INFLUENCE OF EXTERNAL ELECTRIC FIELD ON PARAMETERS OF MECHANICAL WAVES IN SATURATED POROUS MEDIUM

SOFIYA TVARDOVSKA

Centre of Mathematical Modelling of Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of Ukrainian Academy of Science, J. Dudajeva Str. 15, Lviv, 79005, Ukraine
e-mail: tvardovska@cmm.lviv.ua

(Received 9 June 2013; revised manuscript received 18 August 2013)

Abstract: A dispersive equation for plane longitudinal mechanoelectromagnetic waves in a porous medium, saturated by an electrolytic solution and placed in an external electrical field was obtained and investigated. The main attention was paid to the effect of the electric field on the parameters of waves. It was established that the external field could determine the value and sign of the decay coefficient of the waves of the first kind. This allows influencing the mechanoelectromagnetic wave parameters by suitable choosing of the external electric field intensity, which can be useful for seismoelectric investigations of the Earth's crust.

Keywords: porous saturated medium, constant electric field, mechanoelectromagnetic wave, dispersive equation, waves of the first and second kind

1. Introduction

A study of the effect of an external electric field on the propagation of mechanical waves is an important direction in the investigations on mechanoelectromagnetic interactions in saturated porous bodies [1–3]. Electromagnetic equations of the mechanics of a porous medium saturated with an electrolyte solution, developed under various approximations have been presented in [2, 4, 5]. A theoretical study of the influence of an external electric field on the parameters of mechanoelectromagnetic waves in a saturated porous medium has been presented in [4]. The results of investigations on mechanoelectromagnetic waves in porous media are presented in [2, 6, 7]. Electromagnetic equations of mechanics of a porous medium in the presence of an external constant electric field are given in [2, 4]. In addition, the effect of the electric field on the parameters of the waves of the first and second kind is described in [2, 4]. In [3] it is mentioned that an external electric field applied to the Earth's crust may give some insight into seismic phenomena. All the above cited studies are rather fragmentary.

The objective of this work is to study the influence of the characteristics of a porous medium saturated with an electrolyte solution on the propagation of mechanoelectromagnetic waves under the action of the applied electric field. In this paper we focus on longitudinal waves and a longitudinal electric field.

2. Problem formulation

A porous medium saturated with an electrolyte solution referred to in the Cartesian coordinate system $(x,\ y,\ z)$ is considered. The skeletal material is a dielectric, while the interconnected pores (the so called open structure) are filled with an electrolytic solution. The influence of an external electric field on the parameters of the wave motion is related to the force-interaction of the electric field and the charge system of a porous body [8, 5–7].

The medium is statistically homogeneous and isotropic. A uniform electric field of intensity $\vec{E}_0$ is established by external sources within the medium. The corresponding linearized system of equations of electromagnetic mechanics [4], neglecting electroosmosis, can be written as

$$
\begin{aligned}
&\alpha_{10}\rho_0^{(1)}\frac{\partial^2\vec{u}^{(1)}}{\partial t^2}=\frac{\alpha_{10}^2}{\beta}\vec{\nabla}\left(\vec{\nabla}\cdot\vec{u}^{(1)}\right)+\frac{\alpha_{10}\alpha_{20}(1-\nu_f)}{\beta}\vec{\nabla}\left(\vec{\nabla}\cdot\vec{u}^{(2)}\right)+\\
&\qquad A\left(\frac{\partial\vec{u}^{(2)}}{\partial t}-\frac{\partial\vec{u}^{(1)}}{\partial t}\right)+\rho_{12}\left(\frac{\partial^2\vec{u}^{(2)}}{\partial t^2}-\frac{\partial^2\vec{u}^{(1)}}{\partial t^2}\right)+\alpha_{10}f_{E0}^{(1)}\rho_{e0}^{(1)}\vec{E},\\
&\alpha_{20}\rho_0^{(2)}\frac{\partial^2\vec{u}^{(2)}}{\partial t^2}=\alpha_{10}\left(\beta^{-1}\alpha_{20}(1-\nu_f)\widehat{I}\right)\cdot\vec{\nabla}\left(\vec{\nabla}\cdot\vec{u}^{(1)}\right)+\\
&\qquad \alpha_{20}\left(K_f+\frac{4}{3}G_f-\beta^{-1}\alpha_{20}(1-\nu_f)^2\right)\widehat{I}\cdot\vec{\nabla}\left(\vec{\nabla}\cdot\vec{u}^{(2)}\right)+\\
&\qquad \alpha_{20}G_f\Delta\vec{u}^{(2)}-A\left(\frac{\partial\vec{u}^{(2)}}{\partial t}-\frac{\partial\vec{u}^{(1)}}{\partial t}\right)-\rho_{12}\left(\frac{\partial^2\vec{u}^{(2)}}{\partial t^2}-\frac{\partial^2\vec{u}^{(1)}}{\partial t^2}\right)-\\
&\qquad \alpha_{10}f_{E0}^{(1)}\rho_{e0}^{(1)}\left(2\frac{f_{E0}^{(2)}}{f_{E0}^{(1)}-1}\right)\vec{E}\\
&\Delta\vec{E}-\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)=\sigma_{e0}\mu\frac{\partial\vec{E}}{\partial t}+\varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2}+\\
&\qquad \alpha_{10}\mu\gamma_1\rho_{e0}^{(1)}\left(\frac{\partial^2\vec{u}^{(1)}}{\partial t^2}-\frac{\partial^2\vec{u}^{(2)}}{\partial t^2}\right)-\beta^{-1}\alpha_{10}\alpha_{20}\nu_f\mu\gamma_\sigma\sigma_{e0}\vec{E}_0\left(\vec{\nabla}\cdot\frac{\partial\vec{u}^{(1)}}{\partial t}\right)+\\
&\qquad \alpha_{20}\mu\gamma_\sigma\sigma_{e0}\left(K_f-\beta^{-1}\alpha_{20}\nu_f(1-\nu_f)\right)\vec{E}_0\left(\vec{\nabla}\cdot\frac{\partial\vec{u}^{(2)}}{\partial t}\right)\\
&\frac{\partial\vec{B}}{\partial t}=-\vec{\nabla}\times\vec{E}
\end{aligned}
\tag{1}
$$

Here $j=1$ corresponds to the porous fluid, while $j=2$ – to the solid, $\vec{u}^{(j)}$ – the displacement vector of moving phases, $\vec{E}$ – the electric field, $\vec{B}$ – the magnetic induction vector, $\widehat{I}$ – the unit tensor, α_{10} – the initial value of porosity, $\alpha_{20}=1-\alpha_{10}$, $\rho_0^{(j)}$ – initial values of mass densities of phases, ν_f – the consolidation coefficient; $\beta=\alpha_{10}\beta^{(1)}+\alpha_{20}\beta^{(2)}$, $\beta^{(j)}$ – compressibility coefficients of phases,

$A=\alpha_{10}\eta/k_p$, η – the viscosity coefficient of the liquid, k_p – the permeability coefficient of the medium, ρ_{12} – the added mass parameter, K_f and G_f – the effective compressibility and shear moduli, $\rho_{e0}^{(1)}=\sqrt{\frac{2\varepsilon^{(1)}C_0\alpha_{10}}{fRTk_p}}\frac{zF_f\zeta}{T_g}$, C_0 – the electrolyte concentration, ζ – the potential of the surface of the closest ions, T – the absolute temperature, R – the gas constant, F_f – the Faraday constant, f – the pore shape parameter, T_g – the hydrodynamic curvature, z – the valence of ions, $f_{E0}^{(j)}=\frac{\varepsilon-\varepsilon^{(3-j)}}{\alpha_{1j}\left(\varepsilon^{(j)}-\varepsilon^{(3-j)}\right)}$, $\varepsilon=\varepsilon^{(2)}\frac{2\alpha_{20}\varepsilon^{(2)}+\varepsilon^{(1)}}{(2+\alpha_{10})\varepsilon^{(2)}+\alpha_{20}\varepsilon^{(1)}}$ – the dielectric constant of the medium in the natural state, $\varepsilon^{(1)}$ – the dielectric constant of the pore fluid, $\varepsilon^{(2)}$ – the permittivity of the solid; μ – the magnetic permeability of the medium, γ_σ – a parameter characterizing the dependence of the electrical conductivity on the first effective stress invariant σ_f, σ_{e0} – the conductivity coefficient of the medium in the absence of mechanical stress.

We considered a plane mechanoelectromagnetic wave propagating along the axis Ox at $\vec{E}_0=(E_0,0,0)$. In this case, $\vec{f}(x,t)=(f(x,t),0,0)$ for all the functions $\vec{f}=\left(\vec{u}^{(1)},\vec{u}^{(2)},\vec{E}\right)$, where $f(x,t)=fe^{-ikx+i\omega t}$, k – is a wave number, ω – the cyclic frequency. Then, the system of equations (1), ordered in $\vec{u}^{(1)}$, $\vec{u}^{(2)}$, $\vec{E}$ functions, can be written as follows

$$\begin{aligned}&\left(\alpha_{10}\rho_0^{(1)}\omega^2-\rho_{12}\omega^2-Ai\omega-\alpha_{10}^2\beta^{-1}k^2\right)u^{(1)}+\left(\rho_{12}\omega^2+Ai\omega-\right.\\&\qquad\left.\alpha_{10}\alpha_{20}\beta^{-1}(1-\nu_f)k^2\right)u^{(2)}+\alpha_{10}f_{E0}^{(1)}\rho_{e0}^{(1)}E=0\end{aligned}\tag{2}$$

$$\begin{aligned}&\left(\rho_{12}\omega^2+Ai\omega-\alpha_{10}\alpha_{20}(1-\nu_f)\beta^{-1}k^2\right)u^{(1)}+\\&\left(\rho_0^{(2)}\omega^2-\rho_{12}\omega^2-Ai\omega-\alpha_{20}\left(K_f+\frac{4}{3}G_f-\alpha_{20}(1-\nu_f)^2\beta^{-1}\right)k^2\right)u^{(2)}-\\&\qquad\alpha_{10}f_{E0}^{(1)}\rho_{e0}^{(1)}\left(2\frac{f_{E0}^{(2)}}{f_{E0}^{(1)}}-1\right)E=0\end{aligned}\tag{3}$$

$$\begin{aligned}&i\alpha_{10}\left(\omega\gamma_1\rho_{e0}^{(1)}+\alpha_{20}\nu_f\beta^{-1}\gamma_\sigma\sigma_{e0}E_0k\right)u^{(1)}-i\left(\alpha_{10}\omega\gamma_1\rho_{e0}^{(1)}+\right.\\&\left.\alpha_{20}\left(K_f-\alpha_{20}\nu_f(1-\nu_f)\beta^{-1}\right)\gamma_\sigma\sigma_{e0}E_0k\right)u^{(2)}+(\sigma_{e0}+i\varepsilon\omega)E=0\end{aligned}\tag{4}$$

An analysis of the wave properties is conveniently carried out using the dimensionless parameters

$$\begin{aligned}&r_{j0}=\frac{\alpha_{j0}\rho_0^{(j)}}{\rho_0}\quad(j=1,2),\quad\Lambda_\beta=\left(\rho_0v_0^2\beta\right)^{-1},\quad\Lambda_K=\frac{K_f}{\rho_0v_0^2},\\&\Lambda_{KG}=\frac{3K_f+4G_f}{3\rho_0v_0^2},\quad G_e=2\frac{f_{E0}^{(2)}}{f_{E0}^{(1)}}-1,\quad\omega_V=\frac{A}{\rho_0},\quad\varepsilon_m=\frac{\rho_{12}}{\rho_0},\\&\omega_r=\frac{\sigma_{e0}}{\varepsilon},\quad g_e=\frac{\gamma_1f_{E0}^{(1)}\left(\alpha_{10}\rho_{e0}^{(1)}\right)^2}{\sigma_{e0}A},\quad\omega_{E0}=\alpha_{10}f_{E0}^{(1)}v_0\gamma_\sigma\rho_{e0}^{(1)}E_0,\\&\varepsilon_{Ev}=\frac{\omega_{E0}}{\omega_V},\quad\varepsilon_\omega=\frac{\omega}{\omega_V},\quad\xi=\frac{v_0}{\omega}k\end{aligned}\tag{5}$$

Let us now consider the conditions for the existence of a nontrivial solution of the homogeneous system of equations (2)–(4). If the electric field intensity is extracted from (4) and substituted into (2)–(3), then the dispersive equation assumes the following form

$$\xi^4 + a\xi^3 + b\xi^2 + c\xi + d = 0 \tag{6}$$

where

$$\begin{aligned}
a &= \frac{\varepsilon_{ev}\left(a_{11}^{(1)}a_{22}^{(2)} + a_{11}^{(2)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(2)} - a_{12}^{(2)}a_{21}^{(1)}\right)}{\varepsilon_\omega\left(a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}\right)},\\
b &= \frac{\varepsilon_{ev}^2\left(a_{11}^{(2)}a_{22}^{(2)} - a_{12}^{(2)}a_{21}^{(12)}\right)}{\varepsilon_\omega^2\left(a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}\right)} + \frac{a_{11}^{(3)}a_{11}^{(1)} - a_{12}^{(1)}a_{21}^{(3)} + a_{11}^{(3)}a_{22}^{(1)} + a_{11}^{(1)}a_{22}^{(3)}}{\varepsilon_\omega\left(a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}\right)},\\
c &= \frac{\varepsilon_{ev}\left(a_{11}^{(3)}\left(a_{21}^{(2)} + a_{22}^{(2)}\right) - a_{12}^{(2)}a_{21}^{(3)} + a_{11}^{(2)}a_{22}^{(3)}\right)}{\varepsilon_\omega^2\left(a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}\right)} +\\
&\quad + \frac{\varepsilon_{ev}\left(a_{11}^{(0)}a_{22}^{(2)} + a_{11}^{(2)}a_{22}^{(0)} - a_{12}^{(2)}a_{21}^{(0)} - a_{12}^{(0)}a_{21}^{(2)}\right)}{\varepsilon_\omega\left(a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}\right)},\\
d &= \frac{a_{11}^{(0)}a_{22}^{(0)} - a_{12}^{(0)}a_{21}^{(0)}}{a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}} + \frac{a_{11}^{(3)}\left(a_{21}^{(0)} + a_{22}^{(0)}\right) - a_{12}^{(0)}a_{21}^{(3)} + a_{11}^{(0)}a_{22}^{(3)}}{\varepsilon_\omega\left(a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}\right)} +\\
&\quad + \frac{a_{11}^{(3)}\left(a_{21}^{(3)} + a_{22}^{(3)}\right) - a_{12}^{(0)}a_{21}^{(3)} + a_{11}^{(0)}a_{22}^{(3)}}{\varepsilon_\omega^2\left(a_{11}^{(1)}a_{22}^{(1)} - a_{12}^{(1)}a_{21}^{(1)}\right)},
\end{aligned} \tag{7}$$

$$\begin{aligned}
&a_{11}^{(0)} = r_{10} + \varepsilon_m, \quad a_{11}^{(1)} = -\alpha_{10}^2\Lambda_\beta, \quad a_{11}^{(2)} = -i\alpha_{10}\alpha_{20}\nu_f\frac{\Lambda_\beta}{1+i\omega/\omega_r},\\
&a_{11}^{(3)} = -i\left(1 + \frac{g_e}{1+i\omega/\omega_r}\right), \quad a_{12}^{(0)} = -\varepsilon_m, \quad a_{12}^{(1)} = -\alpha_{10}\alpha_{20}(1-\nu_f)\Lambda_\beta,\\
&a_{12}^{(2)} = i\alpha_{20}\frac{\Lambda_k - \alpha_{20}\Lambda_\beta\nu_f(1-\nu_f)}{1+i\omega/\omega_r}, \quad a_{12}^{(3)} = -a_{11}^{(3)},\\
&a_{21}^{(0)} = a_{12}^{(0)}, \quad a_{21}^{(1)} = a_{12}^{(1)}, \quad a_{21}^{(2)} = a_{12}^{(2)}Ge, \quad a_{21}^{(3)} = i\left(1 + \frac{g_e G_e}{1+i\omega/\omega_r}\right),\\
&a_{22}^{(0)} = r_{20} + \varepsilon_m, \quad a_{22}^{(1)} = -\alpha_{20}\left(\Lambda_{KG} + \alpha_{20}(1-\nu_f)^2\Lambda_\beta\right),\\
&a_{22}^{(2)} = -a_{12}^{(2)}Ge, \quad a_{22}^{(3)} = -a_{21}^{(3)}
\end{aligned}$$

In the following sections the mechanoelectromagnetic waves will be investigated using numerical methods.

We will consider $E = E_0 e^{i\omega t}$ and assume the following characteristics of the system

$$\begin{aligned}
&\alpha_{10} = 0.3, \quad \rho_0^{(1)} = 10^3\,\mathrm{kg/m^3}, \quad \rho_0^{(2)} = 2.6\cdot 10^3\,\mathrm{kg/m^3},\\
&\beta^{(1)} = 4.4\cdot 10^{-10}\,\mathrm{m^2/N}, \quad \beta^{(2)} = 10^{-11}\,\mathrm{m^2/N}, \quad K_f = 6\cdot 10^{10}\,\mathrm{m^2/N},\\
&G_f = 10^{10}\,\mathrm{N/m^2}, \quad v_0 = 5.67\cdot 10^3\,\mathrm{m/s}, \quad \eta = 10^{-3}\,\mathrm{Pa\cdot s}, \quad \nu_f = 0.6,\\
&\sigma_{e0} = 10^{-2}\,\mathrm{S/m}, \quad \varepsilon^{(1)} = 7.2\cdot 10^{-10}\,\mathrm{F/m}, \quad \varepsilon^{(2)} = 5.3\cdot 10^{-11}\,\mathrm{F/m},\\
&\gamma_1 = 0.9, \quad \gamma_\sigma = 3\cdot 10^{-8}\,\mathrm{Pa^{-1}}, \quad \rho_{12} = 100\,\mathrm{kg/m^3},\\
&z = 1, \quad C_0 = 100\,\mathrm{mol/m^3}, \quad T_g = 5, \quad f = 2.5, \quad \zeta = 0.1\,\mathrm{V}
\end{aligned} \tag{8}$$

3. Quantitative analysis results

The wave numbers with indices 1 and 2 correspond to the waves of the first kind, while those with indices 3 and 4 – to the waves of the second kind. The calculations have shown that the influence of the electric field on the parameters of the waves of the second kind are relatively insignificant. Therefore, in what follows the main attention will be paid to the waves of the first kind.

The numerical results on the decay coefficient and the phase velocity of the waves of the first kind vs. intensity of the external electric field for various values of the porosity and permeability coefficients of the medium are shown in Figures 1–4. Lines 1–3 in Figures 1, 3 correspond to the values of medium permeability of $k_p = 10^{-13}$, 10^{-12}, $10^{-11}\,\mathrm{m^2}$, while the lines in Figures 2, 4 correspond to the values of porosity $\alpha_{10} = 0.25$, 0.3, 0.35.

As can be seen from Figure 1 the phase velocity depends both on the coefficient of permeability of the porous medium and the intensity of the external electric field. Its minimum value is reached at $E_0 = 0\,\mathrm{V/m}$.

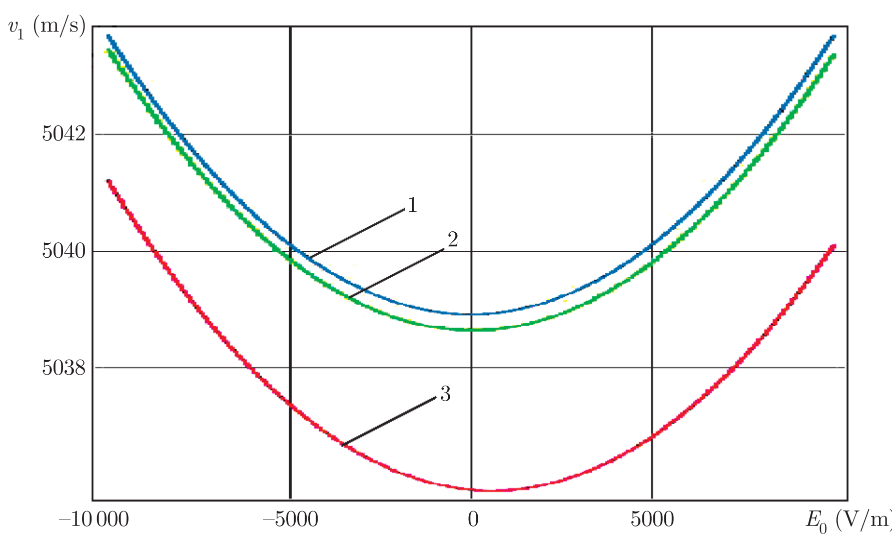

Figure 1. Dependence of phase velocity of the wave of the first kind on external electric fields for $k_p = 10^{-13}$, 10^{-12}, $10^{-11}\,\mathrm{m^2}$ (lines 1, 2, 3, respectively)

It is established that the external electric field, in the considered range (from 10^{-4} V/m to 10^{4} V/m), only insignificantly modifies the phase velocity ($v_1 = 5036$ m/s at $E_0 = 0$ V/m , while $v_1 = 5040.5$ m/s at $E_0 = 10^4$ V/m). Hence, for $k_p = 10^{-11}\,\mathrm{m}^2$ the difference is merely 0.02%.

The above described dependence remains valid in the range of $k_p \in \left[10^{-16}, 10^{-11}\right]\mathrm{m}^2$. The dependence of the decay coefficient of the first-kind wave ($\Im k_1$) on the amplitude of the electric field (E_0) is linear and such that it defines a sign of this coefficient.

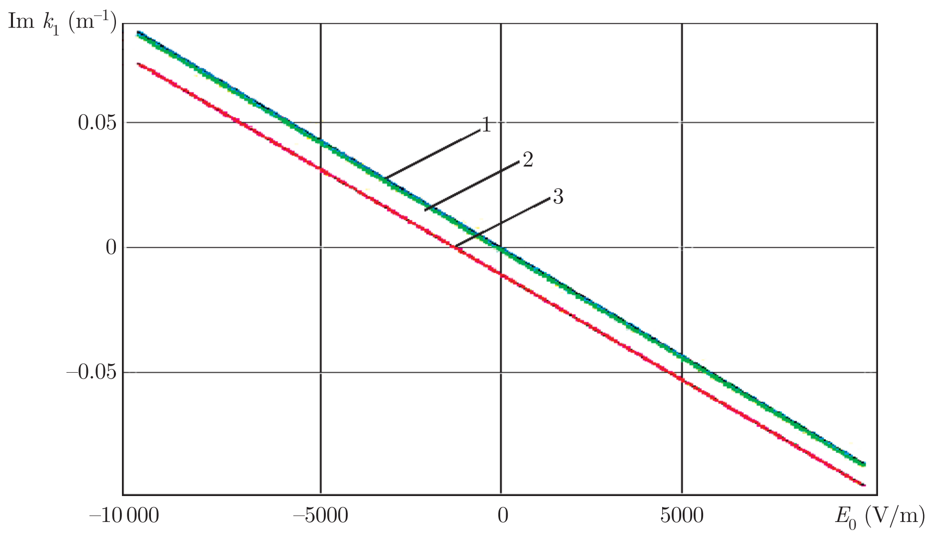

Figure 2. Dependence of decay coefficients of the wave of the first kind on external electric fields for $k_p = 10^{-13}$, 10^{-12}, $10^{-11}\,\mathrm{m}^2$ (lines 1, 2, 3, respectively)

An analysis of the results presented in Figure 3 suggests that the phase velocity of mechanoelectromagnetic wave is strongly dependent on the medium porosity (α_{10}). However, this dependence is more significant for lower porosity coefficients. The influence of the external electric field on the phase velocity of the wave of the first kind decreases with increasing α_{10}.

The influence of the electric field on the decay coefficients of the wave of the first kind for various values of permeability of the porous medium (Figure 4) is determined by the magnitude and sign of the coefficient for frequency $\omega \approx 10^4\,\mathrm{c}^{-1}$.

The changing of the direction of the external electric field changes the sign of the induced field, and therefore the influence of the latter on the fluid motion and the corresponding changes in energy.

The out of phase motion of the liquid and solid phases is the decay mechanism. An external electric field can result in a reduction of the decay coefficient. Growth of the electric field leads to a conversion of the electromagnetic wave energy to the elastic energy. Thus, the motion is intensified and consequently the damping coefficient becomes positive. This is the reason for the change of the

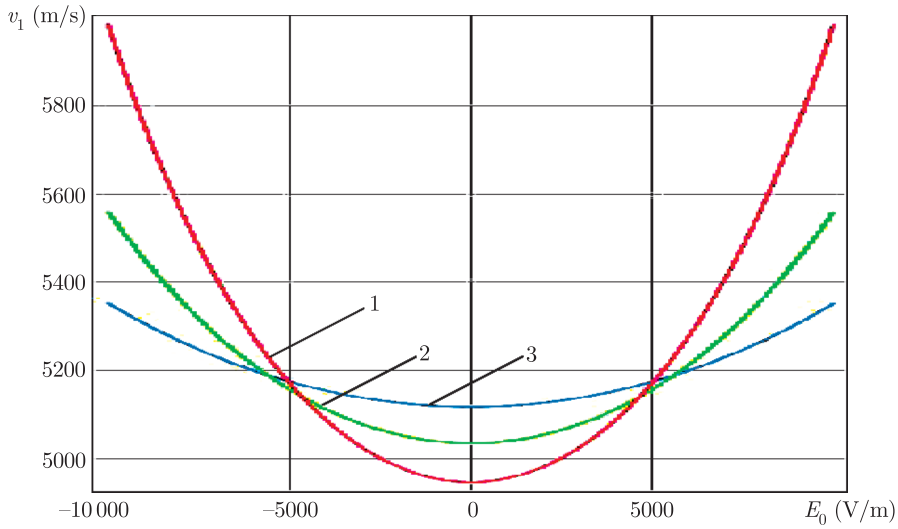

Figure 3. Dependence of phase velocity of the wave of the first kind on external electric fields for $\alpha_{10} = 0.25$, 0.3, 0.35 (lines 1, 2, 3, respectively)

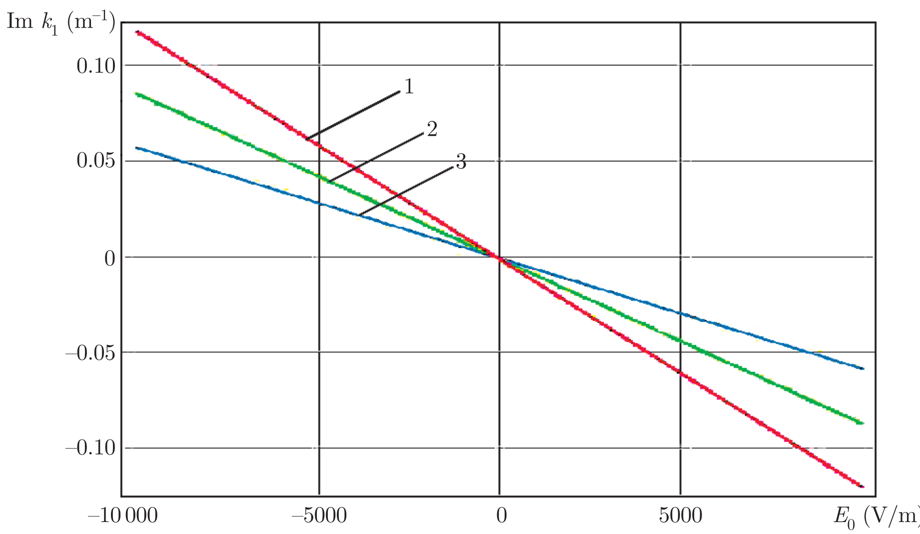

Figure 4. Dependence of decay coefficients of the wave of the first kind on external electric fields for $\alpha_{10} = 0.25$, 0.3, 0.35 (lines 1, 2, 3, respectively)

decay coefficient sign for the wave of the first kind when the direction of the external electric field changes (Figures 2, 4).

The above numerical results were obtained using an iterative method. The accuracy of the input data and the numerical method is higher than the accuracy of the obtained results. Therefore, the results can be considered reliable.

These results are consistent with the results obtained in the field of seismic frequencies modelling [9].

tq217l-c/221 15 V 2014 BOP s.c., http://www.bop.com.pl

4. Conclusion

The dispersive equation for plane longitudinal mechanoelectromagnetic waves in a porous medium, saturated with and electrolytic solution, and placed in an external electrical field was obtained and analyzed. It was established that the external field determined the value and sign of the decay coefficient of the first-kind waves and the character of the phase velocity dispersion. The dependence of the decay coefficient of the waves of the first kind on the external electric field is linear for various values of the medium porosity and permeability. The decay coefficient of the wave of the first-kind decreases with increasing amplitude E_0. The dependence of the phase velocity on the external electric field becomes more pronounced with a decreasing coefficient of porosity, while no similar effect is related to the permeability coefficient.

An analysis of the dispersion equation shows that two longitudinal waves of the first and second kind propagate in a porous medium A strong dependence of the decay coefficients and phase velocities of the waves of the first kind on the electric charge density and structural characteristics of the porous medium were observed. For high frequencies, the decay coefficient of the first-kind wave is greater than seismic frequencies by two orders of magnitude. These results can be used in structural studies of the Earth's crust.

References

[1] Svetov B S 2008 *Fundamentals of Geoelectric*, LKI Publisher, Moskow (in Russian)
[2] Kobranova V N 1986 *Petropnisics*, Nedra, Moskow (in Russian)
[3] Podberejnii M Yu 2009 *Features of interaction of the multiphase microstructured environments with acoustic and electric fields*, Novosibirsk (in Russian)
[4] Potapov O, Lizun S, Kondrat V *et al.* 1995 *Fundamentals of Seismoelectrical Exploration*, Nedra, Moskow (in Russian)
[5] Pride S 1994 *Phys. Rev. B* **50** (21) 15678
[6] Garambois S and Dietrich M 2001 *Geophysics* **66** (5) 1417
[7] Lyashchuk D, Boyko A, Boyko V and Freik N 1983 *Method of Geophysical Exploration* (in Russian)
[8] Burak Y, Chaplia Y, Nahirnyj T *et al.* 2004 *Physical-Mathematical Modelling Complex Systems*, SPOLOM, Lviv (in Ukrainian)
[9] Kondrat V F and Tvardovska S R 2009 *Physicomathematical modeling and informational technologies* **10** 56 (in Ukrainian)