MODELING OF ROUGH WALL BOUNDARY LAYERS WITH AN INTERMITTENCY TRANSPORT MODEL

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Abstract: The paper presents a new extension of the γ-$\mathrm{Re}_{\theta t}$ model to account for both the laminar-turbulent transition and the surface roughness. The new modeling approach takes into account the pressure gradient, turbulence intensity and roughness height and density. In the transition region both the intermittency transport equation and the momentum thickness Reynolds number $\mathrm{Re}_{\theta t}$ transport equation, supplemented by the correlation of Stripf *et al.* (2009) suitable for rough wall boundary layers are used. An additional modification of the SST turbulence model allows for modeling a full turbulent boundary layer over surfaces with sand roughness. A comprehensive validation of the new method using transitional and fully turbulent test cases was performed. Flat plate data with a zero and non-zero pressure gradient test case as well as a high pressure turbine blade case were used for this purpose The studies proved that the new modeling approach appeared to be sufficiently precise and enabled a qualitative prediction of the boundary layer development for the tested flow configurations.

Keywords: surface roughness, boundary layer, turbulence, transition modeling

1. Introduction

Surface roughness has a strong influence on the efficiency, heat transfer, and hence, on the machine maintenance cost. It is known that the roughness could increase the skin friction in the turbulent boundary layer as well as shift the laminar-turbulent transition upstream the flow. Surface roughness generally adversely affects the blade row aerodynamic efficiency due to the thickened boundary layer and an increase in blockage. A decrease in the turbine efficiency has been reported, *inter alia*, by Waigh and Kind [1] and Boynton *et al.* [2]. The impact of surface roughness is however a function of the Reynolds number. Boyle and Senyitko [3] have shown that surface roughness with a high Reynolds number doubles the vane loss, however, roughness improves the aerodynamic efficiency at low Reynolds numbers. The last effect is present in case of airfoils where large laminar separation occurs and the best example are modern high-lift turbine blades. Therefore, it is not surprising that recent studies on high-lift blades suggest that a blade with as-cast surface roughness could have a lower loss than a polished one [4].

An accurate and reliable prediction of the surface roughness effect on the fluid flow and heat transfer is of great interest for designers. Modeling of the flow on a rough surface should cover the whole blade surface, hence, correct computations of the laminar, turbulent and transitional boundary layers are required. However, as shown by Stripf *et al.* [5], the roughness influence on the laminar momentum boundary layer is negligible. Therefore, the major tasks are modeling of the turbulent boundary layer and the transition process. Three different strategies have been employed for modeling the turbulent boundary layers. The first strategy consists in adding roughness sensitivity to the turbulent eddy viscosity near the wall, the second strategy comprises accounting for the roughness blockage and obstruction drag through the discrete-element method (DEM) and the third strategy fully discretizes the roughness features [6]. The most common method is the first one. The majority of the turbulence models in use today are based on the turbulent eddy viscosity, μ_t, thus, the obvious method to account for the surface roughness is to make μ_t a function of the roughness height. There are several modifications of this type, which rely mostly on the equivalent roughness height, K_s^+, a wide review of this subject is given, *inter alia*, by Aupoix and Spallart [7]. The alternative discrete-element method (DEM) which incorporates the roughness by extra terms in the governing equations has limited application as it is not formulated for a three dimensional unsteady flow field and could be used rather for artificial and localized roughness. The best method is to fully resolve the roughness with a computational grid, however, this approach is very expensive and not applicable for complex geometries nowadays.

However, without adequate modeling of the roughness effect on the transition process each of the strategies often fails. There are only a few proposals available today. One of the most interesting proposals formulated recently is the roughness-sensitive correlation of Stripf *et al.* [5] developed based on the comprehensive experimental investigation of a turbine vane.

On the other hand, modeling of the laminar-turbulent transition is one of the challenges even for a smooth surface. The greater need for more accurate flow simulations has resulted in intense development of transition modeling approaches in the last decade. the most popular recent methods for boundary layer modeling on a smooth wall include methods based on the intermittency parameter, γ, where the most representative model is γ-Re_θ proposed by Menter *et al.* [8]. In the recent period a modification of Menter's model has been proposed by Piotrowski *et al.* [9]. This model developed at the Częstochowa University of Technology has been named the Intermittency Transport Model (ITM). In the frame of the current work it was decided, based on the experience gained, to further extend this model to have a possibility of calculating the rough surface boundary layer.

The paper discusses the results of verification of the new approach based on the flat plate data with zero and non zero pressure gradient test cases as well as on the high pressure turbine blade case.

2. Methodology description

The modeling approach applied in this paper is based on the SST turbulence model with a time scale bound according to Medic and Durbin [10] and the γ-Re_θ transition model by Menter *et al.* [8]. The advantage of the latter model is that the transition onset is achieved locally through the use of the vorticity Reynolds number. The momentum thickness Reynolds number $\mathrm{Re}_{\theta t}$ transport equation was introduced for this purpose apart from the intermittency transport equation. This transport equation takes a non-local empirical correlation and transforms it into a local quantity, which is then compared to the local vorticity Reynolds number to detect the transition onset. On top of this advantage, this model may be easily adapted for parallel calculations on unstructured grids and that is why this model is considered as a promising perspective. An extension of Menter's model proposed by Piotrowski *et al.* [9] was made by development of two in-house correlations on the onset location and the transition length, which are confidential in Menter's original model. The great advantage of Piotrowski's approach is the possibility of unsteady calculations of the interaction of upstream wakes with downstream blades what is a basic feature of turbomachinery flows.

The onset parameter F_{onset} is formulated as a function of the critical transition Reynolds number, $\mathrm{Re}_{\theta c}$, and vorticity Reynolds number, Re_V, *i.e.* $F_{\mathrm{onset}} = f(\mathrm{Re}_V, \mathrm{Re}_{\theta c})$. $\mathrm{Re}_{\theta c}$ determines the location where the intermittency starts to increase in the boundary layer, that occurs upstream of the transition Reynolds number, $\mathrm{Re}_{\theta t}$. To determine $\mathrm{Re}_{\theta c}$ it was proposed to tie its value with $\widetilde{\mathrm{Re}}_{\theta t}$, which comes from the transport equation of the momentum thickness Reynolds number, $\mathrm{Re}_{\theta t}$, according to the relation: $\mathrm{Re}_{\theta c} = F_P \widetilde{\mathrm{Re}}_{\theta t}$, where F_P is an unknown function and $\widetilde{\mathrm{Re}}_{\theta t}$ is determined at the wall. An estimation of this function together with a correlation for the F_{length} parameter has been proposed by Piotrowski *et al.* [9]. The F_{length} parameter is located in the production term of the intermittency transport equation, thus, it influences both the length of the transition zone and the onset location. The F_{length} parameter is dependent not only upon the local properties, but also upon the global properties of the flow field, and, in order to account for this fact it was decided to relate F_{length} with $\widetilde{\mathrm{Re}}_{\theta t_{\mathrm{av}}}$, *i.e.* to the mean value of the $\widetilde{\mathrm{Re}}_{\theta t}$ distribution at the wall. Those correlations supplement the transport equations for intermittency, γ, and the Reynolds number, $\widetilde{\mathrm{Re}}_{\theta t}$, and form a complete calculation procedure for the l-t transition referred to as ITM.

According to Perry *et al.* [11], roughness elements can be classified into k-type and d-type, depending on the flow characteristics. For instance, when the roughness shift depends on the roughness height, k, it is called k-type, while for a d-type flow, the cavities between the roughness elements are narrow, and the roughness shift depends on an outer scale (*e.g.* pipe diameter). In the present study, it is only the k-type roughness that is considered. To take into account the roughness effect, according to the statement formulated above, it is necessary to describe the influence of roughness on the turbulent boundary layer and on the transition location. Two modifications to the SST model proposed by Hellsten and Laine [12] were introduced to predict the turbulent boundary layer behavior. The former is a change of the wall boundary conditions for a specific dissipation rate, ω, and the latter is a modified definition of the eddy viscosity, μ_T. For an ideally smooth solid surface, $\omega \to \infty$, while for a rough wall, ω has a finite value of:

$$\omega_w = \frac{u_\tau^2}{\nu} S_R \tag{1}$$

where u_τ is the friction velocity and S_R is a nondimensional coefficient defined as:

$$\begin{array}{ll} S_R = \left[50/\max\left(K_s^+; K_{s\min}^+\right)\right]^2 & \text{for } K_s^+ < 25 \\ S_R = 100/K_s^+ & \text{for } K_s^+ \geq 25 \end{array} \tag{2}$$

K_s^+ is the nondimensional sand grain height defined as:

$$K_s^+ = \frac{u_\tau k_s}{\nu} \tag{3}$$

where k_s is the grain size.

The rationale behind the eddy viscosity modification comes from the necessity to prevent SST limitation, and hence, the modeled shear-stresses from being activated in the near wall region, *i.e.* a sublayer or a rough layer [12]. It was done by redefining:

$$\mu_T = \frac{a_1 \rho k}{\max(A_1; |\Omega| F_2 F_3)} \tag{4}$$

where the new function, F_3, is introduced in such a form that it is zero in the near-wall region and unity elsewhere. a_1 is a constant equal to 0.31 and $|\Omega|$ is the absolute value of vorticity.

Roughness influences the location of the laminar-turbulent transition. It has been already stated that for prediction of the onset location the ITM model uses the information obtained from the transport equation of the momentum thickness Reynolds number, $\mathrm{Re}_{\theta t}$, *i.e.* the $\widetilde{\mathrm{Re}}_{\theta t}$ values determined at the wall. For the purpose of the current investigation it was decided to define new $\widetilde{\mathrm{Re}}_{\theta t_R}$ according to the Stripf correlation [5]:

$$\begin{array}{ll} \widetilde{\mathrm{Re}}_{\Theta t_R} = \widetilde{\mathrm{Re}}_{\Theta t} & \text{for } k_r/\delta^* \leq 0.01 \\ \widetilde{\mathrm{Re}}_{\Theta t_R} = \left[\left(\frac{1}{\widetilde{\mathrm{Re}}_{\Theta t}} + 0.0061 f_\Lambda \left(\frac{k_r}{\delta^*} - 0.01\right)^{f_{\mathrm{Tu}}}\right)^{-1}\right] & \text{for } k_r/\delta^* > 0.01 \end{array} \tag{5}$$

with displacement thickness, δ^*, f_Λ which takes into account roughness topographies and f_{Tu} which is a function of the local free stream turbulent intensity, Tu, expressed as percentage:

$$f_{\mathrm{Tu}} = \max(0.9; 1.61 - 1.15\exp(-\mathrm{Tu})) \tag{6}$$

All the above formulations together with the transport equations for the intermittency, γ, and the Reynolds number, $\widetilde{\mathrm{Re}}_{\theta t}$, form a complete calculation procedure for the *l-t* transition modeling which is referred to as ITM$_R$ in the following part of the paper. The transport equations for intermittency and the momentum thickness Reynolds number as well as for the SST turbulence model were implemented in the commercial package Fluent with the use of User Defined Functions (UDFs).

3. Tests of model for rough wall turbulent boundary layer

In spite of the number of measurements available in the literature it is difficult to chose a proper test case as only some of them are suitable for comparative evaluation. Apart from proper documentation of geometry and inlet conditions, the definition of the rough surface should be detailed enough. Two test cases were chosen for initial verification of the method proposed above. The first one was a flat plate flow with a zero pressure gradient published by Healzer [13]. The test section was 2.4 m long, 0.508 m wide and 0.102 high. The roughness was obtained by means of copper balls with a diameter of $d_0 = 1.27$mm brazed together in a most dense configuration. The equivalent sand roughness needed to model the flow was $k_s = 0.62 \cdot d_0 = 0.79$mm. The inlet turbulence intensity was equal to $\mathrm{Tu} = 0.4\%$ while the inflow velocity was set to be equal to $U_\infty = 27$ and 42m/s.

Figure 1 shows experimental and numerical distributions of skin friction coefficients for both inflow velocities. The results shown in the red lines for the ITM$_R$ model are compared to the results obtained by Stripf with the DEM-TLV model [5] shown as black lines. Additionally, distributions plotted in accordance with the semi-empirical formula proposed in 1983 by Mills and Hang [14] are shown. The formula:

$$c_f = (3.476 + 0.707\ln(x/k_s))^{-2.46} \tag{7}$$

defines the skin friction coefficient on a sand-roughened flat plate which is valid in the full-rough regime. It is seen that the ITM$_R$ model predicts the experimental data with high accuracy. It gives slightly lower values in comparison with the DEM-TLV model and the Mills and Hang [14] correlation for a higher velocity case, but fits better the experimental data.

For a better evaluation of the numerical method the results are overplotted with the correlation proposed by Pimenta (see Figure 2). Based on his own measurements as well as on the data of Healzer he has proposed a relation which links the skin friction coefficient with the momentum thickness, θ, and the length scale characteristic of the roughness elements (sphere radius, r, for this study) [15]:

$$1/2C_f = 0.00328(\theta/r)^{-0.175} \tag{8}$$

The correlation is assumed to be valid within the range of $1 < \theta/r < 10$ and to be independent of the Reynolds number for a zero pressure gradient and a fully rough state. The numerical results with ITM$_R$ show good agreement with the correlation line, although those for a higher velocity deviate slightly for a larger value of θ/r.

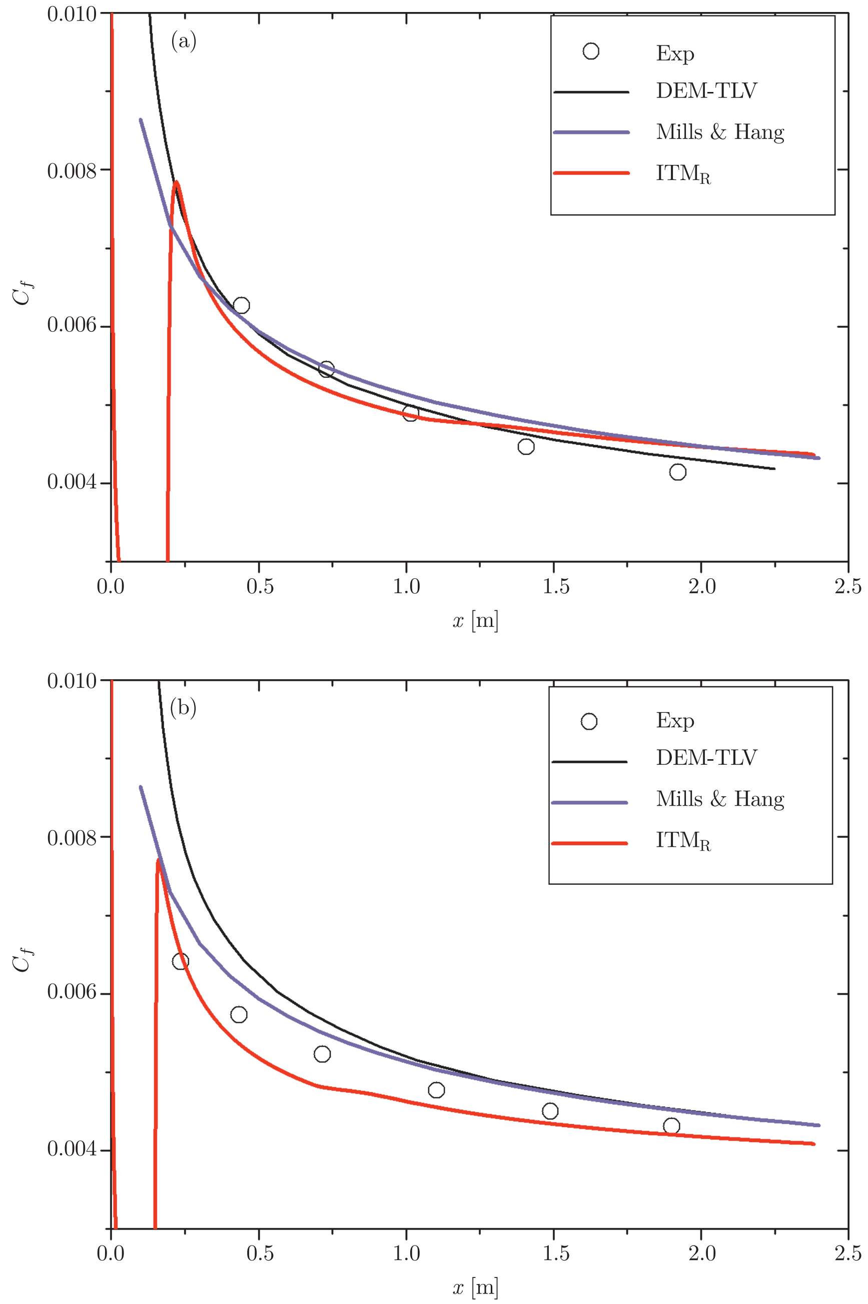

Figure 1. Skin friction coefficient distribution for zero-pressure gradient: (a) $U_\infty = 27\,\mathrm{m/s}$, (b) $U_\infty = 42\,\mathrm{m/s}$

The next test case concerns a non-zero pressure gradient case which was experimentally investigated using several variable freestream velocity distributions along the flat plate by Coleman *et al.* [15] on the same test section. One test case

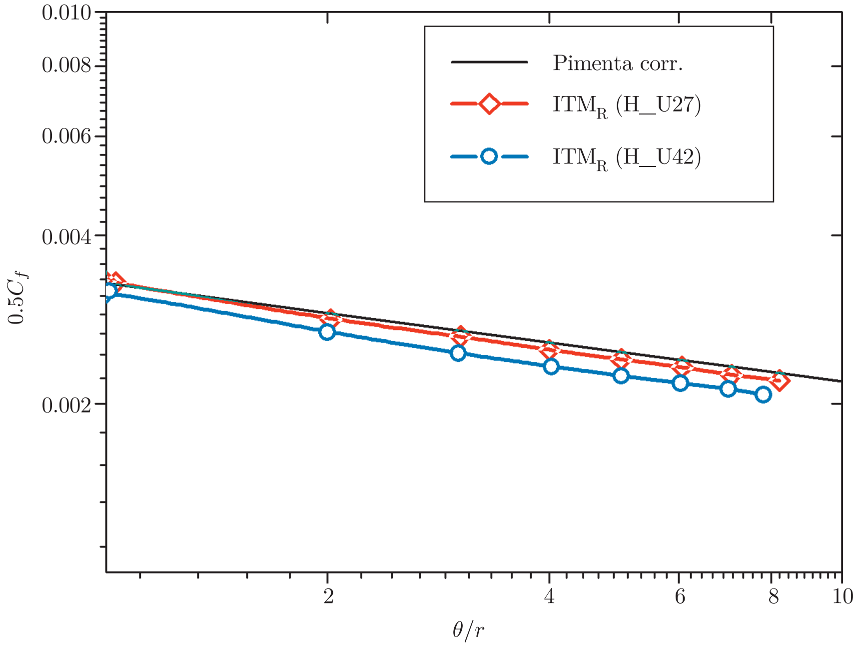

Figure 2. Skin friction coefficient distribution as function of θ/r for zero-pressure gradient

was chosen, where the velocity gradient in the acceleration region was adjusted in such a way as to obtain a constant distribution of the skin friction coefficient. The freestream velocity is given in Figure 3a, while the C_f distributions are presented in Figure 3b. It is seen that the shapes of the velocity curves are perfectly compatible with the experimental data. It is also the calculated skin friction coefficients that correspond well to the measured values, although the resolution of the experimental data seems to be too low. A similar discrepancy as in the case of the zero pressure gradient is however seen with the DEM-TLV results, taking slightly higher values.

Altogether, one can say that the performance of the ITM$_R$ model is sufficient to calculate the rough wall turbulent boundary layer and may be applied for more demanding test cases.

4. Turbine blade calculations

For confirmation that the proposed approach can be used for more complex industrial cases it was validated against the data of a high pressure turbine vane (HPTV) of a chord $c = 93.95$ mm, experimentally and numerically investigated at the Karlsruhe University [5]. Experimentally this turbine profile was investigated in a linear blade cascade at two different inflow Reynolds numbers ($\mathrm{Re} = 1.4\cdot10^5$ and $\mathrm{Re} = 2.5\cdot10^5$) and two turbulence intensities, $\mathrm{Tu} = 3.5\%$ and 8%. The applied deterministic roughness consisted of evenly spaced truncated cones uniformly distributed on the blade surface. The roughness height varied from 20μm, through 37μm, and to 70μm finally. The equivalent sand roughness needed to model the

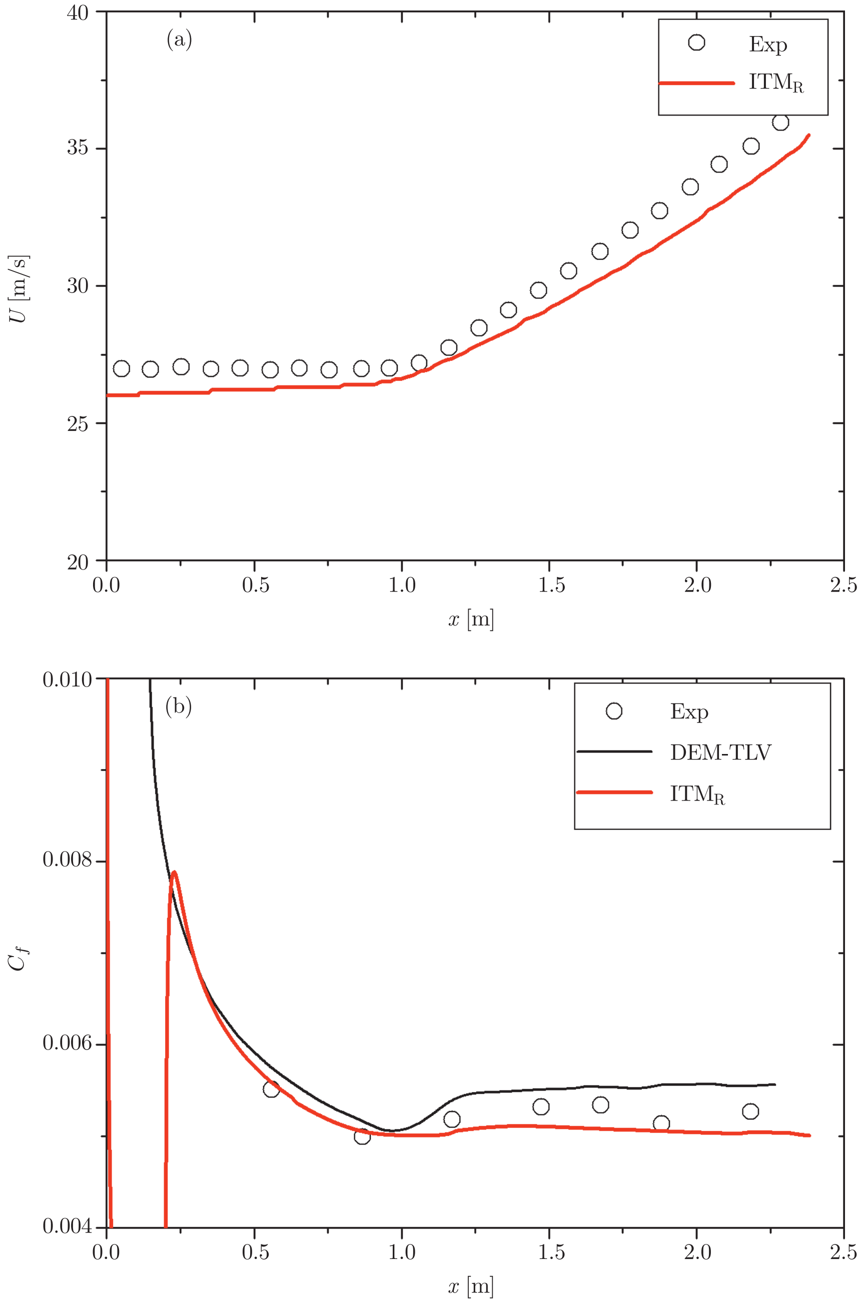

Figure 3. Freestream velocity distribution (a) and friction coefficient distributions (b) for non zero-pressure gradients flow

flow has been calculated by Stripf *et al.* [5] according to the Waigh and Kind correlation [1]. Additionally, a smooth surface was used as a reference case. One should notice that, in accordance with the observations of Zhang and Hodson [16] for the relevant impact on the flow, an equivalent sand roughness, k_s, should be at a level above 0.15% of the chord, and this corresponds to 0.14mm for the analyzed blade. Table 1 contains the basic roughness parameters as well as the boundary layer data needed for flow calculations. The boundary layer parameters (displacement thickness, wall shear stresses, friction velocity and turbulence intensity) contained in Table 1 are related to the *l-t* transition point detected during the calculations. The roughness model was validated on the basis of the Reynolds number, $\mathrm{Re}=1.4\cdot10^5$, and the turbulence intensity, $\mathrm{Tu}=3.5\%$.

Table 1. Roughness parameters and basic boundary layer parameters

Test Case	Roughness parameters			Boundary layer parameters			
	k_s [mm]	K_s^+ [—]	k_s/δ^* [—]	δ^* [mm]	τ [Pa]	u_τ [m/s]	$\mathrm{Tu}_{\mathrm{tr}}$ [%]
HP_01_20a	0.072	22.05	0.46	0.157	39	6.21	0.98
HP_01_40b	0.129	48.13	1.26	0.102	57.9	7.57	1.0
HP_01_70c	0.238	99.03	3.26	0.073	72	8.44	1.19

The pressure distribution around the blade profile presented in Figure 4 reveals that the flow accelerates sharply and reaches the minimum pressure for $S/c=0.7$ on the suction side. Downstream, a moderate deceleration is observed. On the pressure side one observes an almost constant velocity of the flow for most of the first part of the surface with a gradual acceleration towards the trailing edge. For the relative distance between $S/c=0.95$ and 1.0 a typical diffusion zone indicating a laminar separation is noticed.

Figure 5a presents the skin friction coefficient distribution obtained numerically (lines) overploted on the Nusselt number distributions (points) calculated from the experiment of Stripf *et al.* [5]. Such a comparison is possible as both the skin friction and the Nusselt number are good indicators of the transition onset location. The skin friction is proportional to the velocity gradient, while the Nusselt number is proportional to the temperature gradient near the wall and both the quantities rise dramatically on the border of the laminar and turbulent flow.

It is seen that the ITMR model gives a qualitatively good prediction of the boundary layer development, although the increase in the skin friction is more abrupt than the increase in the experimental Nusselt number. A small separation reported by Stripf *et al.* [5] was confirmed for the smooth case at the rear part of the blade. It is the most important region as the state of the boundary layer here determines the magnitude of losses. With an increase in the roughness height the transition location is shifted upstream. Even for the smallest value of k_s (0.072mm) the boundary layer looks to be sensitive to the surface roughness. It is clear that the separation bubble was suppressed for that case. The impact of the roughness could be also clearly evaluated by an analysis of the shape factor, H, Figure 5b, which is among the most precise indicators of the boundary layer state as it is obtained from the entire velocity profile. For a smooth case the shape

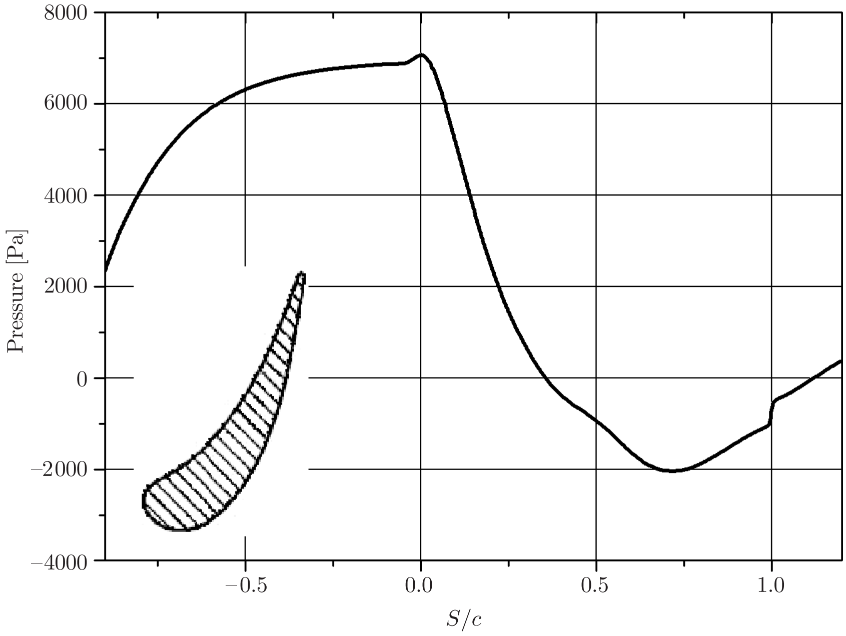

Figure 4. Pressure distribution for HPTV blade

factor reaches the value of $H = 3.4$ in the rear part of the suction side indicating laminar separation, and with an increase in k_s, this peak is damped and for further cases a drop of the H value to a turbulent level (~ 1.6) is located more and more upstream.

The aim of the paper was targeted towards validation of the numerical approach which could be used at the design stage for prediction of the surface roughness impact on the flow properties. Both our own and literature results indicate that the laminar-turbulent transition modeling on rough surfaces is extremely difficult. One can also suppose that some scaling factors in the Stripf correlation are lacking. It should be recalled that these correlations were adjusted based only on artificially generated roughness, which is nothing else that industrial roughness in turbomachinery. A further adjustment of this correlation is therefore necessary. However, such a model could be helpful in a numerical analysis of machine efficiency which could vary as a function of the surface roughness during the turbine's lifetime.

5. Conclusions

The paper presents the results of the tests and validations of a new modeling approach which relies on the γ-$\mathrm{Re}_{\theta t}$ model proposed by Menter *et al.* [8] and is extended by laminar-turbulent transition correlations proposed by Piotrowski *et al.* [9] and Stripf *et al.* [5] which take into account the surface roughness effects. The test cases included a flat plate turbulent boundary layer on rough walls as well as a high pressure turbine profile with various surface roughness.

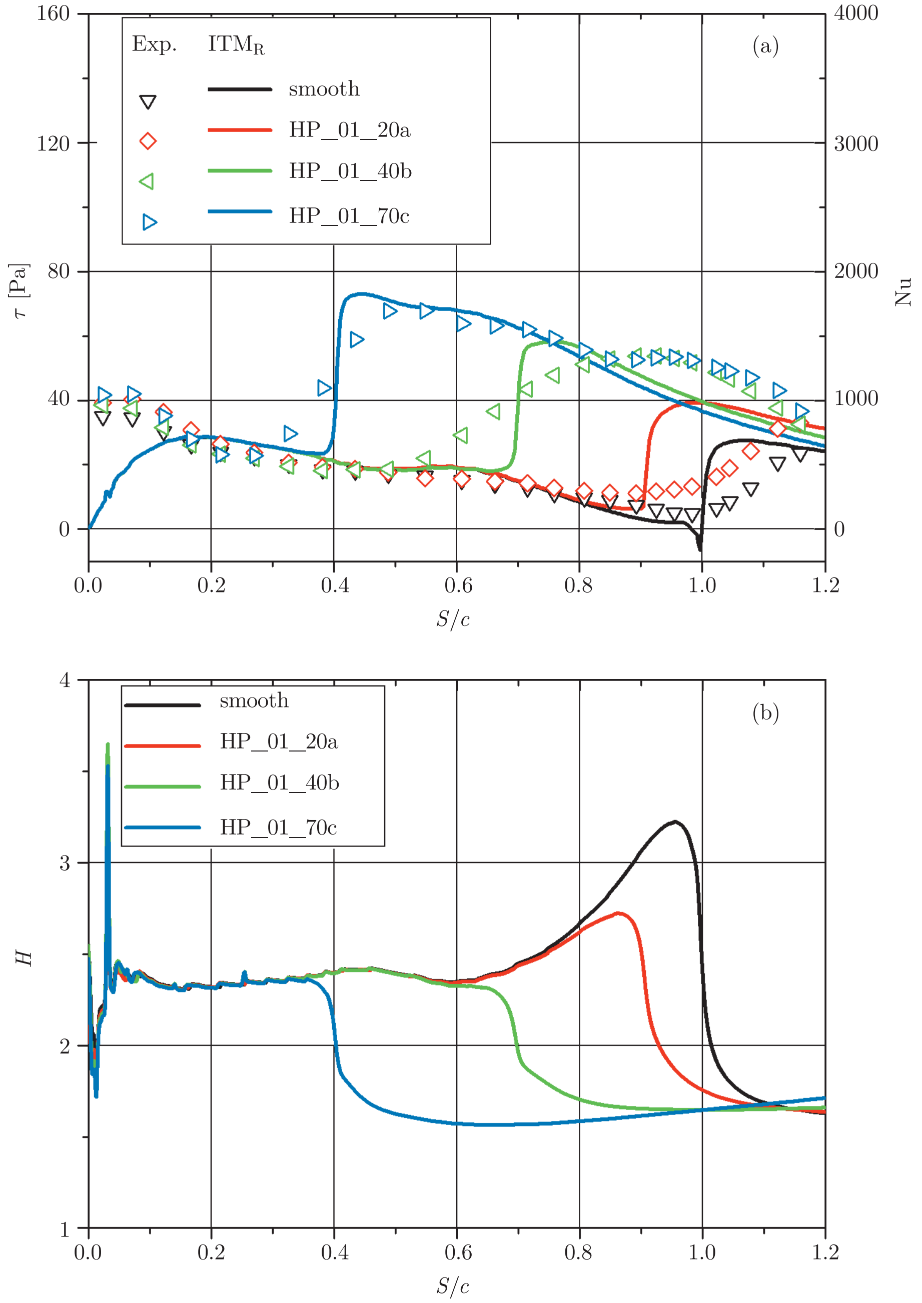

Figure 5. Skin friction (numerical results) and Nusselt number (experimental results) distributions (a), shape factor distributions (b) for various surface roughness for HPTV blade

The studies proved that the new modeling approach appeared to be sufficiently precise and enabled a qualitatively correct prediction of the boundary layer development for the tested flow configurations. Nevertheless, the transition onset location was not predicted with the highest accuracy. The lack of some scaling factors in the Stripf correlation as well as the possible interdependence of the transition correlations and the roughness correlations could be the reason for the observed discrepancy. One should be aware of the complexity of the task, as the modeling of the l-t transition and especially the modeling of the l-t transition on rough surfaces is an extremely difficult problem in the contemporary CFD.

Acknowledgements

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