PARALLEL IMPLEMENTATION OF RAY TRACING PROCEDURE IN ANISOTROPIC MEDIUM

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Abstract: This article describes a parallel implementation of a ray tracing algorithm in a heterogeneous anisotropic geological medium. The shortest path method, which was used for calculations, can give ray path and travel time of seismic wave propagation even for a random and discontinuous velocity field. The high precision required in such calculations was obtained by employing a dense computational grid. This led to a significant increase in the computational effort of the algorithm. The procedure was parallelized using domain decomposition. The results show that the parallel performance of the ray tracing procedure strongly depends on the assumed geological method and differs between media with and without anisotropy of seismic wave propagation.

Keywords: ray tracing, seismic anisotropy, domain decomposition

1. Introduction

The knowledge of seismic attributes is important in coal and hydrocarbon exploitation. Accurate information on the velocity field is a crucial aspect of enhanced oil recovery [1], deep silver mine safety [2] and many methods of seismic data processing, such as migration, static corrections and modeling. Seismic travel time tomography is one of the methods used to estimate velocity parameters. The knowledge of ray paths is essential in the case of inversion tomography data. There are several methods of seismic P-wave ray tracing: from basic straight-line approximations (see for example [3, 4]), through ray tracing using the eikonal equation [5] and the shortest path method [6], to the most advanced methods such as full waveform modeling. One of the most thorough classifications and descriptions is presented in [7]. Further information on the characterization of ray tracing and its application to the inversion of geophysical data can be found in [8].

This work presents a parallel implementation of the ray tracing procedure based on the shortest path method ([9, 10]). The shortest path method, particularly in a real geological and anisotropic medium, is an example of an extremely time-consuming algorithm. The associated computational effort depends on the assumed geological model and is significantly larger for the anisotropic media.

2. Shortest-path ray tracing algorithm

The calculation of the shortest path is an important combinatorial problem [9]. A seismic ray is defined as the path s between points A and B in a velocity field v which minimizes the travel time of a seismic wave [11]:

$$t=\int_A^B \frac{ds}{v(x,y,z)}=\min \tag{1}$$

In the shortest-path algorithm, two discrete computational grids are used. The first one stores the values of the velocity field, while the second is used to save the shortest computed time of seismic wave propagation from the source point to the given computational point. In the second grid, the coordinates of the computational point from which a seismic wave arrives are saved as well. The first step of the presented algorithm concerns the calculation of the travel time from the source of the seismic wave to the nearest computational points, which is saved as the so-called shortest time parameter. Each subsequent step, according to the Huygens's law, involves computations performed for each point whose value of the shortest time changed in the previous step. Subsequently, such a point becomes the new source point and the procedure is repeated recursively. The calculations are performed until the shortest time value does not change in a single iteration in the entire model.

In nature, the shortest ray path connecting two different computational points in a real inhomogeneous anisotropic model is not straight, but curved. This is due to the dependence of the seismic wave velocity on both the position and direction of the wave propagation. In the presented algorithm, the anisotropy of the seismic wave velocity was introduced by three parameters: two components of the velocity vector v_x and v_z, and the angle φ describing the rotation of the velocity vector components about the origin of the assumed coordinate system (Figure 1).

The ray paths are obtained by a recursive procedure, which starts from the receiver point and moves backwards to the points with the shortest time, for which the coordinates are saved as the seismic wave source. The procedure terminates when the source point is reached.

2.1. Data structure

A two-dimensional velocity model of lengths $N_z \times N_x$, in the x and z direction, respectively, was assumed in the presented algorithm. The model employs

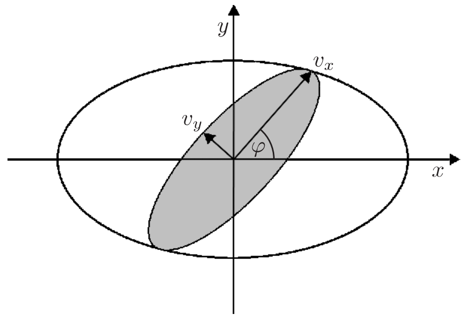

Figure 1. Components of the velocity vector: v_x and v_z define the axes of the anisotropy ellipse and φ – the rotation angle of the axes about the horizontal plane. Velocity can be interpreted as the length of the ellipse radius

rectangular cells, each with an area of $d_z \times d_x$. The velocity parameters are assume to be constant within each cell. Such an assumption allows to use a straight ray between the computational points located at the edges of the rectangular cells. The straight-line propagation inside the cells limits the number of computational points only to the points located at the edges. A highly precise ray tracing algorithm is obtained by placing additional computational points between the cell vertices. The computational points located in the cell vertex are called primary nodes (PNs); the grid points added to the cell edges in order to increase the precision of the algorithm are called secondary nodes (SNs, see Figure 2). SNs are arranged with a certain linear density, termed the node per edge (NPE) ratio. There is a strong dependence between the value of the NPE ratio and the error of ray path estimation (see Figure 3). The relative error presented in Figure 3 was calculated using Equation (2) in PNs for the isotropic, homogeneous model employing 50×50 cells and a constant velocity $v = 1000\,\mathrm{m/s}$. The higher the assumed value of the NPE ratio, the better the precision of the obtained ray tracing algorithm. However, a large value of the NPE ratio significantly increases the computational effort of the ray tracing algorithm:

$$Error = \frac{100\%}{N} \cdot \sum_{i=1}^{N} \frac{|t_i^{est} - t_i^{math}|}{t_i^{math}} \tag{2}$$

where:

$$t_i^{math} = \frac{\|SP,\, PN_i\|}{1000} \tag{3}$$

N – number of primary nodes,
SP – coordinate of the shot point (source of the seismic wave).

2.2. Ray tracing scheme

There are two scenarios for the seismic wave propagation due to a specific parameterization of the computational model described above. The calculations for a point which depends on the localization of the P-wave source are performed for two or four adjacent velocity cells (see Figure 2). If the source point is

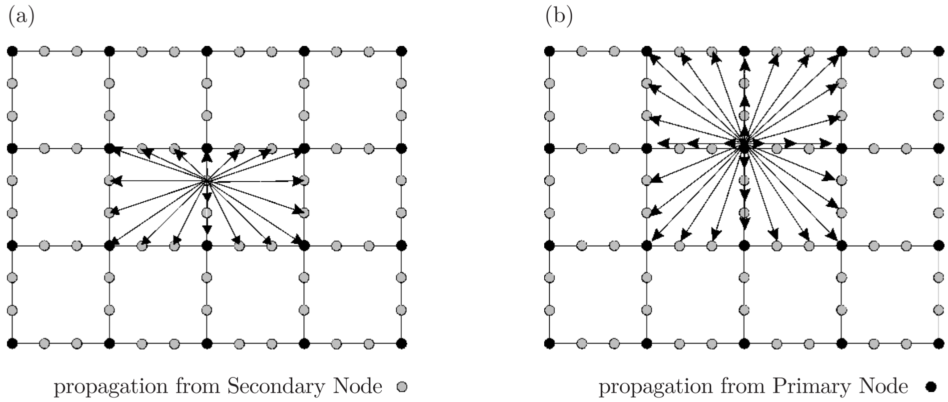

Figure 2. Schematic of seismic wave propagation from the source point, which is (a) a secondary node point; or (b) a primary node point

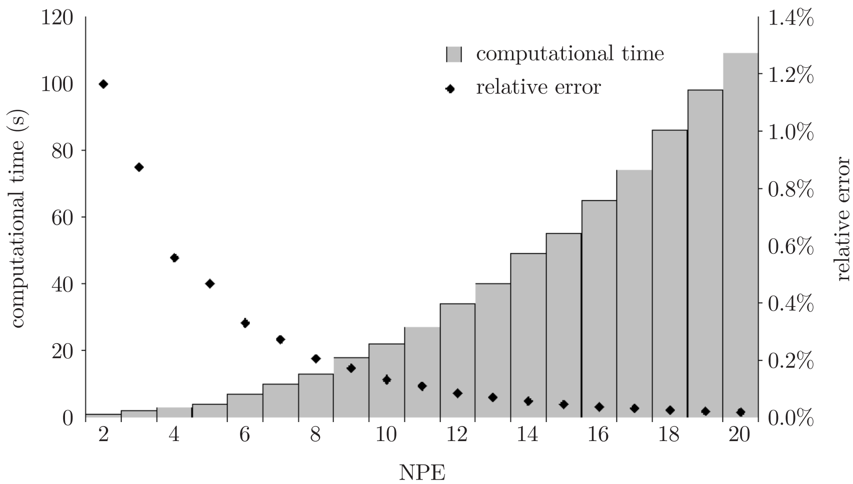

Figure 3. Relationship between the NPE ratio and the relative error (Equation (2)) of the ray path algorithm and the computational effort. The results were obtained for a calculation on a single node of the IBM Blade machine

not a primary but rather a secondary node point, the arrival times to the grid points located at the boundary of the two adjacent velocity cells are calculated (Figure 2, panel (a)). If the source point is a primary node point, the calculation is performed for all grid points that belong to the four adjacent velocity cells (Figure 2, panel (b)).

The results of the ray tracing algorithm for the Legnicko-Glogowski copper district (LGOM; a mining region in Poland) are presented in Figure 4. The computation was performed with a serial algorithm, using 1500×5750 velocity cells with an NPE ratio equal of 5. The calculations were performed on a single node of the IBM Blade computer with a 2 GHz processor and 4 GB of RAM, taking more than 6 days to complete. The program was compiled using gcc on a Fedora 10 system. The plots were made using MathWorks MatLAB 7.9 [12].

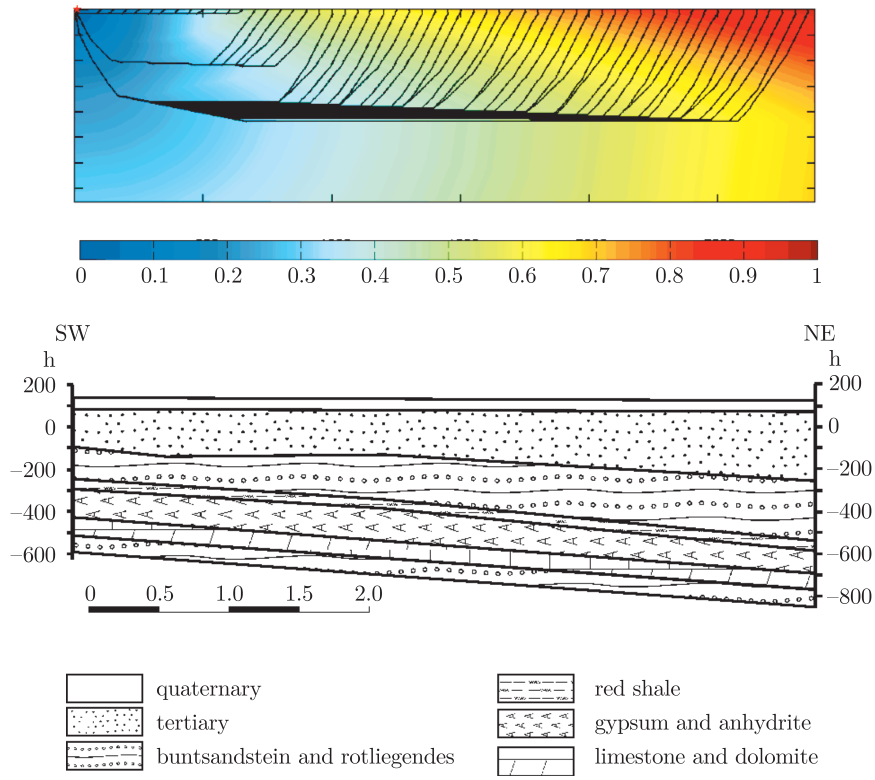

Figure 4. Result of the application of the ray tracing algorithm for the Legnicko-Glogowski copper district (LGOM). The upper panel shows the travel time (in s) of a seismic wave from the source point located in the upper left corner of the assumed model, and the seismic ray path. The lower panel presents a geological sketch of the assumed model (after [13])

3. Parallel implementation

The main disadvantages of the ray tracing algorithm applied to a two-dimensional travel time tomography problem are its associated extensive CPU and memory requirements apparent with large models. As shown in the previous section, this is caused not only by the dimensions of the model, but mainly by the application of the secondary nodes, which improve the accuracy of the ray tracing algorithm. Basically, the more precise information is required, the higher NPE ratio must be assumed. Consequently, such an assumption results in the increase of the total amount of grid points, which must be calculated at each iteration of the shortest path algorithm. One way to overcome the problem of the time-consuming, computationally intensive numerical algorithm is the application of a parallel computing environment. This method is commonly used in solving geophysical problems [14].

Here, parallelism was introduced into ray tracing for the two-dimensional time tomography problem by the decomposition of the computation domain. A parallel algorithm based on domain decomposition was employed. One of the computational nodes, the so-called master node, distributed the velocity model among the remaining nodes of the parallel computational environment. Subsequently, the computational nodes simultaneously executed the ray tracing algorithm. After each iteration of the ray tracing algorithm, the adjacent computational nodes exchanged the calculated travel time for a halo region encompassing the borders of a subdomain (*cf.* Figure 5).

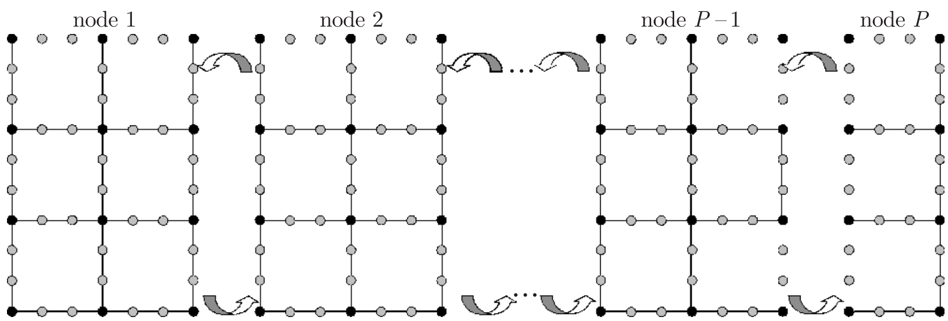

Figure 5. Parallel decomposition of the parallel ray path algorithm

The role of the master node was restricted to management. Apart from dividing the computational domain, the master node was also responsible for controlling the computation and, finally, for collecting the obtained results.

The parallel algorithm was implemented using MPI [15].

4. Models and results

The algorithm was tested on several models, including simple models (homogeneous, isotropic), as well as more complicated models (heterogeneous, anisotropic). The geological model of the Legnicko-Glogowski copper district (LGOM), a model with gradient changes of the velocity field and the model constructed using randomly chosen velocities were also considered. All tested models were of the same size (150×575 velocity cells) and all had the same value of the NPE ratio, equal to 5. The anisotropic parameters assumed in all tested media are presented in Table 1.

For all considered models, we calculated the commonly used parallelization metrics: computational time, speedup and efficiency. The relationship between the parallel algorithm parameters and the number of computational nodes is presented below (Figures 6–8).

The analysis presented in this work shows a different behavior of the parallel ray tracing algorithm for different sets of data. In general, the higher the number of computational nodes used, the shorter the computation time of parallel implementation and the better the speedup of the ray path algorithm. The computation time required to execute the ray path algorithm depended strongly on the geometry and the velocity field of the geological medium and could be several times longer for specific data sets (see Figure 6), particularly the computation

Table 1. Anisotropy of seismic wave propagation parameters for all tested media. In each case, the rotation of the components of the velocity vector about the origin of the assumed coordinate system was zero

Velocity Model	Value of component v_x	Value of component v_z
LGOM geological	fixed	0.8 of component v_x
homogenous isotropy	1000 (m/s)	1000 (m/s)
anisotropic	1000 (m/s)	random $v_z = (0.7; 1.3) \cdot v_x$
random	random $v_x = (1000; 5000)$ (m/s)	random $v_z = (0.7; 1.3) \cdot v_x$
gradient	linear gradation from the top to the bottom of the model $v_x = (1000; 5000)$ (m/s)	random $v_z = (0.7; 1.3) \cdot v_x$

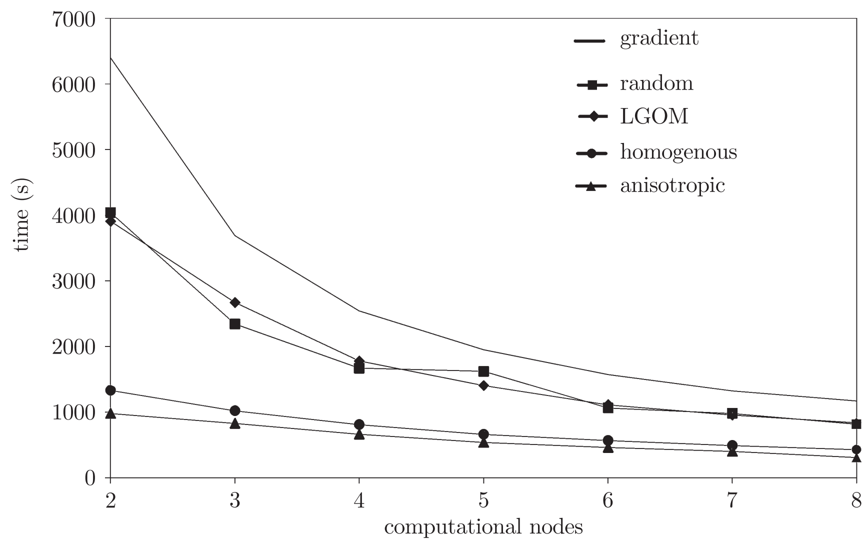

Figure 6. Relationship between the time required to execute the ray path algorithm and the number of computational nodes

time obtained when two computational nodes were employed. The differences in computation times strongly influenced the speedup and efficiency of the ray tracing procedure (Figure 7 and Figure 8). Another factor that caused the differences in speedup and efficiency, was assumption anisotropy of the seismic velocity. There is a constant dependency between speedup and efficiency in the homogeneous and the LGOM geological models. Both models exhibit a fixed dependency between velocity model components. Speedup curves for the media with random velocity and even random ratio of velocity components do not increase monotonically as can be observed for other speedup curves. For the medium with the strongest anisotropy of the seismic wave velocity (random model), both speedup and efficiency decreased with an increase in the number of computational nodes (Figure 7 and Fig-

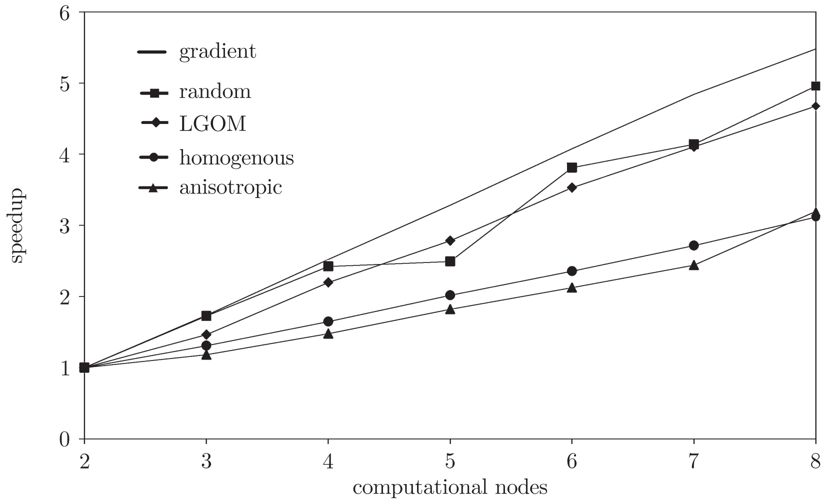

Figure 7. Relationship between the speedup of the parallel ray path algorithm and the number of computational nodes

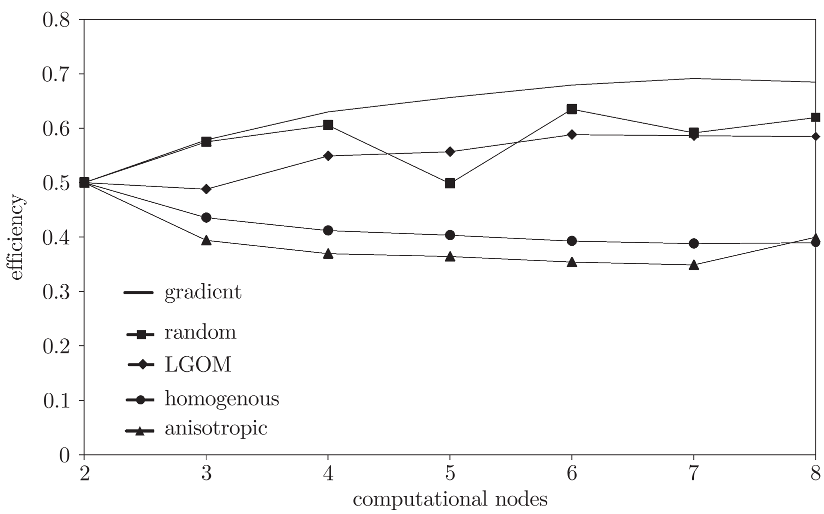

Figure 8. Relationship between the efficiency of the parallel ray path algorithm and the number of computational nodes

ure 8 for 5 and 7 nodes, respectively). A relatively low efficiency, varying from 40 to 70% for all assumed algorithms, was due to the fixed size of the subdomain. Better performance could be obtained by dynamically decomposing the medium, based on the number of points that constitute the source points in the next iteration.

5. Conclusions

Presented algorithm of ray tracing allowed to obtain ray paths and travel times for primary seismic waves in most common geological medium. The method allowed also effective decrease computational time using parallel computing.

It makes that an analysis of more dense grid model is possible in a reasonable computational time.

The obtained results show monotonically decrease of computational time as a function of number of used computational nodes. Effectiveness of parallel computing depends also on velocity distribution in the geological medium. The highest effectiveness is for a gradient velocity model. The worst results were obtained for homogeneous model, both isotropic and anisotropic case.

Acknowledgements

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