# EXPERIMENTAL AND NUMERICAL INVESTIGATION OF SMALL AXIAL TURBINES

PATRYK CHAJA, ADAM GÓRALCZYK
AND TADEUSZ KORONOWICZ

*Institute of Fluid-Flow Machinery PAS,*
*Fiszera 14, 80-952 Gdansk, Poland*
*papik@imp.gda.pl, adam.goralczyk@gmail.com, ttk@imp.gda.pl*

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**Abstract:** The authors conducted a series of experimental studies using a flow tunnel to determine the hydrodynamic characteristics of small axial turbines. The results obtained were confronted with the results obtained from the authors' own numerical codes based on vortex methods. This article describes the experiment and the numerical method and also discusses the results obtained*.

**Keywords:** vortex theory, experiment, turbine

## 1. Introduction

At present the Institute of Fluid-Flow Machinery has had over thirty years of experience in developing software for ship propeller design and analysis. These programs have been based on a vortex model [1], thus benefiting from the small demand for the CPU power and low computation time. One of these programs has been capable of calculating the hydrodynamic characteristic of ship propellers. Experimental research has proved very good conformity of the numerical and measurement results in the propeller working range [2]. Therefore, the authors decided to use this algorithm for calculating the hydrodynamic characteristic in the turbine operation range. Kaplan's model propeller (Figures 1 and 2) was chosen for the experiment, as it is available in the Institute laboratory and can be directly examined as a turbine. This is because their profiles are of a circular shape and they are fully symmetrical with respect to the trailing and leading edge. This characteristic property of model geometry allowed us to obtain the range of

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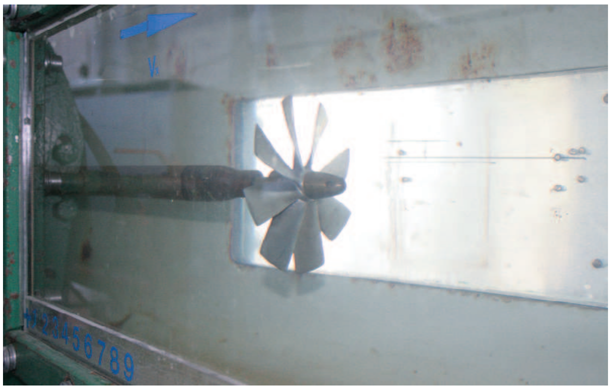

**Figure 1.** Kaplan's model propeller installed in cavitation tunnel

the turbine operation (correct location of the pressure and suction side) only by changing the direction of water flow to the model.

The study of a wind turbine in water is fully reliable. There is a close analogy between models researched in water and in air. While flow cavitation does not occur on a machine in water and the phenomena associated with supersonic flow will not occur in air, while maintaining the geometric and kinematic similarity, the coefficients obtained in the studies of the dimensionless dynamic are the same in water and in air. Relevant experiments [3] have been conducted on such a research analogy in both aquatic and aerial environment, which fully confirms the validity of this thesis. The results presented in this paper have one main purpose – verification of a computer program for calculation of the hydrodynamic characteristic in the turbine operation range and in this way we wanted to obtain a reliable tool for the turbine design and performance control purposes.

## 2. Laboratory research

While developing methods for model studies, the main feature being taken into account was the applicability (after some upgrades) of an existing test bench, without the need for its complete reconstruction. The method of research was developed in such a way that the measurement time was short in order not to allow excessive overheating of the generator, which would cause a malfunction of the motor control circuits. The essence of the experimental method was to measure the torque, $Q^*$, and the thrust, $T$, at a certain speed, $V$, for a known value of both the influx of water and the rotor speed, $n$. With these parameters the dimensionless coefficients are calculated:

$K_Q = \frac{Q^* - Q_0}{\rho n^2 D^5}$ – torque coefficients,

$K_T = \frac{T}{\rho n^2 D^4}$ – thrust coefficients,

$C_P = \frac{2\pi n (Q^* - Q_0)}{\frac{1}{2}\rho V^3 \pi \frac{D^2}{4}} = 16 \frac{K_Q}{J^3}$ – power coefficients,

where $Q_0$ – shaft torque resistance, $\rho$ – water density, $D$ – rotor diameter.

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The calculated coefficients of the torque, thrust and energy are functions of an advance coefficient:

$$J = \frac{V}{nD} \tag{1}$$

Research was conducted on marine screws of the "Kaplan" type having the spring coefficient of $P/D = 1.0$ and a different number of blades $Z = 4$, 5, 8. The diameter in all cases under consideration was $D = 148$ mm. The cross-section of the screw blade was of a circular sector shape. The geometry used in turbine blades is presented in Figure 2.

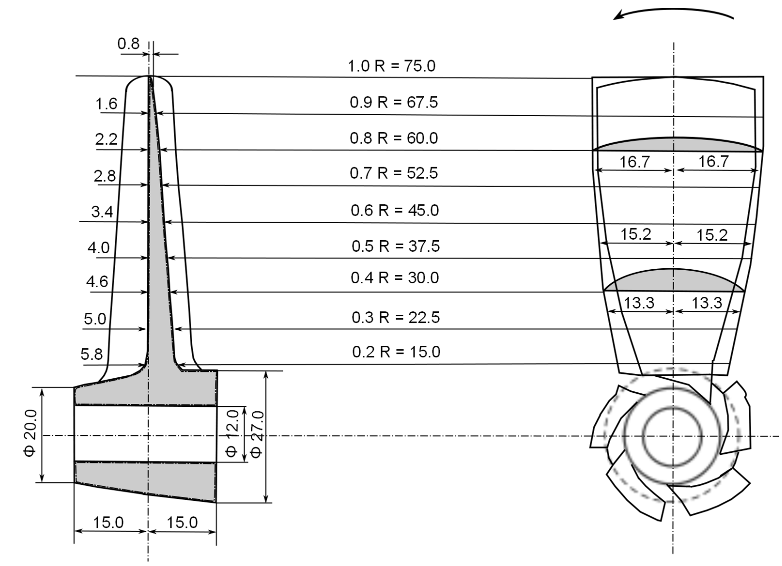


**Figure 2.** Geometry of tested axial turbine blades

## 3. Numerical algorithm of the program

The numerical algorithm of the program for calculating the hydrodynamic characteristics is based on the vortex model. The essence of the model is to seek for vorticity on the body flow surface (turbine blades, nozzle, hub), assuming it to be a grid of vortices made up of so-called, horseshoe vortices. Each of these vortices consists of a bound vortex and a free vortex (Figure 3).

General assumptions of the vortex model:

- Viscous forces are important only in the boundary layer. The following is true in the entire area of flow outside the layer: $\mathrm{rot}\, V = 0$.
- The boundary layer is replaced with the vortex surface and then digitized to the vortex filaments.
- The vortex filament circulation is determined based on the condition of the zero normal component of velocity to the flow profile wall.

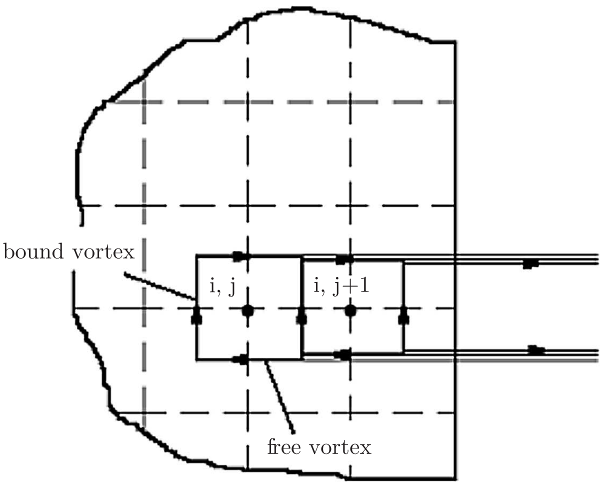


**Figure 3.** Vortex grid spread over wing surface

$$V_n = \overrightarrow{V}_0 \cdot \overrightarrow{n}_i + \overrightarrow{V}_\Gamma \cdot \overrightarrow{n}_i \tag{2}$$

$$-\overrightarrow{V}_0 \cdot \overrightarrow{n}_i = \sum_{j=1}^{N} \Gamma_j \left( \frac{1}{4\pi} \int_l \frac{\overrightarrow{r}_{ij} \times \overrightarrow{dl}}{r_{ij}^3} \right) \cdot \overrightarrow{n}_i \tag{3}$$

With this condition a set of equations is obtained:

$$Wsn_{ij}\Gamma_j = B_i \tag{4}$$

where the matrix of influence coefficients is determined basing on the Biot-Savart equation:

$$Wsn_{ij} = \frac{1}{4\pi} \int_l \frac{\overrightarrow{r}_{ij} \times \overrightarrow{dl}}{r_{ij}^3} \cdot \overrightarrow{n}_i \tag{5}$$

and free elements $B_i$:

$$B_i = \overrightarrow{V}_{0i} \cdot \overrightarrow{n}_i \tag{6}$$

$$\overrightarrow{V}_{0i} = \overrightarrow{V}_{np} + \overrightarrow{\omega} \times \overrightarrow{R}_i \tag{7}$$

where

$\overrightarrow{V}_0$ – undisturbed flow velocity,
$\overrightarrow{V}_\Gamma$ – vortex induced velocity,
$\overrightarrow{n}_i$ – normal vector with respect to the wall at a given point,
$\Gamma_j$ – circulation of the vortex at a given point,
$\overrightarrow{\omega}$ – rotation velocity,
$\overrightarrow{R}_i$ – the distance of the ith control point to the axis of rotation,
$N$ – the number of control points.

After calculating the circulation, $\Gamma_j$, in the next step the values of the tangential velocity are calculated:

$$V_{si} = \sum_{j=1}^{N} Wst_{ij}\Gamma_j + \vec{t}_i \cdot \vec{V}_{0i} \pm \frac{\Gamma_i}{2dx_i} \tag{8}$$

$$Wst_{ij} = \frac{1}{4\pi}\int_l \frac{\overrightarrow{r}_{ij} \times \overrightarrow{dl}}{r_{ij}^3} \cdot \overrightarrow{t}_i \tag{9}$$

where

$\overrightarrow{t}_i$ – vector tangent to the surface at the $i^{\mathrm{th}}$ checkpoint,
$\Gamma_i$ – circulation in the $i^{\mathrm{th}}$ checkpoint,
$dx_i$ – width of the airfoil which was attributed to a vortex element of circulation $\Gamma_i$,
'+' – suction side of the rotor blade,
'−' – pressure side of the rotor blade.

Using the tangential velocity, $V_{si}$, calculated by means of the Bernoulli equation the dimensionless coefficient of pressure and the pressure distribution on the blade are calculated:

$$Cp_i = \frac{p_i - p_0}{\frac{1}{2}\rho V_0^2} = 1 - \frac{V_{si}^2}{V_0^2} \tag{10}$$

where:

$p_i$ – pressure in $i^{\mathrm{th}}$ point on the surface,
$p_0$ – pressure in the undisturbed flow,
$\rho$ – density.

The calculated distribution of pressure is used to calculate the torque, $Q$, and the dimensionless coefficient, $K_Q$:

$$Q = \int_s \Delta p(s)\ (n_Y\ r_Z - n_Z\ r_Y) \tag{11}$$

where

$s$ – surface of the rotor blade,
$\Delta p$ – pressure difference between suction and pressure sides of the blade.

Knowing the value of the coefficient, $K_Q$, we are able to determine the energy coefficient, $C_P$.

## 4. Results of experimental studies and calculations

The results of experimental studies and the results obtained from the program considering the characteristics for the three axial turbine models with a different number of blades are presented below (Figures 4–6).

The presented results of theoretical and experimental research prove their good correlation. Significant differences occur only for high values of $J$ coefficients. Axial turbines (wind turbines) are not used in this area. The results indicate

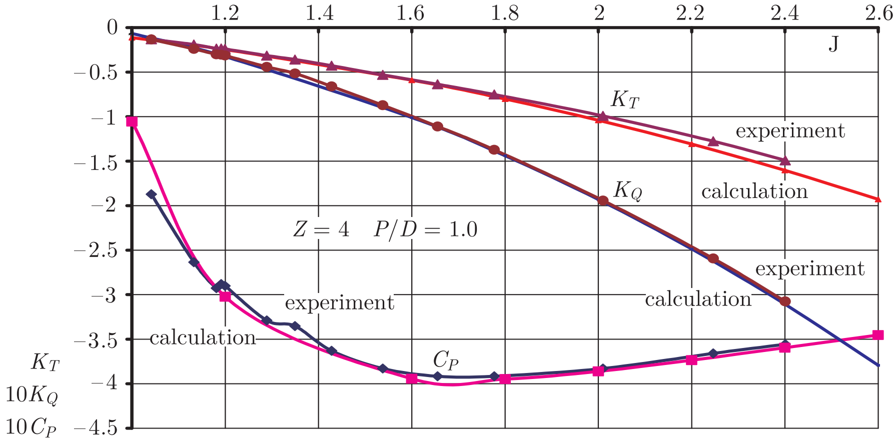


**Figure 4.** Results obtained for 4-blade turbine

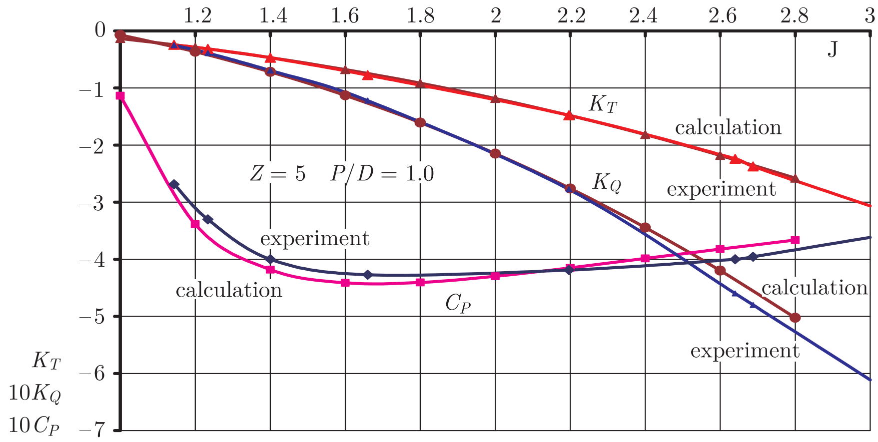


**Figure 5.** Results obtained for the 5-blade turbine

a strong correlation with the $C_P$ coefficient and the number of blades $Z$. The agreement between the calculations and the experiment seems to be the best for the 4-blade turbine and decreases with the increasing number of blades. This is the result of deformations of the researched turbines with respect to the original test turbine geometry which is shown in (Figure 2). The turbines analyzed in a laboratory test were constructed over 30 years ago. During this long period they have been extensively researched in various studies and may have some distortion of the original geometry (the models are made of a tin alloy which is easily deformed).

## 5. Conclusions

The article presents the results of model tests carried out on a tunnel flow compared with the results obtained from a program for calculating the hydrodynamic characteristics. The very good agreement of the results obtained

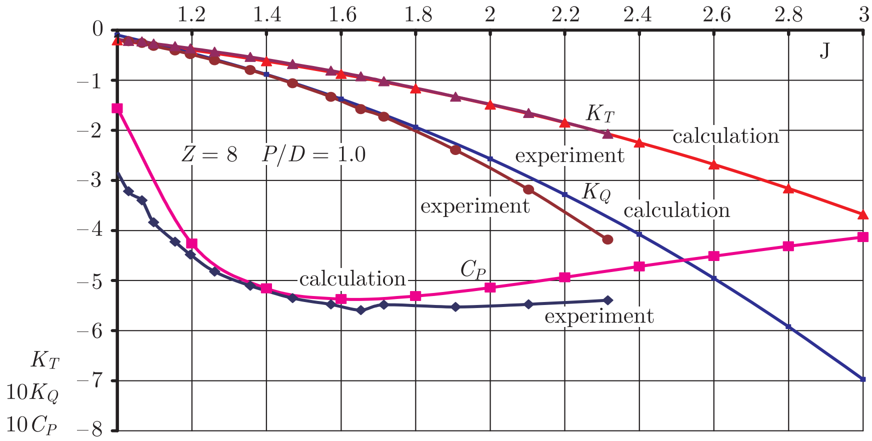


**Figure 6.** Results obtained for the 8-blade turbine

in the experiment and the results computed with our numerical code leads us to the conclusion that we have a numerical tool capable of quickly determining the hydrodynamic characteristics of axial turbines. Therefore, we are able to quickly specify the work parameters of the existing axial turbines operating both in water and in air. The program also makes it possible to correct the geometry and improve the operation of the existing turbines. It also allows us to design a new turbine for specific performance. The program determining the hydrodynamic characteristics of the geometry of a given turbine geometry is, as in the case of propellers, a tool for analysis and optimization of turbines. The main advantages of the program are the speed and low requirements considering the computational power. In addition, this program does not require any expensive and time consuming models or computational grids. It fully deserves to be called an engineering program capable of providing the information necessary for designing an axial turbine.

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