p/p^*$ through layer, where:

$$\sigma^* = -\mu'_{\pi 0}\frac{a_{e\rho}}{a_\rho^\mu}\left[1 - \frac{a_1^\sigma}{a_1^\sigma + 2a_2^\sigma - a_{e\rho}^2/(\rho_0 a_\rho^\mu)}\left(1 - \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\right], \tag{37}$$

$$\varphi^* = \mu'_{\pi 0}\frac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}, \qquad p^* = \mu'_{\pi 0}\frac{\varepsilon_0\lambda a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}. \tag{38}$$

As can be seen, thin layers (curves 1–3 in Figure 1) are characterized by an overlay of the surface inhomogeneity while there is a well-defined bulk region

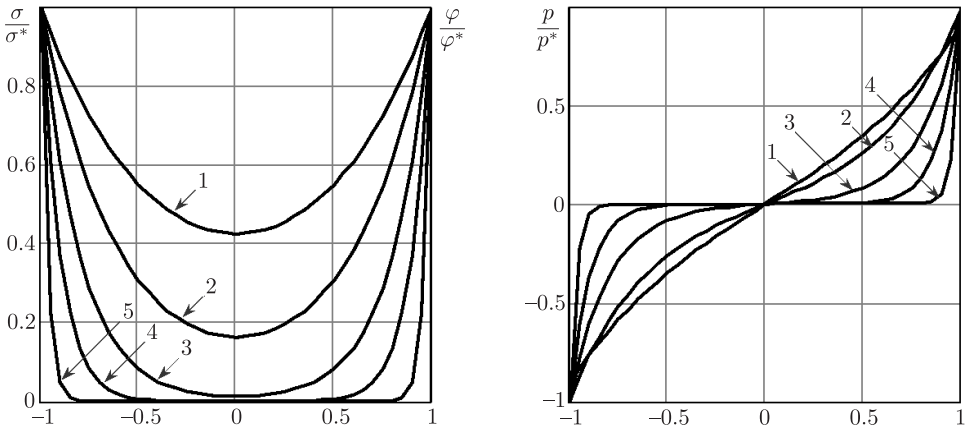


**Figure 1.** The distribution of the stress $\sigma/\sigma^*$, electric potential $\varphi/\varphi^*$, and electric polarization $p/p^*$ throughout the layer for $\lambda l = 1.5$, 2.5, 5, 10, 30 corresponding to the curves 1-5, respectively

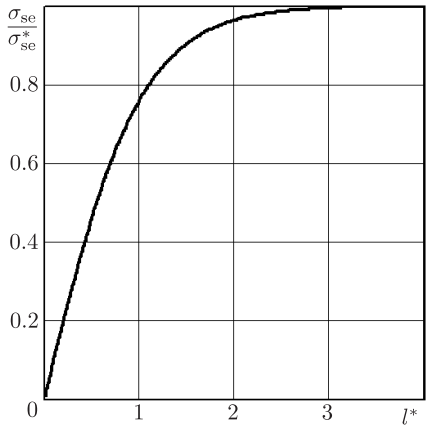


**Figure 2.** The dependence of the normalized surface charge density $\sigma_{se}/\sigma^*_{se}$ on the normalized layer thickness $l^* = \lambda l$

characterized by a uniform (constant) profile for thicker layers (curves 4 and 5). This effect manifests itself in the dependence of the surface charge density $\sigma_{se}/\sigma^*_{se}$, where $\sigma^*_{se} = \mu'_{\pi 0}\lambda a_{E\mu}\varepsilon_0\rho_0/(\varepsilon_0 - \rho_0 a^p_E)$, on the layer thickness, as is apparent from Figure 2.

The bounded charge (36) is induced at the surfaces of the layer while in the vacuum the electric field momentum arises and propagates from $x = \pm l$ to $\pm\infty$. Thus, the proposed model permits a description of the interface inhomogeneity of the stress-strained state and the surface polarization in dielectrics, the appearance of an electrical charge at surfaces as well as an electromagnetic signal caused by the surface formation.

### *3.2. The Mead anomaly*

Let us consider a layer of a dielectric with traction-free surfaces at $x = \pm l$. It is supposed that the electric potential $\varphi = \pm V$ is kept fixed on the corresponding surfaces

of the dielectric. In such a case, Equations (24) and (25) become one-dimensional and are reduced to:

$$\begin{gathered}\left(a_1^\sigma+2a_2^\sigma-\frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\frac{\partial^2 u}{\partial x^2}+\frac{a_{e\rho}}{a_\rho^\mu}\frac{\partial \tilde{\mu'}_\pi}{\partial x}=0,\\ (\varepsilon_0-\rho_0 a_E^p)\frac{\partial\varphi}{\partial x}+\rho_0 a_{E\mu}\frac{\partial \tilde{\mu'}_\pi}{\partial x}=0.\end{gathered}\tag{39}$$

The boundary conditions for a traction-free dielectric layer with the voltage applied at $x=\pm l$ are:

$$\left(a_1^\sigma+2a_2^\sigma-\frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\frac{\partial u}{\partial x}+\frac{a_{e\rho}}{a_\rho^\mu}\tilde{\mu'}_\pi=0,\qquad \varphi=\pm V.\tag{40}$$

Equations (26) and (39) need to be supplied with an additional boundary condition. Similarly to the boundary problem formulation by Mindlin [9, 10], it is assumed that at the layer surfaces $x=\pm l$ the specific polarization $p$ is proportional to the value of the specific polarization calculated within the classical theory framework $p_c=-(\varepsilon_0\chi/\rho_0)\,V/l$, namely:

$$a_E^p\frac{\partial\varphi}{\partial x}-a_{E\mu}\frac{\partial \tilde{\mu'}_\pi}{\partial x}=-k\frac{\varepsilon_0\chi}{\rho_0}\frac{V}{l}\quad\text{for}\quad x=\pm l,\tag{41}$$

where $\chi$ is the dielectric susceptibility, $k$ is the phenomenological constant, $0\le k\le 1$. The classical condition if $k=1$ is obtained from the expression (41), while $k=0$ corresponds to the polarization continuity across the interface [2, 9, 10].

After solving the boundary problem we obtain for non-zero components of the stress tensor $\sigma_{yy}=\sigma_{zz}\equiv\sigma(x)$, potential $\tilde{\mu'}_\pi$, electric potential $\varphi$, and polarization $p$:

$$\begin{gathered}\sigma(x)=\frac{2a_2^\sigma a_{e\rho}\left(\rho_0 a_\rho^\mu\right)^{-1}}{a_1^\sigma+2a_2^\sigma-a_{e\rho}^2(\rho_0 a_\rho^\mu)^{-1}}\cdot\frac{\varepsilon_0}{a_{E\mu}}\cdot\frac{V(k-1)(1+\chi)}{1+\chi^{-1}\lambda l\coth(\lambda l)}\cdot\frac{\sinh(\lambda x)}{\sinh(\lambda l)},\\ \tilde{\mu'}_\pi=\frac{\varepsilon_0}{\rho_0 a_{E\mu}}\cdot\frac{V(k-1)(1+\chi)}{1+\chi^{-1}\lambda l\coth(\lambda l)}\cdot\frac{\sinh(\lambda x)}{\sinh(\lambda l)},\\ \varphi(x)=V\frac{x}{l}+\frac{V(k-1)}{1+\chi^{-1}\lambda l\coth(\lambda l)}\left[\frac{x}{l}-\frac{\sinh(\lambda x)}{\sinh(\lambda l)}\right],\\ p(x)=-\chi\frac{\varepsilon_0}{\rho_0}\cdot\frac{V}{l}\left[1+(k-1)\frac{1+\chi^{-1}\lambda l\cosh(\lambda x)\sinh^{-1}(\lambda l)}{1+\chi^{-1}\lambda l\coth(\lambda l)}\right].\end{gathered}\tag{42}$$

The capacitance $C$ is defined as a ratio of the surface charge to the voltage drop across the dielectric layer:

$$C=\frac{1}{2V}\left(\varepsilon_0\frac{d\varphi}{dx}-\rho_0 p\right)\Big|_{x=\pm l}.\tag{43}$$

Then

$$C^{-1}=\frac{2l}{\varepsilon_0(1+\chi)}\cdot\frac{1+\chi(\lambda l)^{-1}\tanh(\lambda l)}{1+k\chi(\lambda l)^{-1}\tanh(\lambda l)}.\tag{44}$$

are obtained for the inverse capacitance $C^{-1}$ from the expressions (43) and (42).

We have taken advantage of the fact that $a_E^p=-\chi\varepsilon_0/\rho_0$.

The results of our calculations are shown in Figure 3. The solid lines are obtained using Equation (44) while the dashed lines correspond to the classical

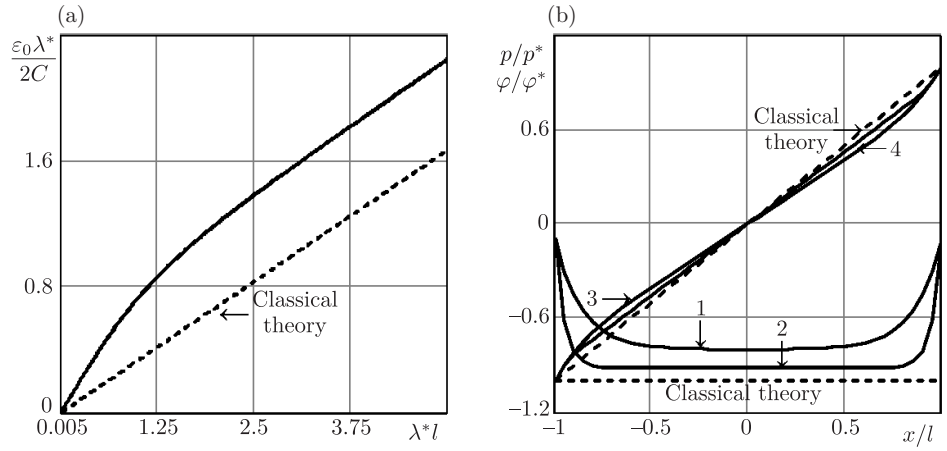


**Figure 3.** (a) Dependence of inverse capacitance on normalized film thickness. The solid line is obtained using Equation (44) and the dashed line corresponds to the classical theory. (b) Polarization (curves 1 and 2) and electric potential (curves 3 and 4) as functions of distance $x/l$ for $\lambda l = 5$ (curves 1 and 3) and $\lambda l = 20$ (curves 2 and 4). The line code as in (a)

theory of piezoelectrics. Figure 3(a) displays the relation between normalized inverse capacitance and normalized thickness. As can be seen, the linear part of the plot in Figure 3(a) is well above the curve obtained from the classical theory which is in agreement with the experimental results [3]. Figure 3(b) illustrates the non-linearity of polarization (curves 1 and 2) and the electric potential (curves 3 and 4) in films of different thickness (curves 1, 3 and 2, 4 correspond to $\lambda l = 5$ and $\lambda l = 20$, respectively). It is noted that similar results have also been obtained in [6, 9] where the gradient model of piezoelectrics has been used.

## 4. Conclusions

Using the recently proposed model of the electro-magneto-thermo-mechanics of dielectric materials which takes into account the local *mass* displacement (see [11, 12] for details), the surface inhomogeneity of the stress-strained state of a dielectric film, the surface polarization and the electric charge as well as the electromagnetic signals induced by the surface formation and their dependence on the film thickness and material characteristics have been studied. We have shown that the formation of a new surface in the dielectric leads to an electromagnetic "signal" which can be used, for instance, for passive (electromagnetic) diagnostics of structures built on dielectric materials.

It has also been shown that such a model adequately describes the anomalous dependence of the electric capacitance of a thin dielectric film on its thickness and the non-linearity of the electric field distribution inside the film. The results obtained herein can be useful in studies of the strength parameters of thin dielectric films and the electromagnetic emission at destruction of dielectric materials.

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