METHOD OF CHARACTERISTICS FOR DESIGN OF CENTRIFUGAL PUMP GEOMETRY

ROMUALD PUZYREWSKI, KAZIMIERZ ŻOCHOWSKI AND PAWEŁ FLASZYŃSKI

Turbomachinery and Fluid Mechanics Department,
Gdansk University of Technology,
Narutowicza 11/12, 80-233 Gdansk, Poland
rpuzyrew@pg.gda.pl

(Received 28 July 2010; revised manuscript received 31 August 2010)

Abstract: The method of characteristics leads to the blade geometry of a centrifugal pump. The method is built taking the advantage of the governing equations of fluid mechanics written in a non-orthogonal coordinates system. The coordinate system is based on an analytically described boundary of a centrifugal pump. Some of the information concerning the designed geometry should be introduced in advance. The mass conservation equation needs the information of the blockage factor resulting from the blading thickness. In the momentum conservation equation the body force replaces the blading force together with the friction force. In the energy conservation equation the dissipation effects are represented by a loss coefficient. It is shown that while simplifying the body force vector, the set of equations reduces to a hyperbolic system which allows applying the method of characteristics. The shapes of surfaces representing the designed blading can be built from the trajectories of fluid particles.

Keywords: pumps, inverse problem, method of characteristics

1. Introduction

The method described in the paper belongs to the category of two-dimensional (2D) methods. The origin of this method can be found in the first publication from 1905 by H Lorenz [1]. The 2D model in the version of an inverse method leading to the blading surface shape has been presented in reference [2]. The recent paper focuses the attention on the application of the method to the centrifugal pump design. The crucial assumptions of the model are axis-symmetry, stationariness and incompressibility of the flow field. The model, although simplifies the real flow in the centrifugal pump type, is located between 1D and 3D models commonly used nowadays in the turbomachinery flow calculation. The advantages of the presented model can be sought in the fact of simplicity and an inverse version. The inverse version means that it allows obtaining the shape of a blade surface. This is particularly important when the leading edges of blading have to match the inlet conditions.

2. Geometry description

An essential point of the method is a description of the main features of geometry. When designing the stage, certain limits of dimensions come from a general layout of the designed pump. The points denoted in Figure 1 are the main constraints. They are (from right to left) the inlet to the rotor, the inlet to the diffuser and the outlet from the diffuser. The the pump stage end walls may be considered approximated by the following functions:

$$r_{b,d} = r_2 - (r_2 - r_1)\ h_{b,d} \tag{1}$$

where for the outer end wall (subscript b)

$$h_b = n\left(\frac{x^{(3)}}{L}\right)^{n-1} - (n-1)\left(\frac{x^{(3)}}{L}\right)^{n} \tag{2}$$

or the inner end wall (subscript d)

$$h_d = n\left(\frac{1-x^{(3)}}{L}\right)^{n-1} - (n-1)\left(1-\frac{x^{(3)}}{L}\right)^{n} \tag{3}$$

(L is optionally L_d or L_r as in Figure 1).

Parameter n, in functions (2) or (3), can be established using Equation (6) and the condition for the ratio, for given $x^{(1)}$, of meridional velocity component U_{x3}. The stream surfaces should be defined between the end walls. The example of stream surfaces in the rotor is shown in Figure 2. They satisfy the equation:

$$f = r_d + (r_b - r_d)g \tag{4}$$

where the stream surfaces distribution can be controlled by function $g\big(x^{(1)}\big)$ of the shape:

$$g = M_d x^{(1)} + (3 - 2M_d - M_b)x^{(1)2} + (M_b + M_d - 2)x^{(1)3} \tag{5}$$

The parameters M_d, M_b in the above function control, if necessary, the density of the stream lines as is shown in Figure 2. Note that if $M_d = 1$, and $M_b = 1$ then $g = x^{(1)}$ which represents the so called homotopic transformation given by relation (4). Transformation (5) is more suitable to change the flow field to a more uniform shape.

Parameters M_d, M_b may depend on coordinate $x^{(3)}$ as well. Function g has to be monotonic in the flow region and vary between 0 and 1. In this way the function of two arguments $f = f\big(x^{(1)}, x^{(3)}\big)$ has been established. The arguments $x^{(1)}$, $x^{(3)}$ can be treated as coordinates in the meridional cross-section whereas $x^{(2)}$ is the angle (circumferential) coordinate, constant for the meridional cross-section plane as in Figure 1 or Figure 2. The coordinate $x^{(1)} = \text{const.}$ describes the stream surface as in Figure 2. It can be also considered as an intersection of

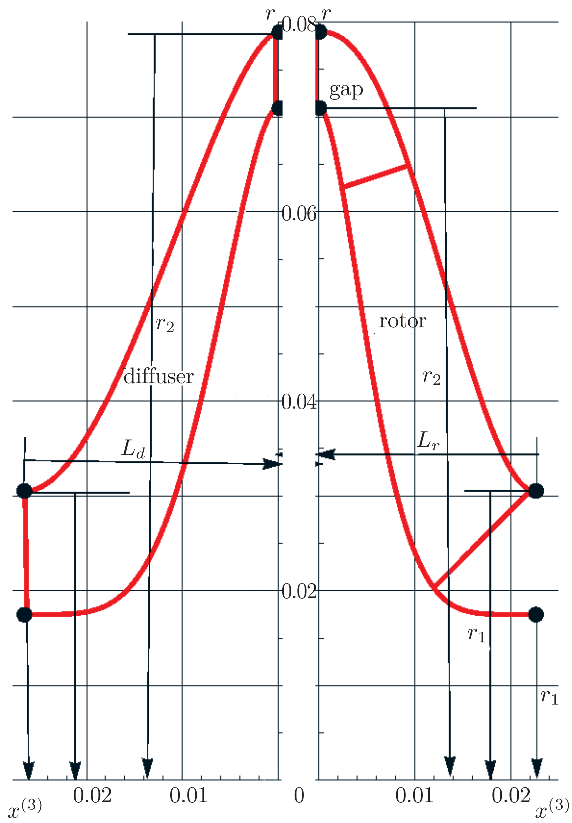

Figure 1. Pump stage contour

the axis-symmetrical surface with the meridional cross-section. In the $x^{(1)}$, $x^{(2)}$, $x^{(3)}$ coordinate system, the limiting stream surfaces of the pump stage correspond to $x^{(1)} = 0$ or $x^{(1)} = 1$. The axis-symmetrical condition can be noted as $\frac{\partial}{\partial x^{(2)}} = 0$, and the stationary flow as $\frac{\partial}{\partial t} = 0$.

3. Governing equations

The conservation equations in the non-orthogonal system of coordinates which is introduced here have the form:

a) mass conservation equation:

$$\left(1-\tau(x^{(1)},x^{(3)})\right)\rho U_{x3}\frac{f\frac{\partial f}{\partial x^{(1)}}}{\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}} = m\left(x^{(1)}\right) \tag{6}$$

where ρ – density, U_{x3} – the meridional velocity component tangential to lines $x^{(1)} = \text{const}$, $\tau(x^{(1)},x^{(3)})$ – the blockage factor due to the profile thickness.

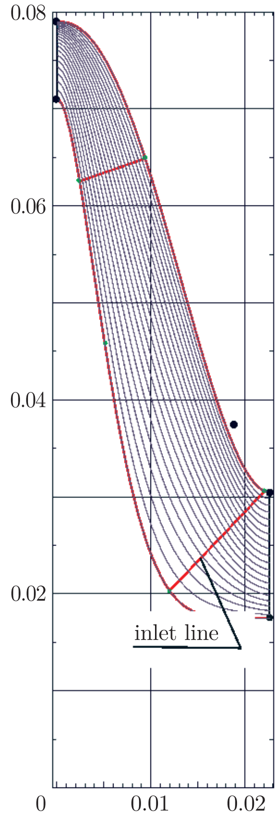

Figure 2. Stream surfaces in rotor domain

b) momentum conservation equations:
– in direction $x^{(1)}$

$$-\frac{\rho U_{x2}^2}{f}+\frac{\rho U_{x3}^2\frac{\partial^2 f}{\partial x^{(3)2}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}=-\frac{\partial p}{\partial x^{(1)}}\frac{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}{\frac{\partial f}{\partial x^{(1)}}}+\frac{\partial p}{\partial x^{(3)}}\frac{\partial f}{\partial x^{(3)}}+\rho f_{x1} \tag{7}$$

– in direction $x^{(2)}$

$$\frac{\rho U_{x3}}{f\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}}\frac{\partial}{\partial x^{(3)}}(fU_{x2})=\rho f_{x2} \tag{8}$$

– in direction $x^{(3)}$

$$\begin{aligned}\frac{\rho U_{x3}}{\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}}\left(\frac{\partial U_{x3}}{\partial x^{(3)}}-\frac{U_{x3}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\frac{\partial f}{\partial x^{(3)}}\frac{\partial^2 f}{\partial x^{(3)2}}\right)=\\ \rho f_{x3}+\frac{\partial p}{\partial x^{(1)}}\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}-\frac{\partial p}{\partial x^{(3)}}\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\end{aligned} \tag{9}$$

c) energy conservation equation:

$$\frac{U_{x2}^2+U_{x3}^2}{2}+\frac{p}{\rho}\pm g\Delta x_3+\zeta\frac{U_{x2}^2+U_{x3}^2}{2}-U_{\mathrm{rot}}U_{x2}=e_{\mathrm{inlet}}(x_1) \tag{10}$$

$$e_{\mathrm{inlet}}(x_1)=\frac{U_{x2\mathrm{inlet}}^2+U_{x3\mathrm{inlet}}^2}{2}+\frac{p_{\mathrm{inlet}}}{\rho}-U_{\mathrm{rot}}U_{x2\mathrm{inlet}} \tag{11}$$

where U_{x2} is the circumferential component of velocity, U_{rot} is the rotor velocity ($U_{\mathrm{rot}}=0$ for vanes or the return channel), ζ is a loss coefficient defined as a fraction of the kinetic energy, p pressure, f_{x1}, f_{x2}, f_{x3} are components of the body force, $g\Delta x_3$ – potential energy due to gravity.

We have 5 equations with 8 unknowns p, U_{x2}, U_{x3}, f_{x1}, f_{x2}, f_{x3}, τ, ζ. The additional assumptions ought to be introduced in order to make a task of an algorithmic type. Assuming that the body force from the blade side acts in a tangential plane to stream-surfaces $x_1=\mathrm{const.}$ we can neglect component f_{x1}. The 2D model (similarly to the 1D model) is not suitable to calculate the loss coefficient ζ. This coefficient has to be introduced arbitrary based on the experience. A similar assumption has to be made for blockage factor τ. These assumptions close the set of equations and enable us to formulate the algorithm.

4. Algorithm

The above set of equations can be reduced to a hyperbolic type of equations with two families of characteristics. The first family of characteristics is:

$$x^{(1)}=\mathrm{const.} \tag{12}$$

Along these characteristics function $m(x^{(1)})$ is constant what determines velocity component U_{x3} according to Equation (6).

The second family of characteristics is given by:

$$\frac{dr}{dx^{(3)}}=-\frac{1}{\frac{\partial f}{\partial x^{(3)}}} \tag{13}$$

and represents the lines perpendicular to the stream surfaces shown in Figure 2. In the coordinates $(x^{(1)},x^{(3)})$ it is given by the equation:

$$\frac{dx^{(1)}}{dx^{(3)}}=-\frac{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}{\frac{\partial f}{\partial x^{(1)}}\frac{\partial f}{\partial x^{(3)}}} \tag{14}$$

Along the second characteristic lines the ordinary differential equation for pressure has to be fulfilled:

$$\frac{dp}{dx^{(1)}}=-\frac{\rho U_{x2}^2}{f\frac{\partial f}{\partial x^{(3)}}}+\rho U_{x3}^2\frac{\frac{\partial^2 f}{\partial x^{(3)2}}}{\frac{\partial f}{\partial x^{(3)}}\left(1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2\right)} \tag{15}$$

The solution of (15) needs an initial condition for the pressure at the border where the characteristics (13) or (14) start.

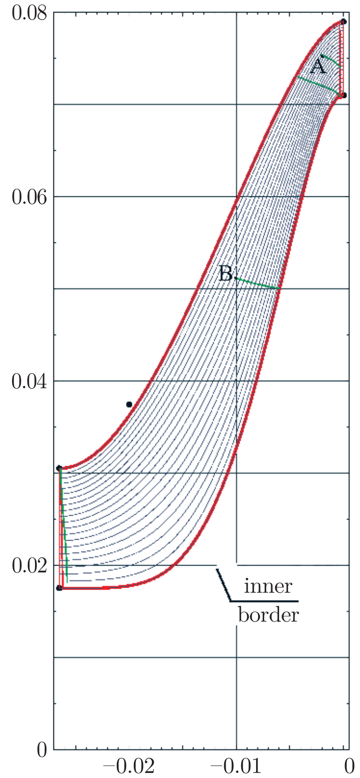

Figure 3. Examples of characteristics in stator

Two examples of characteristics are shown in Figure 3. To point A, the characteristic starts at the stator. The pressure distribution should be given at the inlet. To point B, the characteristic starts at the inner border of the stator where the pressure should be also given.

5. Examples of solutions

For the rotor shown in Figure 2 and the given inlet velocity and pressure distribution, the streamlines starting from the inlet line create the surface shown in Figure 4. The shape of this surface can be identified with the rotor blading shape. Usually the correction due to a finite number of blading can be introduced to the shape obtained for an infinite number of blading as is in the case of the 2D axis symmetrical flow model. The surface shown in Figure 4 is modified according to the Pfleiderer correction, well known in the pump technique. The rotor surface in Figure 4 is shown in the form of a thin blade. It is a common technique not to complicate the rotor blade shape because it simplifies the technology.

Contrary to this simplification, the stator blading shape is reasonable to design with a variable thickness as is shown in Figure 5. The reason for that comes from the fact that a large area of separation is indicated in the stator channel. In order to reduce the separation at the inner border (Figure 3) the

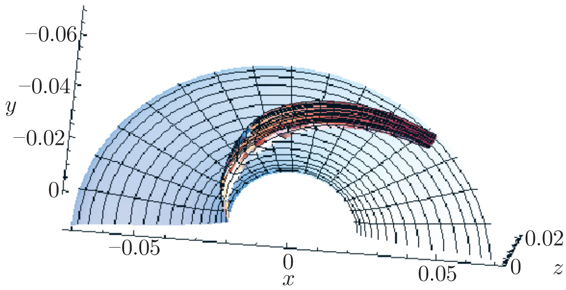

Figure 4. Shape of rotor blade surface located at inner border

increased blading thickness is a reasonable solution. This fact has been supported by the 3D computation. It has been found that the stator shaping is much more difficult that the rotor shaping. The problem cannot be solved on the basis of a 2D model computation only. The 2D model gives only a first approximation to further elaborating of the stator shape. The blockage factor such as the loss coefficient comes from outside the 2D model. In Figure 5, the stator blading shape is shown together with the stator end wall (inner border).

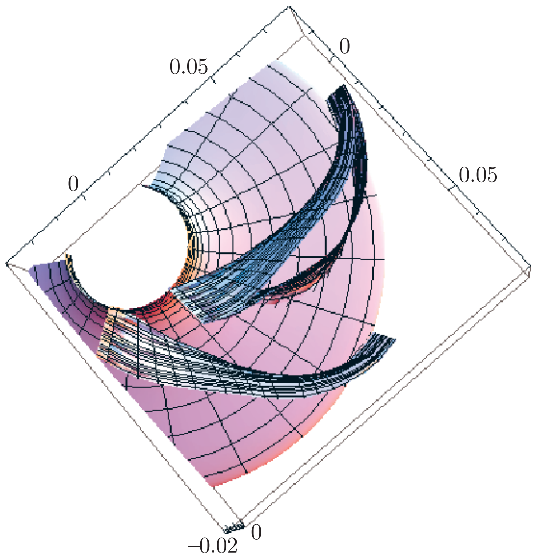

Figure 5. Surface of stator blade located at inner end wall

An appreciable reduction of the space due to the blockage factor is visible. The flow space reduction near the inner border is higher that at the outer border. The empty space between the two blade surfaces separated in the middle of the passage represents the thickness of the blade introduced by the blockage factor. One of the surfaces comes from the solution given by the flow field on the surfaces $x^{(1)} = \text{const}$. The second surface can be obtained by a shift in the circumferential direction by the distance given by the blockage factor. The reduction of the space by the blockage is not unique. The optimisation due to the losses can be solved with the help of the 3D computation, starting with the 2D results as the first approximation. The example of optimisation in this respect is presented in [3]. In this sense one can consider the gain of the 2D model in the chain of procedures leading to an optimal final design with respect to the given characteristic parameter as efficiency or the pressure head.

6. Conclusion

The idea of a 2D model in the inverse approach presented above can be considered as a starting point of the pump blading design. The further improvement of geometry, if necessary, can be done with the help of a higher model such as *e.g.* 3D. However, the first approximation based on the results of the 2D model seems to be helpful, provided that realistic assumptions concerning the factors necessary for a 2D model are introduced.

References

[1] Lorenz H 1905 *Zeitschrift des Vereines Deutscher Ingenieure* **49** (41) 1610

[2] Puzyrewski R 2005 *Two Dimensional Inverse Method of Turbomachinery Stage Design, Developments in Mechanical Engineering*, Gdansk University of Technology Publishing, pp. 57–72 (ISBN 83-7348-113-3)

[3] Banaszek M and Tesch T 2010 *Proc. 2nd Int. Conf. on Engineering Optimization (EngOpt 2010)*, Lisbona, Portugal (on CD)