NON-STATIONARY THERMAL SELF-ACTION OF ACOUSTIC BEAMS CONTAINING SHOCK FRONTS IN THERMOCONDUCTING FLUID

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Abstract: Non-stationary thermal self-action of a periodic or impulse acoustic beam containing shock fronts in a thermoconducting Newtonian fluid is studied. Self-focusing of a saw-tooth periodic and impulse sound is considered, as well as that of a solitary shock wave which propagates with the linear sound speed. The governing equations of the beam radius are derived. Numerical simulations reveal that the thermal conductivity weakens the thermal self-action of the acoustic beam.

Keywords: non-stationary thermal self-focusing of an acoustic beam, shock acoustic wave, thermal lens

1. Introduction

Self-action effects in the wave theory started to attract considerable interest after discovery of the self-focusing of optic beams. The self-action of optic waves manifests itself by means of the dependence of the local wave speed on the wave's intensity [1–4]. Theoretical works on self-focusing of optic waves had considerable impact on studies in the field of nonlinear acoustics. The fact that acoustic beams may manifest thermal self-action similarly to laser beams has first been pointed out in [5]. The typical attenuation specific for Newtonian fluids always causes the background temperature to rise. That influences the local sound speed in different domains on the beam's path, and, as a consequence, yields refraction of the sound rays in a thermally inhomogeneous medium. This alters the width of the sound beam, its focal area and distribution of the peak acoustic pressure. In gases, where sound velocity increases with an increase in temperature T, the acoustic beam undergoes defocusing, while in liquids (except for water) with negative thermal coefficient $\delta = (\partial c/\partial T)_p/c < 0$, it undergoes focusing (c denotes the infinitely small-signal sound speed in a fluid). The first theoretical results concerning self-action of sound beams are reviewed in [6], and the first experiments confirming the theory are discussed in [7, 8].

Optic waves are strongly dispersive, therefore it is possible to consider propagation of harmonic compounds of a waveform individually. On the contrary, acoustic media reveal typically weak dispersion. The spectrum of intense sound waves is quite rapidly spread due to nonlinear generation of higher harmonics, and their profiles become distorted [9, 10]. The nonlinear self-action is especially significant in the case of ultrasound waves in weakly attenuating media, when diffraction is small. Nonlinear distortions of sound may be weak but they accumulate with enlargement of the distance from a transducer. Eventually, these distortions lead to formation of profiles with a universal shape. For example, a single impulse transforms into the N-wave form due to the joint action of the quadratic nonlinearity and absorption. The stationary solitary shock wave which propagates with the linear sound speed or with the different speed, may propagate in a Newtonian fluid. The waveforms with shock fronts are of great importance not only in the theory of nonlinear acoustics, but also in technical and medical applications of ultrasound. The comprehensive review by Rudenko and Sapozhnikov [11] focuses on the thermal self-action of periodic beams containing shock fronts in media with quadratic and cubic nonlinearities. The only parameters of these steady acoustic waveforms are the peak values of acoustic pressure and, in the case of a pulse, its duration. These parameters vary along with the path of propagation. As it has been discovered, the governing equations for peak pressure in the paraxial area of a slightly diffracting beam may be considerably simplified in the case of periodic waveforms including short shock fronts. They do not include the Newtonian total attenuation any longer, but pure nonlinear attenuation [11]. The theory uses the approach of geometric acoustics, which implies weak diffraction.

The statement of the problem of thermal self-action consists in fact of two parts, one to describe the dynamics of acoustic pressure, and the other to evaluate slow background temperature variations in the course of sound propagation and their influence on the sound beam itself. The initial equations were derived by O. V. Rudenko and co-authors [12–14]; the thermal self-action of strictly periodic waves with discontinuities, stationary and non-stationary (without an account for the thermal conduction of a Newtonian fluid), were also studied by this group of authors. The aim of this study is the non-stationary thermal self-action of a periodic sawtooth wave, a single sawtooth impulse or the integer number of sawtooth impulses in a thermoconducting medium. A solitary shock wave is also considered. The mathematical content in this study is close to the developments of Rudenko et al. in the investigations of the self-action of periodic sound beams. There is a significant distinction which is connected with aperiodicity of impulses under study: the instantaneous acoustic force of heating, not averaged over the sound period, should be used. It was derived by the author in [15]. The account for thermal conduction requires expansion of excess temperature in series in the even powers of the transversal coordinate and a solution of system of equations resulting from equating coefficients by their different powers.

2. The governing equations and starting points

The system of equations describing thermal self-action in an axially symmetric flow of a Newtonian fluid, takes the form [12, 13, 11]

$$\frac{\partial}{\partial \tau}\left(\frac{\partial p}{\partial x}-\frac{\delta T}{c_0}\frac{\partial p}{\partial \tau}-\frac{\varepsilon}{c_0^3\rho_0}p\frac{\partial p}{\partial \tau}-\frac{b}{2c_0^3\rho_0}\frac{\partial^2 p}{\partial \tau^2}\right)=\frac{c_0}{2}\Delta_\perp p \tag{1}$$

$$\frac{\partial T}{\partial t}-\frac{\chi}{\rho_0 C_P}\Delta_\perp T=F \tag{2}$$

where x and r are cylindrical coordinates, the Ox axis coincides with the axis of the beam, p is the acoustic pressure, ρ_0 is the unperturbed density of a fluid, $c_0=\sqrt{\frac{C_P}{C_V\beta\rho_0}}$ denotes the linear speed of sound at an unperturbed thermodynamic state ($\beta=\rho_0^{-1}\left(\frac{\partial \rho}{\partial p}\right)_T$ is the isothermal compressibility, C_P and C_V are specific heats under constant pressure and volume, respectively), $\tau=t-x/c_0$ is the retarded time in the reference frame which moves with the sound speed c_0, $\Delta_\perp$ is the Laplacian with respect to the radial coordinate r, ε is the parameter of nonlinearity of a medium, and χ denotes its thermal conductivity. The total attenuation b is a sum of terms responsible for the shear (μ), bulk (η) viscosity and thermal conductivity,

$$b=\frac{4}{3}\mu+\eta+\left(\frac{1}{C_V}-\frac{1}{C_P}\right)\chi \tag{3}$$

Equation (1) describes the acoustic pressure in a beam which propagates in the positive direction of the axis Ox. In contrast to the famous Khokhlov-Zabolotskaya-Kuznetsov equation (KZK) [6], it accounts for variations in the wave speed due to a slow increase in the fluid temperature (the second term) [14]. Equations (1), (2) account for acoustic heating, that is, irreversible enhancement of the chaotic motion of a fluid's molecules due to loss in acoustic energy. As usual, "the acoustic force" of heating F is a quantity averaged over the integer number of sound periods. It is well established for periodic sound [10]:

$$\langle F\rangle=\frac{b}{c_0^4\rho_0^3 C_P}\left\langle\left(\frac{\partial p}{\partial \tau}\right)^2\right\rangle \tag{4}$$

In the theory of nonlinear self-action of periodic sound beams,$\langle F\rangle$ replaces F in the right-hand side of Equation (2). The angular brackets denote averaging over the integer number of sound periods. The form of Equations (2), (4) imposes that (a) acoustic heating is a slow process as compared to the fast variations of sound perturbations, and (b) sound is periodic at any distance from a transducer. The former condition is always valid, however, the latter is not valid any longer in the case of aperiodic sound, impulses or wavepackets. Strictly speaking, it is not valid for transmission of periodic sound at a transducer, which starts at some time and has a finite duration, that is, which is periodic inside some temporal domain. The instantaneous acoustic force takes the form which has been derived by the author in [15]:

$$F=\frac{1}{c_0^6\rho_0^3}\left(\frac{\left(\frac{1}{C_V}-\frac{1}{C_P}\right)\chi}{\alpha}p\frac{\partial^2 p}{\partial\tau^2}+\left(\frac{c_0^2}{C_P}\left(\frac{4\mu}{3}+\eta\right)-D\frac{\left(\frac{1}{C_V}-\frac{1}{C_P}\right)\chi}{\alpha}\right)\left(\frac{\partial p}{\partial\tau}\right)^2+\right.$$
$$\left.\left(-\frac{\varepsilon\left(\frac{1}{C_V}-\frac{1}{C_P}\right)\chi}{2\alpha}+\frac{(\varepsilon-1)\chi}{C_V\alpha}+\frac{1}{2}\frac{\chi C_P}{\beta^2C_V^2}\frac{\partial^2 T}{\partial p^2}+\frac{1}{2}\frac{\chi}{C_P}\frac{\partial^2 T}{\partial\rho^2}+\frac{\chi}{\beta C_V}\frac{\partial^2 T}{\partial p\partial\rho}\right)\frac{\partial p^2}{\partial\tau^2}\right) \tag{5}$$

where $\alpha=-\rho_0^{-1}\left(\frac{\partial\rho}{\partial T}\right)_p$ is the thermal expansion, and D is expressed in terms of partial derivatives of the internal energy, e:

$$D=\frac{\alpha c_0^2}{C_P}\left(-1+c_0^2\rho_0^2\frac{\partial^2 e}{\partial p^2}+\rho_0^2\frac{\partial^2 e}{\partial p\partial\rho}\right) \tag{6}$$

The temperature and internal energy are considered as functions of pressure and density. Actually, Equation (5) is rearranged Equation (18) from [15] accounting for the fact that the isobaric perturbations of temperature and density are correlated by equality $T'=-\rho'/(\alpha\rho_0)$.

In this study, we focus on the aperiodic nonlinear pulses, including one (or some) periods of the sawtooth wave that is of the most interest. These waveforms are defined by the peak acoustic pressure which depends on the distance from the transducer and plays a similar role as the amplitude of single harmonics in optics. Another important waveform is the solitary shock wave, infinite in time, which may propagate with the sound speed or with a velocity different from the linear sound speed. When the acoustic nonlinearity is important, and the beam is slightly divergent, the approximation of the geometrical acoustics is successful. For the validity of approximation of geometrical acoustics, diffraction should be insignificant over the characteristic length of self-focusing. Acoustic pressure may be found in the form which follows from the theory of geometrical acoustics [11],

$$p=p(x,r,\theta) \tag{7}$$

where

$$\theta=\tau-\psi(x,r)/c_0 \tag{8}$$

and ψ denotes eikonal. Substituting it into Equation (1), we arrive at the following system in the limit of short wavelengths, small compared with the scale of thermal inhomogeneities:

$$\frac{\partial p}{\partial x}-\frac{\varepsilon}{c_0^3\rho_0}p\frac{\partial p}{\partial\theta}-\frac{b}{c_0^3\rho_0}\frac{\partial^2 p}{\partial\theta^2}+\frac{\partial\psi}{\partial r}\frac{\partial p}{\partial r}+\frac{\Delta_\perp\psi}{2}p=0 \tag{9}$$

$$\frac{\partial\psi}{\partial x}+\frac{1}{2}\left(\frac{\partial\psi}{\partial r}\right)^2+\delta T=0 \tag{10}$$

This set of equations, along with Equation (2), is the famous starting point in studies of thermal self-focusing of acoustic beams in Newtonian fluids [11–13].

3. Thermal self-action of sawtooth waves

One period of the periodic wave profile may be thought as a sum of a stationary jump and straight sawtooth portions [10], and it is described by the formula

$$p(x,r,\theta)=A(x,r)\left(-\frac{\omega\theta}{\pi}+\tanh\left(\frac{\varepsilon\theta}{b}A(x,r)\right)\right) \tag{11}$$

where ω is the sound frequency. This is an acoustic pressure within one period, that is, at times $-\pi\le\omega\theta\le\pi$. In fact, Equation (11) is the exact solution of the Burgers equation for planar nonlinear waves with A being a function of coordinate x [10]:

$$A(x)=\frac{P_0}{1+x/x_s} \tag{12}$$

where

$$x_s=\frac{\pi c_0^3\rho_0}{\varepsilon\omega P_0} \tag{13}$$

denotes the distance at which a break of the initially sinusoidal planar wave occurs, P_0 is the initial peak acoustic pressure at the axis of a beam at $x=0$. Equation (11) has a sawtooth profile in the limit when $b\omega/(c_0^2\rho_0)$ tends to zero. Substitution of Equation (11) into Equation (9) and allowing $b\omega/(c_0^2\rho_0)\to 0$ results in the transport equation for $A(x,r)$:

$$\frac{\partial A}{\partial x}+\frac{A^2}{x_sP_0}+\frac{\partial\psi}{\partial r}\frac{\partial A}{\partial r}+\frac{\Delta_\perp\psi}{2}A=0 \tag{14}$$

An acoustic force for the symmetric periodic shock wave in the limit $b\omega/(c_0^2\rho_0)\to 0$ has been obtained by Rudenko and co-authors [13]:

$$\langle F\rangle=\frac{2\varepsilon\omega}{3\pi\rho_0^3c_0^4C_P}A^3 \tag{15}$$

The value averaged over the sound period, $\langle F\rangle$ was used in evaluations of the acoustic peak pressure and the beam width in [13].

3.1. Periodic sawtooth wave

The equations which follow are readily simplified by assuming the parabolic wave front in the eikonal described by Equation (10)

$$\psi(x,r,t)=\psi_0(x,t)+\frac{r^2}{2}\frac{\partial}{\partial x}\ln f(x,t) \tag{16}$$

and with allowance for power series of T over the transversal coordinate,

$$T=T_0-\frac{r^2}{2}T_2(x,t)-\frac{r^4}{4}T_4(x,t) \tag{17}$$

The solution of Equation (14) accounting for Equation (16) has the form [14]

$$A(x,r,t)=\frac{P_0}{f}\Phi\left(\frac{r}{a_0f}\right)\left[1+\frac{1}{x_s}\Phi\left(\frac{r}{a_0f}\right)\int_0^x\frac{dx'}{f(x')}\right]^{-1} \tag{18}$$

The function Φ describes the initial transverse distribution, $A(x=0,r,t=0)=P_0\Phi\left(\frac{r}{a_0}\right)$, where a_0 denotes the initial beam radius at $x=0$. We will consider initially Gaussian beams with $\Phi(\xi)=\exp(-\xi^2)$.

Equation (16) reflects the sphericity of the wave front, only its curvature may vary during the beam propagation. The unknown function of two variables f is responsible for these variations, and $\psi_0(x,t)$ is a phase shift of the wavefront at the beam axis . In accordance with Equations (10), (16), (17), the evolution of eikonal ψ is described by the equation

$$\frac{1}{f}\left(\frac{\partial^2 f}{\partial x^2}\right)=\delta T_2 \tag{19}$$

The diffusion Equation (2) accounting for the acoustic source, Equation (4), and Equation (17), rearranges as a system of three equations (they are actually multipliers by different powers of r: r^2 and r^4):

$$\begin{aligned}\frac{\partial T_2}{\partial t}-\frac{8\chi}{C_P\rho_0}T_4&=\frac{4\varepsilon\omega P_0^3}{\pi a_0^2c_0^4C_P\rho_0^3f^5\left[1+\frac{1}{x_s}\int_0^x\frac{dx'}{f(x')}\right]^4}\\ \frac{\partial T_4}{\partial t}&=\frac{4\varepsilon\omega P_0^3\left[-3+\frac{1}{x_s}\int_0^x\frac{dx'}{f(x')}\right]}{\pi a_0^4c_0^4C_P\rho_0^3f^7\left[1+\frac{1}{x_s}\int_0^x\frac{dx'}{f(x')}\right]^5}\end{aligned} \tag{20}$$

The system of Equations (19), (20) determines the unknown function f. In order to eliminate T_2, T_4 from the system, one should integrate the second equation from the Equation (20). The equation which describes the behavior of function f, takes the form

$$\frac{\partial}{\partial\theta}\left(\frac{1}{f}\frac{\partial^2 f}{\partial z^2}\right)=\pm\left(\frac{1}{\left[1+\int_0^z\frac{dz'}{f(z')}\right]^4f^5}+\frac{8\chi t_0 f^5}{a_0^2C_P\rho_0}\int_0^\theta d\theta'\frac{\left[-3+\int_0^z\frac{dz'}{f(z',\theta')}\right]}{f^7\left[1+\int_0^z\frac{dz'}{f(z',\theta')}\right]^5}\right) \tag{21}$$

where $\theta=t/t_0$ with the characteristic time t_0 and dimensionless coordinate z:

$$t_0=\frac{a_0^2C_P\varepsilon\omega}{4|\delta|M\pi c_0^4},\qquad z=\frac{x}{x_s} \tag{22}$$

The sign plus in the right-hand side of Equation (21) corresponds to positive δ (defocusing), and the sign minus corresponds to the case of self-focusing medium with negative δ. Rudenko et al. have considered also the non-stationary self-action, for which thermal conduction is unimportant. It occurs at times much smaller than t_0. The equation derived by Rudenko and co-authors takes the form (we reproduce Equation (22) from [11]):

$$\left[1+\int_0^z\frac{dz'}{f(z')}\right]^4 f^5\frac{\partial}{\partial\theta}\left(\frac{1}{f}\frac{\partial^2 f}{\partial z^2}\right)=\pm 1 \tag{23}$$

The initial and boundary conditions for both Equations (21), (23), are as follows

$$f(z=0)=f(\theta=0)=1,\qquad \frac{\partial f}{\partial z}=\frac{x_s}{R} \tag{24}$$

if the initial curvature of the beam front is $1/R$. Equation (23) rearranges into Equation (21), if thermal conduction tends to zero. The validity of Equation (23) may be readily evaluated in the approximation of the thin lens, $f \approx 1$. The second term in the brackets is much smaller than the first one; at $z=0$ that corresponds to

$$\frac{8\chi t_0}{a_0^2 C_P \rho_0}\theta \ll \frac{1}{3} \tag{25}$$

The inequality is certainly valid for larger z. In the non-stationary numerical simulations in accordance with Equation (21), the beam is planar initially: $x_s/R=0$. The thin lines in Figure 1 refer to the case without thermal conduction which is described by the dynamic Equation (23). The bold lines in Figure 1 represent numerical simulations of Equation (21) for $\frac{8\chi t_0}{a_0^2 C_P \rho_0}=0.3$. The pictures are plotted for focusing ($\delta<0$) and defocusing media ($\delta>0$). The plots represent the dimensionless magnitude of an acoustic pressure at the axis of a beam

$$\frac{A(r=0,z)}{P_0} = \frac{1}{f}\left(1+\int_0^z \frac{dz'}{f(z')}\right)^{-1} \tag{26}$$

and its characteristic width (referring to the level where the magnitude decreases e times),

$$\frac{a(z)}{a_0} = f\sqrt{\ln\left(e+(e-1)\int_0^z \frac{dz'}{f(z')}\right)} \tag{27}$$

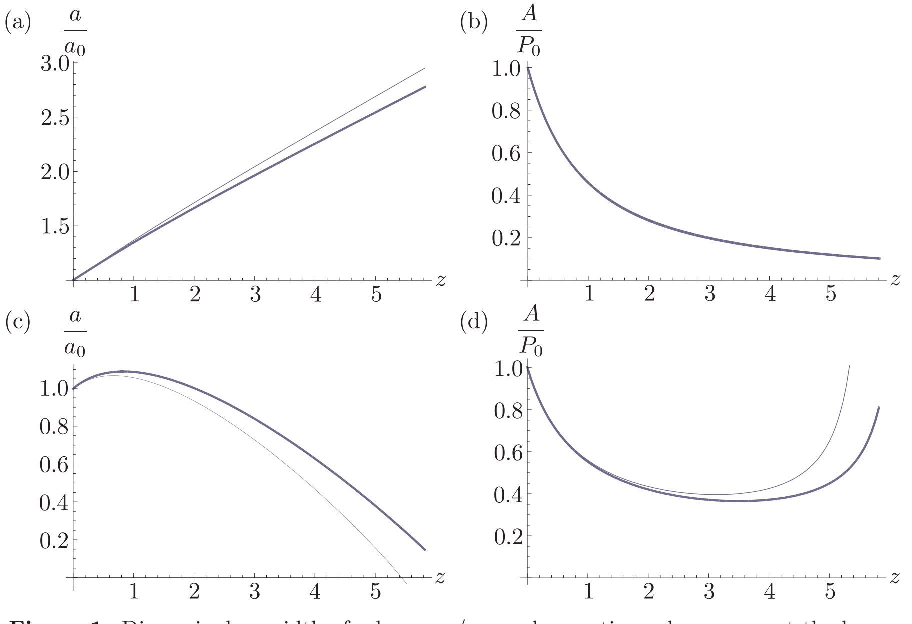

Figure 1. Dimensionless width of a beam, a/a_0, and acoustic peak pressure at the beam axis, $A(r=0,z)/P_0$; curves (a,b) correspond to self-defocusing, and curves (c,d) to the self-focusing of an initially planar wave; bold lines relate to numerical solutions of Equation (21) for a periodic sawtooth waveform and parameter $\frac{8\chi t_0}{a_0^2 C_P \rho_0}=0.3$, and thin lines relate to numerical solutions of Equation (23) for periodic everywhere sound; all series are plotted at dimensionless time $\theta=t/t_0=0.5$; the curves at Figure 1(b) almost overlap

3.2. Single sawtooth impulse or integer number of impulses

One "period" of acoustic pressure is also a solution of the one-dimensional Burgers equation [10] (in the case of a single impulse, $2\pi/\omega$ denotes its duration), but cannot be considered any longer as periodic at any time. Equations (4), (15) are not valid any longer. In order to consider relative thermal self-action, we insert Equation (11) into Equation (5) and establish T_2 and T_4 equating the coefficients standing by r^2 and r^4. For simplicity, thermal and caloric equations for an ideal gas are used:

$$T=\frac{p}{(C_P-C_V)\rho}, \qquad e=\frac{p}{(\gamma-1)\rho} \tag{28}$$

The coefficients are

$$\begin{aligned} T_2&=\frac{8\varepsilon P_0^3}{a_0^2c_0^2C_Pf^5\rho_0^3\left(1+\frac{1}{x_s}\int_0^x\frac{dx'}{f(x')}\right)}+\frac{8\chi T_4\cdot t}{C_P\rho_0},\\ T_4&=\frac{8\varepsilon P_0^3\left(-3+\frac{1}{x_s}\int_0^x\frac{dx'}{f(x')}\right)}{a_0^4c_0^4C_Pf^7\rho_0^3\left(1+\frac{1}{x_s}\int_0^x\frac{dx'}{f(x')}\right)} \end{aligned} \tag{29}$$

Along with the approximation of the thin lens, $f\approx 1$, that results in equation governing the unknown function f:

$$\left[1+\int_0^z\frac{dz'}{f(z')}\right]^4 f^4\left(\frac{\partial^2 f}{\partial z^2}\right)=\Pi\left(1+\frac{8\chi t}{a_0^2C_Pf^2\rho_0}\frac{\left[-3+\int_0^z\frac{dz'}{f(z',\theta')}\right]}{\left[1+\int_0^z\frac{dz'}{f(z',\theta')}\right]^5}\right) \tag{30}$$

where

$$\Pi=\frac{8\delta M\pi^2c_0^4}{a_0^2\varepsilon C_P\omega^2} \tag{31}$$

and $M=P_0/\rho_0c_0^2$ is the initial Mach number. Equation (30) describes variations in the single impulse magnitude and its width as a beam propagates by means of function f. In the case of the waveform containing n shock fronts, Π should be replaced by Π_n,

$$\Pi_n=\frac{8n\delta M\pi^2c_0^4}{a_0^2\varepsilon C_P\omega^2} \tag{32}$$

The results of numerical simulations of Equations (30), (31) are shown in Figure 2 for the planar at a transducer wave. The smallness of the second term in the brackets in the right-hand side of Equation (30) implies, in approximation of the thin lens, that $\frac{8\chi t}{a_0^2C_Pf^2\rho_0}\ll\frac{1}{3}$.

4. Thermal self-action of stationary shock wave which propagates with the linear sound speed

The waveform which recalls the stationary solution of the planar Burgers equation takes the form [10]:

$$p(x,r,\theta)=A(x,r)\tanh\left(\frac{\varepsilon\theta A(x,r)}{b}\right) \tag{33}$$

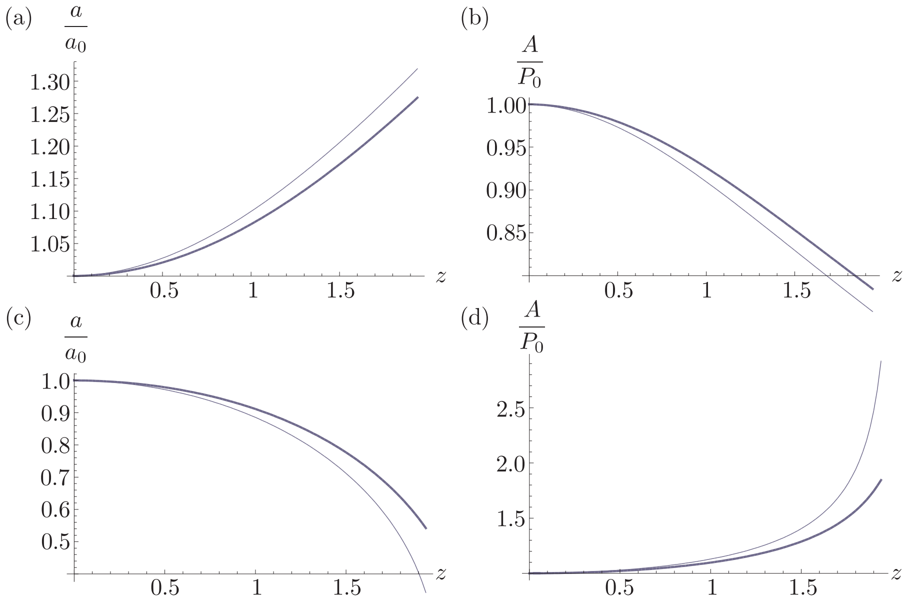

Figure 2. Dimensionless width of a beam, a/a_0 and magnitude of acoustic pressure at the beam axis, $A(r=0,z)/P_0$; curves (a,b) correspond to self-defocusing ($\Pi=0.2$), and curves (c,d) to the self-focusing of an initially planar wave ($\Pi=-0.2$); bold lines relate to numerical solutions of Equation (30) for a single shock wave and dimensionless time $\frac{8\chi t}{a_0^2 C_P \rho_0}=0.1$, and thin lines relate to numerical solutions of Equation (30) at $t=0$

where θ is defined by Equation (8). The relative transport equation for the amplitude $A(x,r)$ allowing durations of a shock front to tend to zero, $b/(\varepsilon P_0)\to 0$, is:

$$\frac{\partial A}{\partial x}+\frac{\partial \psi}{\partial r}\frac{\partial A}{\partial r}+\frac{\Delta_\perp \psi}{2}A=0 \tag{34}$$

It has a solution

$$A(x,r)=\frac{P_0}{f}\Phi\left(\frac{r}{a_0 f}\right) \tag{35}$$

A Newtonian thermoconducting fluid, which obeys equations of state specific for an ideal gas, is considered. We repeat the steps undertaken in the previous section:

1) perturbation of temperature is expanded in the Equation (17);
2) the acoustic source is calculated by the use of Equation (5) with acoustic pressure in the form of Equation (33);
3) the system of equations similar to Equations (20) are solved when the characteristic duration of a shock front tends to zero, $b/(P_0\varepsilon)\to 0$.

Equations (19), (35) yield the following equalities in approximation of the thin lens $f\approx 1$ (they in fact represent the coefficients standing by different powers of r: r^2 and r^4):

$$T_2=\frac{8\varepsilon P_0^3}{a_0^2 c_0^4 C_P f^5 \rho_0^3}+\frac{8\chi T_4\cdot t}{C_P\rho_0},\qquad T_4=-\frac{24\varepsilon P_0^3}{a_0^4 c_0^4 C_P f^7 \rho_0^3} \tag{36}$$

which in turn define the equation for the unknown function f:

$$f^4\left(\frac{\partial^2 f}{\partial z^2}\right)=\Pi\left(1-\frac{24\chi t}{a_0^2 C_P\rho_0 f^2}\right) \tag{37}$$

where Π, z are determined by Equations (31), (22). Dimensionless quantities Π and z are chosen analogously with the previous subsections, they include characteristic frequency ω, which does not denote the sound frequency any longer. It may be chosen arbitrarily. Equation (37) is valid at dimensionless times satisfying the inequality

$$\frac{24\chi t}{a_0^2 C_P\rho_0 f^2}\ll 1 \tag{38}$$

The curves in Figure 3 represent the beam amplitude at the axis and its characteristic width,

$$\frac{A(r=0,z)}{P_0}=\frac{1}{f},\qquad \frac{a(z)}{a_0}=f \tag{39}$$

In the simulations, the beam is initially planar with $x_s/R=0$.

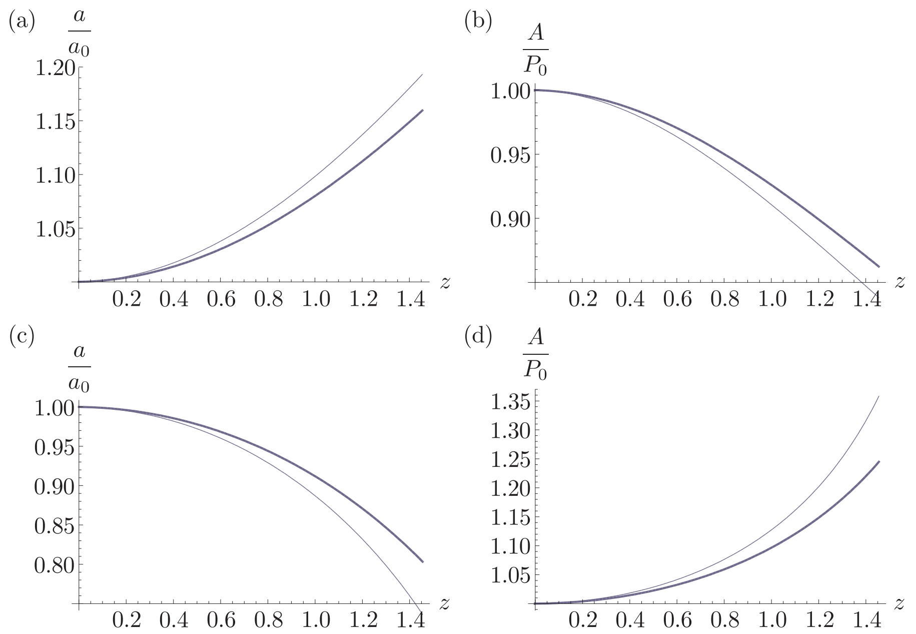

Figure 3. Dimensionless beam width, a/a_0 and acoustic pressure amplitude at the beam axis, $A(r=0,z)/P_0$; the curves (a,b) correspond to self-defocusing ($\Pi=0.2$), and curves (c,d) to the self-focusing of an initially planar wave ($\Pi=-0.2$); the bold lines relate to the numerical solutions of Equation (37) for a single shock wave and dimensionless time $\frac{24\chi t}{a_0^2 C_P\rho_0}=0.2$, and the thin lines relate to numerical solutions of Equation (37) at $t=0$

5. Concluding remarks

This study considers the thermal self-action of some waveforms with shock fronts in a Newtonian fluid with thermal conduction. Three types of waveforms are considered:

1) periodic in time saw-tooth wave,
2) single saw-tooth impulse or some integer number of impulses, and
3) solitary symmetric shock wave.

The conclusions are valid for sawtooth impulses with temporal profiles containing discontinuities or steep shock fronts of finite width much smaller than the characteristic duration of a pulsed signal ($b\omega/(c_0^2\rho_0) \ll 1$), where $2\pi/\omega$ is the impulse duration, and for a solitary waveform with a narrow shock front. In all cases, the parabolic wavefront in the paraxial area is assumed. Expanding the excess temperature in series in the vicinity of the axis and equating coefficients by similar powers of r result in the leading-order equation describing the acoustic beam width and the acoustic pressure magnitude in the paraxial area. Equations (21), (30), (37) are the main results of this study. They describe the beam width dynamics and the corresponding acoustic pressure magnitude at the beam axis by means of function f. In the case of impulses and a solitary shock wave, the dynamic equations are valid in approximation of the thin lens, when $f \approx 1$. In the case of a periodic acoustic beam, the acoustic source is a quantity, averaged over the sound period (Equation (4)), but in the two first cases, it should be calculated using the instantaneous formula, Equation (5), and thermal and caloric equations of state of the fluid.

All the numerical evaluations were performed in *Mathematica*. They reveal the influence of thermal conduction of a medium on the self-focusing and self-defocusing of a sound beam. It makes these nonlinear phenomena weaker. As for the periodic sound, the focal distance of self-focusing shifts far from the transducer as compared to the case without thermal conduction. As it has been established in the non-stationary focusing in a fluid without thermal conduction, the periodic beam radius increases as the wave propagates, *i.e.*, nonlinear broadening of the beam is evidently observed. This effect can be explained by the flattening of the transverse beam profile due to the stronger absorption near the axis [11]. The magnitude of acoustic pressure at small distances decreases. This is caused by nonlinear absorption which competes with the self-focusing of the wave front. Near the nonlinear focus, the magnitude becomes infinitely large. In this case, the description becomes inadequate as it does not account for the divergence due to diffraction. As for the single sawtooth impulse, or the integer number of these impulses, the beam width does not initially reduce during the self-focusing. The conclusion about the "smoothening" effect of thermal conduction in all the considered examples is evident: thermal conduction makes the temperature field more uniform thus weakening the effects originating from the non-uniformity of temperature, like nonlinear self-focusing of sound beams.

All the conclusions of Section 3 are valid also for periodic or single pulses with the acoustic pressure in the form

$$p(x,r,\theta)=A(x,r)\left(-\frac{\omega\Theta}{\pi}+\tanh\left(\frac{\varepsilon\Theta}{b}A(x,r)\right)\right)+LP_0, \tag{40}$$

where $\Theta=\theta+\frac{\pi(x/x_s+G)}{\omega}L=\theta+\left(\frac{P_0}{A(x,r)}+G-1\right)\frac{\pi}{\omega}L,$ L, G are some constants, and θ is given by Equation (8). In fact, Equation (40) originates from the exact solution of the Burgers equations for planar nonlinear waves where A is a function of x. One period of this planar wave is determined in the temporal domain

$$-\pi+\pi(1-G)L<\omega\theta<\pi+\pi(1-G)L \tag{41}$$

The domain of distances where a shock is within the interval $[-\pi+\pi(1-G)L,\pi+\pi(1-G)L]$, is determined by inequality $|LP_0|\leq A(x)$. In the case of $L=0$, the impulse is symmetric, it propagates with the speed c_0. It may be readily discovered that the shock speed given by Equation (40) equals $c_0+\frac{\varepsilon LP_0}{\rho_0 c_0}$.

In this study, we assume that the thermal self-action occurs in a static medium. The effects associated with the inertial self-action of the sawtooth waves in Newtonian fluids are discussed in [12]. Acoustic streaming always leads to additional divergence as the drift caused by streaming causes the wave velocity to increase in the central part of the beam; that happens to any waveform in the course of propagation in a viscous fluid, periodic or impulse.

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