NUMERICAL MODELING OF TSUNAMI WAVE DESTRUCTION AND TURBULENT MIXING AT TSUNAMI WAVE CLASH ON THE SHORE

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(received: 4 January 2016; revised: 12 February 2016;
accepted: 22 February 2016; published online: 21 March 2016)

Abstract: A numerical model of propagation of internal gravity waves in a stratified medium is applied to the problem of tsunami wave run-up onto a shore. In the model, the ocean and the atmosphere are considered as a united continuum in which the density varies with height with a saltus at the water-air interface. The problem solution is sought as a generalized (weak) solution; such a mathematical approach automatically ensures correct conditions of matching of the solutions used on a water-air interlayer. The density stratification in the ocean and in the atmosphere is supposed to be described with an exponential function, but in the ocean a scale of the density stratification takes a large value and the density changes slightly. The initial wave running to a shore is taken in the form of a long solitary wave. The wave evolution is simulated with consideration of the time-varying vertical wave structure. Near the shore, the wave breaks down, and intensive turbulent mixing develops in the water thickness. The wave breakdown effect depends on the bottom shape. In the case when the bottom slope is small and the inshore depth grows slowly with the distance from the shore, mixing happens only in the upper stratum of the fluid due to the formation of a quiet region near the bottom. When the bottom slope takes a sufficiently large value, the depth where fluid mixing takes place goes down up to 50 meters. The developed model shows that the depth of the mixing effects strongly depends on the bottom shape, and the model may be useful for investigation of the impact strong gales and hurricanes on the coastline and beaches.

Keywords: tsunami, stratified fluid, internal wave, numerical modeling, wave breakdown, mixing, turbulence, ocean

1. Introduction

Many monographs (see, for example, [1–11]), Internet resources [12–14] are devoted to the modeling of the generation and propagation of tsunami waves in the ocean. Advances in the study of tsunami waves are significant. Nevertheless, the tsunami waves are very interesting objects of research. When a tsunami wave arrives at shoal-water, the wave breaks, and very intensive turbulent mixing of the liquid originates. These mixing processes are evolved not only in water, but also in the atmosphere. This stage of development of the tsunami waves, characterized by the emergence and development of turbulent mixing in the liquid, is rather difficult for numerical simulation, and success in its study is much less significant. At the same time, these processes of intensive turbulent mixing of the liquid are very important because large devastating effects of tsunami waves are due not only to the water level rise, but they are also a consequence of involving of all bodies and subjects in intensive mixing and movement. In this study, a numerical model designed to simulate and study the tsunami wave destruction and the formation of liquid mixing is developed and applied.

The up-to-date numerical models of tsunami wave propagation are described, in particular, in [15–17]. A series of TSUNAMI programs is described in [15]; the numerical model MOST is given in [16]; the refined variant of the FUNWAVE program is suggested in [17]. The majority of models of tsunami wave propagation are based on versions of shallow water equations. Shallow water equations are written for fluid variables averaged over a vertical variable. This approach simplifies fluid equations significantly and allows simulation of propagation of waves in the ocean up to long distances with excellent accuracy. However, because of averaging over a vertical variable, the shallow water equations do not describe fluid mixing effects. Consequently, we cannot use shallow water equations for simulation of tsunami wave breaking and occurrence of fluid mixing.

The various Korteweg models with variable coefficients as well as Gardner's equations describe the coming of a tsunami wave onto a shoal-water, with consideration of the interior vertical structure of the wave (see [18] and the literature listed there). However, these models are asymptotic, and they have a restricted field of applicability.

In the analysis of applicability of a specific numerical model to simulate the processes of wave breaking and fluid mixing, we must keep in mind an important point. The spatial scale of the wave can change by orders of magnitude following the breaking of an incoming wave and the development of turbulent mixing.

Under such circumstances, the exact implementation of fundamental conservation laws of the numerical method is very important. The energy law conservation follows automatically from differential equations for density and momentum. However, finite-difference equations differ from differential equations and the energy conservation law does not follow from the conservation of mass and momentum for the finite-difference equations. The corresponding checking of fulfillment of the exact conservation laws is not made for the existing numerical free-surface models [19–21]; and the formulas [19–21] are very difficult for analyzing the conservation laws, since the calculations are multistage and contain a lot of corrective additives.

In the work [19], the TVD-scheme has been proposed for modeling of wave propagation in a heavy fluid with a free surface over an uneven bottom. The numerical method [19] has been verified by comparing computer simulations with laboratory experiments in a tank with the dimensions $40\,\mathrm{sm} \cdot 20\,\mathrm{sm} \cdot 4\,\mathrm{sm}$, and good agreement of numerical simulations with experimental data has been demonstrated. The spatial scale of the waves in the tank is not large, and therefore viscosity substantially enough affects the wave processes in the tank [19]. It is well known that the viscosity smoothes the solution and it simplifies the numerical integration of equations. In particular, the implementation of fundamental conservation laws is facilitated due to the smoothing action of viscosity. The spatial scale of real ocean waves traveling to a coast is at least a thousand times greater than the scale of the waves in the tank. The breaking up of ocean waves has to undergo many stages before the scale of new very small-scale waves become comparable to the scale of the waves in the tank. Obviously, the effects of viscosity are small at these first stages of propagation and destruction of ocean waves. Thus, the idea of the model [19] to take viscosity into account in order to ensure the solution smoothness is unlikely to work in the modeling of real ocean processes; and we have to consider the general case and to assume that the solution can be non-smooth and apply the appropriate complex mathematical methods, developed on the assumption that the problem solution can be non-smooth.

In [21], some improvements of the numerical scheme [19], giving methods useful for modeling the processes in coastal areas are proposed. The models [20, 21] do not take into account the viscosity, since the viscosity influence is negligible for the processes under consideration, but they take into account the diffusion of turbulent mixing or turbulent viscosity, which are significant

The numerical models that use semi-empirical turbulence models (k-ε model as [20], or turbulent viscosity, as [21]) require an empirical choice of the turbulence coefficient. Due to the complex dynamics of the waves, turbulent coefficients may depend on time and coordinates. Natural processes, in contrast to the experiments in tanks, are not reproducible. It gives us some problems when we set up the turbulence models.

In [22–24], some problems with the free liquid surface have been solved under the assumption of the potential movement of the liquid.

The numerical model used in this study is a two-dimensional non-hydrostatic model designed to simulate the propagation of internal gravity waves in a stratified fluid. Initially, this numerical model was developed to simulate the propagation of internal waves in a medium with a continuous density stratification. Nevertheless, the model is universal enough, and in this paper, we apply our numerical model to simulate the waves in a medium with a density jump. In our model we interpret the ocean and the atmosphere as a single continuum. The density of this continuum has a jump at the air-water interface.

Since the atmosphere is taken into account in our model, along with a wave in the ocean, some atmospheric disturbance induced by a wave in the ocean is also simulated. The relation of our model to the models [19–21] is the following: if we substitute the expression $\rho(x,z,t) = \rho_{\text{water}} \cdot \eta\big(-(z-\sigma(x,t))\big)$ for density into our Equations (1), where η is a unit step function and $\sigma(x,t)$ is a function describing the water-air interface, then our equations (1) turn into equations similar to [19–21]. Hence, we can treat our model (1) as some generalization of the models [19–21].

Unlike in [19], viscosity is not taken into account in our Equations (1), because it is negligible for the considered waves. Unlike in [20], we do not use the k-ε model to account for the turbulent diffusion mixing, and unlike in [21], we do not consider turbulent viscosity in our model. Modeling of the appearance of turbulent mixing of the liquid and the research of mixing is one of the main interests of our work, and therefore, the use of any parameterization of mixing processes contradicts the aims of our study.

In [25], our model has been used to simulate mixing processes in fluids with a continuous stratification, and the numerical calculations have been compared with laboratory experiments. It has been shown that our numerical method provides a good description of early stages of mixing fluids. However, analyzing the numerical model [25], we have to note that vortexes are destroyed by development of fluid mixing, and since some instant the resolution of our mesh becomes insufficient to resolve small formed vortexes and the viscosity effects become significant. Since that time, the turbulent mixing has been modeled only qualitatively, and we cannot guarantee an accurate quantitative description. However, in the early stages of wave destruction, the scales of emerging new vortexes are not very small, and the accuracy of our model is quite satisfactory. In this paper, we are interested only in fairly general characteristics of the studied process: at what distance from the coast the fluid mixing takes place; to what depth this fluid mixing penetrates; and how the mixing depth depends on the bottom form. The proposed model is quite adequate for the study of these fairly general characteristics.

Versatility is an important feature of the considered model. The model allows taking into account the details of the density stratification of ocean water.

Our model takes into account the interaction of oceanic and atmospheric waves. As the atmospheric gas density is low, the atmosphere has little effect on the propagation of long waves in the ocean. The atmosphere is incorporated into our model only due to difficulties in describing the water surface at the break of waves. From observations, we know that small-scale structures with thin scales up to individual water droplets often occur when a wave breaks. Foam often appears on the water surface. The function describing the water surface can become ambiguous in the considered process. The water surface is not explicitly involved in our model and we have no problems with the water surface description, but the inclusion of air into consideration may cause some trouble. However, we are not going to describe the process in detail, down to the individual droplets possibly formed. The energy of individual droplets or foam is not significant. Thus, we can take air into consideration, but we do not need to describe the evolution of the water surface in detail.

Nevertheless, the interaction of ocean waves with the atmosphere is interesting to explore . The simulation shows that an atmospheric disturbance induced by the wave in the ocean exists and propagates together with the ocean wave. In the atmosphere, wind is generated over the ocean wave; the wind direction is opposite to the direction of water movement in the ocean wave. This velocity shift creates conditions for the occurrence of secondary small waves on the water surface, resulting from the interaction of the oceanic wave with the atmosphere. Atmospheric gas can also influence the wave disintegration. Although the air density is low, but air can exacerbate instability. For example, the water flow moves in vacuum without breaking, but in air this water flow disintegrates and can fall apart to droplets due to instabilities arising at the air-water interface.

In [25], the author has shown that simultaneous implementation of all the fundamental conservation laws and certain inequality for density given in [25], is a prerequisite for stability and convergence of the numerical method. Simultaneous implementation of all the fundamental conservation laws and the inequality for density has been proven in [25] for our numerical model.

The same numerical approach has previously been used for modeling of propagation and destruction of waves in the atmosphere and has allowed simulating the formation of turbulence at the turbopause altitudes in the atmosphere (about 100 km) due to destruction of internal gravity waves, propagated upward from land-based sources [26, 27].

2. Problem Statement

We consider the two-dimensional movement of an incompressible fluid placed in a gravity field; the fluid moves over an irregular bottom and the flow is described by the set of equations

$$\begin{gathered}\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho\frac{\partial\Psi}{\partial z}\right)-\frac{\partial}{\partial z}\left(\rho\frac{\partial\Psi}{\partial x}\right)=0\\ \frac{\partial}{\partial t}\left[\frac{\partial}{\partial z}\left(\rho\frac{\partial\Psi}{\partial x}\right)+\frac{\partial}{\partial x}\left(\rho\frac{\partial\Psi}{\partial z}\right)\right]+\\ \frac{\partial^2}{\partial x\partial z}\left[\rho\left(\left(\frac{\partial\Psi}{\partial z}\right)^2-\left(\frac{\partial\Psi}{\partial x}\right)^2\right)\right]-\left(\frac{\partial^2\Psi}{\partial z^2}-\frac{\partial^2\Psi}{\partial x^2}\right)\rho\frac{\partial\Psi}{\partial x}\frac{\partial\Psi}{\partial z}-\frac{\partial}{\partial x}(\rho g)=0\end{gathered}\tag{1}$$

followed by Euler's equations for an incompressible fluid. Here ρ is the medium density; Ψ is a flow function; t is time; g is the acceleration of gravity; x is a horizontal coordinate; and z is a vertical coordinate. $u=\frac{\partial\Psi}{\partial z}$ is a horizontal velocity; $w=-\frac{\partial\Psi}{\partial x}$ is a vertical velocity.

The behavior of atmospheric parameters is governed by the same equations, but with the density value equal to the atmospheric gas density. That is, the atmosphere is described in the model by incompressible fluid equations. The use of an incompressible fluid approximation for an atmospheric gas is justified by the fact that the atmospheric gas density is small; thus, the energy of waves in the atmospheric part of our model is small, and high accuracy of the atmospheric part of the model is not required. The characteristic time of the processes of tsunami wave propagation is estimated about 100 s; and one can show that an incompressible fluid approximation well describes these slow processes, even in case of gas.

At $t=0$, the density $\rho\ \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$ is given by the formula:

$$\rho(x,z,0)=\begin{cases}1000\cdot\exp\left(-\frac{z}{H_W}\right) & z<0\\ 1.2\cdot\exp\left(-\frac{z}{H_A}\right) & z>0\end{cases} \tag{2}$$

The formula (2) describes the real change of atmospheric gas density with altitude ($z>0$), and model behavior of the ocean water density ($z<0$). $H_A=8\,\mathrm{km}$ is a scale of the air density stratification, $H_W=100\,\mathrm{km}$ is a scale of the water density stratification. At $t>0$, the density behavior is calculated by solving the system of equations (1).

Difference approximation of equations always introduces some errors in a computer model. These approximation errors are equivalent to the appearance of some additional sources of mass, momentum, energy in equations. It is impossible to avoid these approximation errors, but we can find such difference schemes where these additional sources of mass, momentum, energy offset each other in the average. To do this, the original equations are rewritten in an integral form, in the form of fundamental laws of conservation of mass, momentum and energy. We need to construct a numerical scheme, so that the fundamental conservation laws in their integral form are satisfied. That is, we require that changes with time of integrals of conserved quantities over any volumes be equal to the flows of these quantities through the volume surfaces (in a discrete model, the integrals are approximated with integral sums over corresponding mesh points). The numerical schemes, supporting the fundamental laws of conservation of mass, momentum, and energy in the form of integral sums, are called conservative. The integral equations do not contain differentiation; thus, the requirement of differentiability of the solution is removed. The solutions that meet the fundamental conservation laws in an integral form are called generalized or weak solutions. The conservative numerical method has the advantage that, although discretization of the equations introduces errors into the model, the fundamental conservation laws are carried out with high accuracy for large volumes with diameters larger the spatial mesh step; thus the fluid flow is properly modeled on scales exceeding the mesh step.

In case when a clear interface between fluids with different densities exists and when the interface can be described with a differentiable function, the standard conditions of matching of the solution used on the interface automatically follow from the definition of a generalized solution. It is important that the technique of generalized solutions is applicable also to the cases when the interface between fluids is so complicated that the interface behavior cannot be described within the framework of differentiable functions. Such a very complicated behavior of the interface between fluids can occur when the wave overturns and breakdown of the wave takes place.

A conservative numerical method of second-order accuracy in space and time is used to solve the equations (1). The difference scheme is explicit-implicit. The spatial mesh "cross" is used. The numerical method was designed and programmed with the help of a program of symbolic computations. The computational formulas of completely conservative numerical methods are very cumbersome: the finite-difference equation for the stream function takes more than a page of text. Therefore, this finite-difference equation is not written in the paper. The applied difference grid, the derivation of the finite-difference formulas, the proof of complete conservativeness of the method, the test results are published in [25].

The problem is solved in a rectangular domain of 2 km altitude and 30 km wide (Figure 1). The atmospheric gas is in the upper part, at altitudes $0\,\mathrm{km} < z < 1\,\mathrm{km}$; either ocean water or ground is at the bottom, at altitudes $-1\,\mathrm{km} < z < 0\,\mathrm{km}$. The condition $\Psi|_{\partial\Omega} = 0$ of impermeability of liquid is imposed along the boundary $\partial\Omega$ of the domain Ω. The condition $\Psi|_{\mathrm{ground}} = 0$ is imposed everywhere on the ground field. The field of the ground is artificial; this field is included in the problem in order to simplify the work with boundary conditions and programming of calculations. The ocean shore meets the mark 0.0 on the horizontal axes. To the left of mark 0.0, when $x < 0$, the liquid is absent, and there is only ground and atmospheric gas above it.

A solitary wave travelling to the shore has been set as an initial condition of the problem. This initial solitary wave is constructed as follows. From the theory of long surface waves, we borrow an exact solution for a solitary gravity wave travelling to the shore. This solitary-type stream function is applied only to water. We define the stream function in the atmospheric part of our model in such a way that it is continuous at the water-air interface, and it is zero at the boundary of our region Ω. We obtain:

$$\begin{aligned} \Psi(x,z,0) &= A\frac{g}{c(x)}\exp\left(-\left(\frac{x-x_0}{\lambda}\right)^2\right)\varphi(z,x) \\ \rho(x,z,0) &= \rho_0\left(z - A\exp\left(-\left(\frac{x-x_0}{\lambda}\right)^2\right)\right) \\ \varphi(z,x) &= \begin{cases} \left(z+h_W(x)\right) & z<0 \\ \frac{h_w(x)}{h_A}(h_A - z) & z>0 \end{cases} \end{aligned} \tag{3}$$

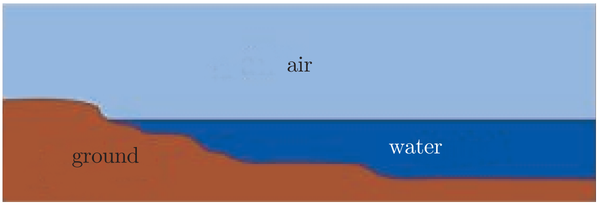

Figure 1. Rectangular domain in which the equations are solved

Here $h_W(x)$ is the ocean depth; $c(x) = \sqrt{g h_W(x)}$; $h_A = 1\,\mathrm{km}$ is the upper boundary of the atmosphere in our model; $A = 10\,\mathrm{m}$. The calculations were performed for two variants: (a) the angle of the bottom slope near the coast is 45° (b) the bottom slope angle near the coast is 10°. In case (a), parameters: $x_0 = 14\,\mathrm{km}$ is the distance from the center of the initial wave up to the shore; $\lambda = 5\,\mathrm{km}$ is a half-width of the wave; the inclined bottom turns into a flat horizontal plateau at the depth of 1 km. In case (b), parameters: $x_0 = 14\,\mathrm{km}$; $\lambda = 4\,\mathrm{km}$, the bottom turns into a horizontal plateau at the depth of 650 m. Case (a) is calculated to test the model and to understand what the incoming wave represents in our model.

The initial solitary wave propagating in the ocean and in the atmosphere, is a moving vortex, a portion of which is in the ocean, and the other part is in the atmosphere. Although we have defined the atmospheric flow function by artificial means, the occurrence of a single vortex, which is partly in the ocean, and partly in the atmosphere, is a natural phenomenon. This is a consequence of the boundary conditions and the requirement of continuity of the stream function on the interface.

3. Outcomes of numerical modeling

One of the goals of this work is to show that the numerical model l [25] not only describes the propagation of internal gravity waves in a continuous stratification, but also describes the propagation of a wave in a stratification with a jump in density. In particular, the model allows us to simulate the propagation of tsunami waves.

The applied difference mesh uses variable steps. In case (a) (the angle of the bottom slope is 45°), the horizontal mesh step is up to 400 meters at large distances from the shore, and the vertical mesh step is up to 10 meters at altitudes far from the ocean surface. The horizontal mesh condenses when reaching the shore, and the vertical mesh condenses when approaching the ocean surface. The coastal horizontal mesh step is reduced to 0.5 m, and near the ocean surface the vertical mesh step is reduced to 0.25 m.

In Figure 3, 4, the propagating wave in the atmosphere-ocean system at $t = 60\,\mathrm{s}$ is shown. The simulations show that the vortex is stable, and it propagates to the shore without breakdown. The speed of wave propagation is approximately equal 100 m/s; it well corresponds to the theory of surface gravity waves. Within the approach of small-amplitude long waves propagating over a plain horizontal bottom, it is simple to construct an analytical solution for equations (1) for initial conditions (3). The outcomes of numerical modeling well coincide with the analytical solution, and Figure 3 may be considered as a test for our numerical model.

At $t = 60\,\mathrm{s}$, the wave front already reaches the shore, and the wave starts to come under the shore influence. The water level rising begins near the shore. In the ocean, the fluid flow in the wave is being directed to the shore, but in

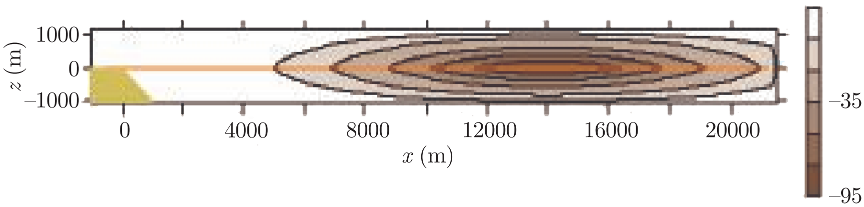

Figure 2. The flow function Ψ for a wave running to a shore, $t = 0$ s (variant a)

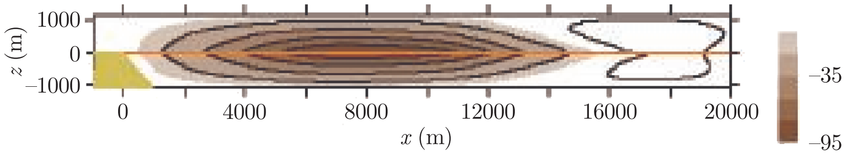

Figure 3. The flow function Ψ for a wave running to a shore, $t = 60$ s (variant a)

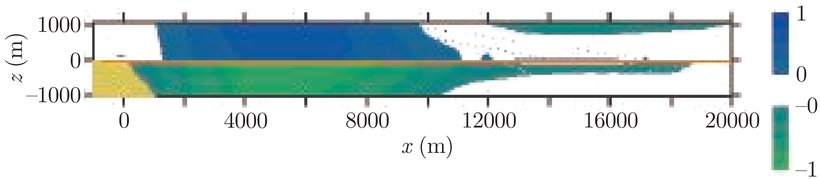

Figure 4. The horizontal velocity, $t = 60$ s (variant a) (intensity of colours corresponds to values of velocity)

the atmosphere, on the contrary, the gas is moving from the shore. At the water-air interface, there is a velocity jump. It creates conditions for development of instability, and to generate a secondary, smaller surface wave associated with the influence of the atmosphere on the wave propagated in the ocean.

At $t = 90$ s, the influence of the shore on the wave behavior becomes significant. The wave stops. Behind the head vortex wave, the vortex pair with opposite rotation arises. The rising water level near the coast reaches up to 7 meters.

At $t = 120$ s, a small derived vortex comes off from the head of the vortex wave and goes to the shore (Figure 5). The vortex moving to the shore is strongly deformed, and at some distance from the coast, there are small vortices counter-rotating with respect to the oncoming vortex. Waves reflected from the coast have formed.

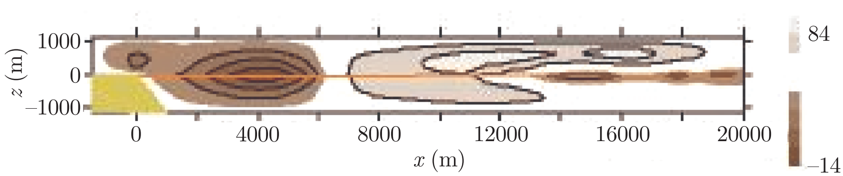

Figure 5. The flow function Ψ, $t = 135$ s (variant a)

At $t = 135$ s, the water level has risen up to about 16 meters near the coast. There are small-scale fluctuations on the surface of the density jump; these small waves have arisen on the interface due to instability of the velocity shift, and they exist because of the influence of the atmosphere on the main propagated wave.

The impact of the shore on the incoming wave leads to intensive mixing of the liquid near the shore. Figure 6 shows a horizontal velocity of the fluid; we can see intense mixing of ocean water and air near the coast. The water mixing processes extend down to the depth of 50 meters. The mixing process results in the formation of a small-scale dynamic vortex structure, which however is not visible in Figure 6 because this picture shows only a general pattern.

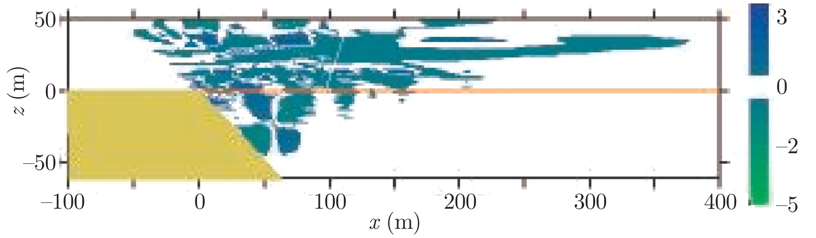

Figure 6. The horizontal velocity, $t = 135$ s (variant a) (intensity of colours corresponds to values of velocity)

For comparison, the arrival of a wave onto the shoal-water and shore with a slight bottom slope is shown in Figure 7 (case b). The wave disturbance propagates onto the shoal-water by the same mechanism as in case (a). The incoming vortex wave is deformed when it arrives close to the shore. Then a child vortex is separated from the main vortex wave and comes onto the shoal-water. Nevertheless, the separated head vortex is larger than in case (a), and is strongly stretched along the horizontal direction. The incoming wave stops far from the shore. The center of the main vortex slightly rises above the sea level. This effect is achieved by the water level rising near the coast.

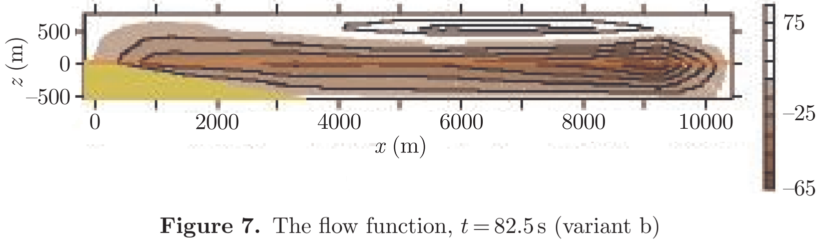

Figure 7. The flow function, $t = 82.5$ s (variant b)

In Figure 8, the water level rise and inundation in case (a) is shown. The water level rising in case (b) is about the same as in case (a) and in both cases it is about 16 meters.

The water level rising depends substantially on the momentum and energy of an incident wave. In both cases, the energies and momentums of the incident

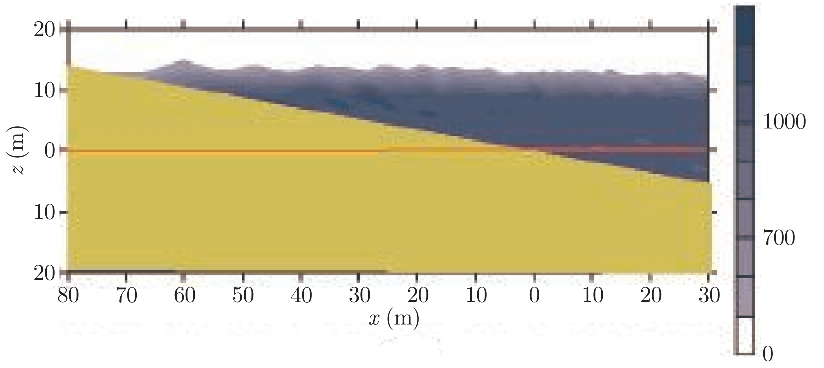

Figure 8. The coastal flood, $t = 82.5\,\mathrm{s}$ (variant b)

waves are approximately equal. Thus, the water level rises in both cases (a) and (b) are identical

4. Conclusions

The propagation of waves onto the shore for the bottom slope 45° and 10° is simulated numerically. It is shown that in this model the tsunami represents a steady moving vortex, which is partly located in the ocean, and partly in the atmosphere.

In the ocean-atmosphere system, the movement of the ocean water is close to a potential movement, and the single vortex in the ocean-atmosphere system is obtained by means of closure of the flow function through the atmosphere.

The wave propagation velocity is approximately equal to the speed of propagation of long surface gravity waves.

In the atmosphere, the direction of movement of air is opposite to the direction of movement of water in the ocean, and a shear of the velocity field exists on the ocean-atmosphere interface. This velocity shear creates conditions for the development of instability and leads to the formation of secondary small-scale waves on the ocean-atmosphere interface over the propagating tsunami wave.

The water level rising in both cases is approximately equal to 16 meters.

The mechanism of penetration of the tsunami wave onto the shore (case a) or onto the shoal-water (case b) is universal. From the main vortex propagating to the coast, a derivative vortex is separated and this derivative vortex propagates onto the shore or onto the shoal-water. As a whole, the wave is reflected from the shore or shoal-water.

Our simulations allowed us to estimate the depth to which intensive mixing of the liquid extends near the shore. In case (a), the mixing depth is of about 50 meters. In case (b), the mixing depth does not exceed 20 meters. In case (b), decreasing of the depth to which the mixing extends, takes place due to the occurrence of a quiet zone near the bottom under the propagating wave near the shore. As a result, the main water movement occurs in the top layer, with a corresponding increase in the flow velocity in the upper layer.

The water level rise near the shore is caused by the incoming wave energy. To simulate the processes, a numerical method that supports the fundamental conservation laws is used; and therefore, we can expect that the inundation of the shore is calculated correctly. However, the model accuracy has to be further explored by comparing the calculated results with laboratory experiments and observations.

Analyzing possible shortcomings of the numerical model, we focus on smoothing the density profile in the vicinity of the water-air interface. This imperfection does not affect the overall quality of the simulation because the main wave energy is concentrated in the water column; and the processes occurring in the water column determine the behavior of the water-air interface, while the reverse influence of the water-air interface on the wave as a whole is slight. However, the problem of more detailed modeling of the water-air interface deserves further consideration. Perhaps simulation of the behavior of the water-air interface can be easily improved through the use of a finer mesh, or by improving the finite-difference approximation of the equation for density.

The authors believe that the experience of modeling of the propagation and the breakdown of tsunami waves within a two-dimensional combined air-water model is successful as a whole.

We hope that the developed model can be of some interest in connection with the problem of destruction of shores and beaches due to severe storms and hurricanes. Intensive mixing of water during heavy storms and hurricanes can cause lifting of sand from the bottom, and this effect creates the conditions when sand can be carried away into the sea or ocean. The coastline stability is highly dependent on the bottom shape near the shore. This study shows that if the depth slowly increases with the distance from the water's edge, the fluid mixing depth is low due to the formation of a quiet zone near the bottom along the shore, and in this case, the bottom and coastline should be more stable. If the bottom slope near the shore exceeds a certain critical level, the intensive mixing of the liquid during strong waves reaches the bottom near the shore, and washing the bottom and erosion of the coast can be natural consequences of this phenomenon.

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