MAGNETOACOUSTIC HEATING AND STREAMING IN A PLASMA WITH FINITE ELECTRICAL CONDUCTIVITY

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Abstract: Nonlinear effects of planar and quasi-planar magnetosound perturbations are discussed. Plasma is assumed to be an ideal gas with a finite electrical conductivity permeated by a magnetic field orthogonal to the trajectories of gas particles. The excitation of non-wave modes in the field of intense magnetoacoustic perturbations, *i.e.*, magnetoacoustic heating and streaming, is discussed. The analysis includes a derivation of instantaneous dynamic equations independent of the spectrum and periodicity of sound.

Keywords: plasma, nonlinear acoustics, magnetoacoustic heating

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1. Introduction

There has been much discussion in the literature during the past decades concerning magnetohydrodynamic phenomena in conducting ionized fluids, which are of importance in many applications of geophysics, plasma physics, cosmic physics and hypersonic aerodynamics. Magnetohydrodynamic (MHD) waves play an important role in the formation and dynamics of the solar atmosphere. There exist four MHD eigenmodes in the uniform magnetized plasma, disregarding the magnetic field direction: Alfvén, fast and slow magnetoacoustic modes, and the entropy mode [1–3]. In the non-planar flows, the vortex mode appears. The magnetic field alters the compressibility of the fluid, and hence, the speed of sound in the medium. The sound speed varies with the dependence on the the angle between the wave vector and the magnetic strength vector, achieving a maximum (corresponding to the fast magnetoacoustic wave) when the vectors are perpendicular.

The imaging and spectroscopic instruments with a high spatial and temporal resolution have rapidly advanced during the last years. Observations have revealed that perturbations fitting the MHD spectrum are present in most, if not in all, coronal structures and have substantiated that MHD waves waves carry a considerable part of the energy required to heat the solar corona [3]. Loss in the MHD wave momentum may drive the solar wind. A comprehensive overview of MHD waves and their role in coronal heating may be found in Refs [4–6]. The identification of widespread acoustic modes in the solar corona has revived interest in their application to coronal seismology. It is difficult to explain the dissipation of these modes by linear damping (see [7] and the references therein). The observations of disturbances traveling along coronal structures make some authors argue that the low propagation speed (approx. 40km/s) makes an interpretation in terms of (MHD) wave modes implausible but requires the presence of flows [8, 9]. These bulk flows may be in turn excited by intense MHD waves.

The coupling of acoustic waves with non-wave modes leads to their damping due to nonlinear losses in acoustic energy (acoustic heating, that is, the entropy mode enhancement) and the acoustic momentum (acoustic streaming, that is, the vortex flow setting). These two phenomena which originate from both nonlinearity and attenuation, are well-understood with regard to the periodic sound in Newtonian fluids [10, 11]. A periodic sound in the role of a source of instantaneous acoustic streaming and heating in Newtonian fluids and fluids which differ from Newtonian, was considered by the author in a number of studies [12–14]. We will neglect the Newtonian attenuation in plasma as well as its thermal conduction, focusing exclusively on the attenuation which arises from its finite electrical conductivity. The attenuation and dispersion which follow from finite electrical conductivity as well as the magnetoacoustic speed are frequency-dependent [15, 16]. The understanding that the sound velocity in an electrically conducting fluid should vary in the presence of a magnetic field comes from the 1950s [17, 18]. It has been established that the finite conductivity introduces absorption associated with the dispersion of sound waves the propagation direction of which is perpendicular to the direction of the magnetic field [19]. Along with nonlinearity, this frequency-dependent absorption will be considered in this study as the reason for the transfer of the magnetoacoustic energy and momentum into the non-wave modes.

In general, we do not have the luxury of complete exact solutions to nonlinear PDEs (partial differential equations) describing perturbations in a fluid flow. The nonlinear interaction of MHD waves has been considered by numerous authors [20–22]. Three-wave interactions of Alfvén and magnetosound waves are studied in Reference [23]. The review by Ballai summarizes the knowledge on nonlinear waves in solar plasmas [24]. As usual, attention is paid to three-wave resonant interactions of strictly harmonic waves. The method, which has been applied by the author in studies of hydrodynamic perturbations in fluids with various attenuation, gives a possibility to derive equations accounting for the interaction of modes independently of their spectrum (see, for example, Reference [12]). The mode is determined according to the links between specific perturbations of an infinitely small magnitude. Once the modes are established, it is possible to evaluate the projectors which distinguish the specific perturbation but eliminate all foreign ones in the total vector of perturbation. They also distinguish specific dynamic equations in their linear part and distribute nonlinear terms between individual equations in the proper manner when applied at a system of conservation PDEs. This allows deriving a set of weakly nonlinear evolution equations, as well as correcting links of specific perturbations in a weakly nonlinear flow. The equations describe interactions of all modes, not only the wave ones. On the whole, the procedure is approximate but appoints a recurrent sequence of actions to obtain the results as series in powers of the Mach number M with any desired accuracy. We focus on the quadratic nonlinearity in this study. This coincides with the concept of a weakly nonlinear flow of a magnetic gas.

The structuring of plasma brings a characteristic spatial scale and the appearance of guided magnetoacoustic modes. Magnetoacoustic modes with wave lengths comparable with (or larger than) the characteristic scale of the plasma inhomogeneity are highly dispersive [25, 26, 2]. The dispersion originating from the structuring of plasma, as well as that originating from boundaries, is not considered in this study. It is an unbounded volume of a gas in a magnetic field that is under consideration. External sources of energy (heating/cooling function per unit volume and time) are not considered. They may considerably influence the dynamics of a gas, making it inhomogeneous and acoustically active, and its flow unstable [27, 28]. Another issue that is not considered is the radiative loss function which is important at high gas temperatures between 10^4K and 10^7K [29–31]. These issues have been investigated in many studies.

2. Decomposition of sound and non-wave modes in a planar flow

2.1. PDEs describing a planar flow of a conducting gas

We consider a planar flow of a gas whose velocity $v(x,t)$ is perpendicular to the magnetic field strength $\vec{H}=(0,0,H(x,t))$, where t and x designate the time and the coordinate which indicates the axis orthogonal to the magnetic field. The magnetic field is evidently solenoidal, $\vec{\nabla}\cdot\vec{H}=0$. The conservation equations of the MHD flow sound [32]:

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho v}{\partial x}=0 \tag{1}$$

for the mass,

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)+\frac{\partial p}{\partial x}+\frac{\partial h}{\partial x}=0 \tag{2}$$

for the momentum,

$$\frac{\partial s}{\partial t}+v\frac{\partial s}{\partial x}=0 \tag{3}$$

for entropy s, and

$$\frac{\partial h}{\partial t}+v\frac{\partial h}{\partial x}+2h\frac{\partial v}{\partial x}+\beta\left(\frac{1}{2h}\left(\frac{\partial h}{\partial x}\right)^2-\frac{\partial^2 h}{\partial x^2}\right)=0 \tag{4}$$

for the magnetic pressure h, where

$$h=\mu H^2/2 \tag{5}$$

ρ, p are the density and pressure of a gas, respectively. In Equation (4), $\beta=(\mu\sigma)^{-1}$, μ is the magnetic permeability, and σ is the electrical conductivity of a fluid. Equation (4) readily follows from the electrodynamic equation [15]

$$\frac{\partial\vec{H}}{\partial t}-\vec{\nabla}\times(\vec{v}\times\vec{H})=\beta\Delta\vec{H} \tag{6}$$

2.2. Projecting of total perturbation into specific modes

Equations (1)–(4) should be completed by the caloric equation of state and the thermodynamic identity for equilibrium thermodynamic processes, $Tds=de+pd(\rho^{-1})$ (T designates the temperature of a gas). We make use of the internal energy e of an ideal gas:

$$e=C_vT=\frac{p}{(\gamma-1)\rho} \tag{7}$$

where γ is the ratio of specific heats under constant pressure and constant density, and C_V is the heat capacity under constant volume. The unperturbed quantities will be marked by subscript 0, and all disturbances will be primed. Perturbations are developed against the motionless background with $v_0=0$. In terms of velocity and perturbations in density, pressure and magnetic pressure, Equations (1)–(4) take the leading-order form:

$$\frac{\partial\psi}{\partial t}+L\psi=\psi_{nl} \tag{8}$$

where

$$\psi=\begin{pmatrix}\rho'\\ v\\ p'\\ h'\end{pmatrix},\quad L=\begin{pmatrix}0 & \rho_0\frac{\partial}{\partial x} & 0 & 0\\ 0 & 0 & \frac{1}{\rho_0}\frac{\partial}{\partial x} & \frac{1}{\rho_0}\frac{\partial}{\partial x}\\ c_0^2\rho_0\frac{\partial}{\partial x} & 0 & 0 & 0\\ 2h_0\frac{\partial}{\partial x} & 0 & 0 & -\beta\frac{\partial^2}{\partial x^2}\end{pmatrix}$$
$$\psi_{nl}=\begin{pmatrix}-\rho_0'\frac{\partial v}{\partial x}-v\frac{\partial \rho'}{\partial x}\\ -v\frac{\partial v}{\partial x}+\frac{\rho'}{\rho_0^2}\frac{\partial p'}{\partial x}+\frac{\rho'}{\rho_0^2}\frac{\partial h'}{\partial x}\\ -v\frac{\partial p'}{\partial x}-\gamma p'\frac{\partial v}{\partial x}\\ -v\frac{\partial h'}{\partial x}-2h'\frac{\partial v}{\partial x}-\frac{\beta}{2h_0}\left(\frac{\partial h'}{\partial x}\right)^2\end{pmatrix} \tag{9}$$

where ψ_{nl} consists of quadratic nonlinear terms, c_0 is the infinitely-small sound speed in an ideal gas at an unperturbed thermodynamic state (p_0,ρ_0) in the absence of a magnetic field,

$$c_0=\sqrt{\frac{\gamma p_0}{\rho_0}} \tag{10}$$

The linear system

$$\frac{\partial \psi}{\partial t} + L\psi = 0 \tag{11}$$

determines four roots of the dispersion relation, ω_n ($n = 1,...,4$), and four eigenvectors corresponding to $-i\omega_n$. Establishing the dispersion relations is the primary procedure in all linear fluid flows. The dispersion relations describing all the independent modes follow from Equations (8). Studies begin by representing all perturbations as a sum of planar waves:

$$f'(x,t) = \int_{-\infty}^{\infty} \tilde{f}(k)\exp(i\omega(k)t - ikx)dk \tag{12}$$

$\tilde{f}(k)\exp(i\omega(k)t) = \tilde{f}(k,t)$ denotes the Fourier transform of $f'(x,t)$:

$$\tilde{f}(k,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x,t)e^{ikx}dx \tag{13}$$

In a planar flow of a magnetic fluid, the dispersion relations take the leading-order forms:

$$\omega_{1,2} = \pm c_0 k \pm \frac{(c_0 \pm i\beta k)h_0 k}{\rho_0(c_0^2 + \beta^2 k^2)}, \quad \omega_3 = 0, \quad \omega_4 = i\beta k^2 - \frac{2i\beta h_0 k^2}{\rho_0(c_0^2 + \beta^2 k^2)} \tag{14}$$

The first two roots ω_1, ω_2 correspond to the magnetosonic waves of different directions of propagation (fast MHD waves). They were derived by the author in the limiting cases of high and low frequencies in [15]. The third root ω_3 corresponds to the entropy mode, and the last one, ω_4, corresponds to the Alfvén wave in the flow where magnetic field is orthogonal to the particle velocity. The only restriction which has been made during the evaluation is the smallness of the equilibrium magnetic strength as compared with the unperturbed thermodynamic pressure of a gas,

$$h_0 \ll \rho_0 c_0^2 = \gamma p_0 \tag{15}$$

It is remarkable that the two last dispersion relations in Equations (14) are zero in a fluid without electrical conductivity. In this case, there are two degenerate eigenvalues and more than one linearly independent eigenvectors corresponding to each of them. This degeneracy is eliminated by electrical conductivity and the inherent to it dispersion. The corresponding eigenvectors in the space of Fourier transforms look as follows:

$$\begin{aligned}
\tilde{\psi}_1 &= (\tilde{\rho}_1 \quad \tilde{v}_1 \quad \tilde{p}_1 \quad \tilde{h}_1)^T = \begin{pmatrix}1 & \frac{c_0}{\rho_0} + \frac{h_0}{\rho_0^2(c_0 - i\beta k)} & c_0^2 & \frac{2c_0 h_0}{\rho_0(c_0 - i\beta k)}\end{pmatrix}^T \tilde{\rho}_1 \\
\tilde{\psi}_2 &= (\tilde{\rho}_2 \quad \tilde{v}_2 \quad \tilde{p}_2 \quad \tilde{h}_2)^T = \begin{pmatrix}1 & -\frac{c_0}{\rho_0} - \frac{h_0}{\rho_0^2(c_0 + i\beta k)} & c_0^2 & \frac{2c_0 h_0}{\rho_0(c_0 + i\beta k)}\end{pmatrix}^T \tilde{\rho}_2 \\
\tilde{\psi}_3 &= (\tilde{\rho}_3 \quad \tilde{v}_3 \quad \tilde{p}_3 \quad \tilde{h}_3)^T = (1 \quad 0 \quad 0 \quad 0)^T \tilde{\rho}_3 \\
\tilde{\psi}_4 &= (\tilde{\rho}_4 \quad \tilde{v}_4 \quad \tilde{p}_4 \quad \tilde{h}_4)^T = \begin{pmatrix}1 & \frac{i\beta k}{\rho_0} - \frac{2i\beta k h_0}{\rho_0^2(c_0^2 + \beta^2 k^2)} & c_0^2 & -c_0^2 - \beta^2 k^2 + \frac{2h_0}{\rho_0}\end{pmatrix}^T \tilde{\rho}_4
\end{aligned} \tag{16}$$

The eigenvectors in the x space may be readily evaluated from Equations (16). The first eigenvector takes the form:

$$\psi_1 = (\rho_1' \quad v_1 \quad p_1' \quad h_1')^T = \tag{17}$$

$$\left(1 \quad \frac{c_0}{\rho_0} + \frac{h_0}{\rho_0^2\beta}\int\limits_{-\infty}^{x} dx' \exp(-\frac{c_0(x-x')}{\beta}) \quad c_0^2 \quad \frac{2c_0 h_0}{\rho_0\beta}\int\limits_{-\infty}^{x} dx' \exp(-\frac{c_0(x-x')}{\beta})\right)^T \rho_1'$$

and so on. The lower limit of integration is $-\infty$ if the perturbations tend to zero when x tends to $-\infty$. It may be set differently in accordance with the physical conditions of a flow. These links are independent of time and describe the dispersion exactly. The total perturbation is represented by a sum of specific disturbances (we can say, eigenvectors inherent to these eigenvalues). We may also establish the leading-order operator row which distinguishes the specific excess density correspondent to the entropy mode from the total vector of perturbations:

$$P_3(\rho' \quad v \quad p' \quad h')^T = \rho_3', \quad P_3 = (1 \quad 0 \quad -1/c_0^2 \quad 0) \tag{18}$$

When P_3 applies at the linearized system (11), it reduces all the terms of the foreign modes and yields the linear dynamic equations for ρ_3'.

The application of P_3 at the system (8), which includes quadratic nonlinear terms, distributes them between dynamic equations in the correct manner. As usual, nonlinear effects of an intense sound are of interest, so that it is only acoustic terms that are considered among a whole variety of nonlinear cross ones. They form the "acoustic forces" exciting the secondary entropy mode.

3. Weakly nonlinear equations for magnetoacoustic perturbations

For a proper description of the nonlinear effects of sound, the linear links which are determined by eigenvectors of the correspondent matrix operator, should be completed by the leading-order nonlinear terms, quadratic in the leading order. Without any loss of generality, it is only the magnetoacoustic mode propagating in the positive direction of axis x that will be considered. It corresponds to ω_1 from Equations (14). The relative eigenvector in the case $\beta = 0$ takes the form:

$$\psi_1 = (v_1 \quad \rho_1' \quad p_1' \quad h_1')^T = \left(1 \quad \frac{\rho_0}{c_{m,0}} \quad \frac{\rho_0 c_0^2}{c_{m,0}} \quad \frac{\rho_0(c_{m,0}^2 - c_0^2)}{c_{m,0}}\right)^T v_1 \tag{19}$$

where

$$c_{m,0} = c_0 + \frac{h_0}{c_0\rho_0} \tag{20}$$

is the speed of a fast magnetosound wave at small magnetic strength h_0 in accordance with Equation (14). The vector with unknown constants K, L, N,

$$\psi_{1,n} = (1 \quad K \quad L \quad N)^T v_1^2 \tag{21}$$

should be added to ψ_1 in order to yield four equivalent leading-order dynamic nonlinear equations for magnetoacoustic perturbations, when substituted into Equations (8). Solving algebraic equations, one arrives at following quantities

$$K=\frac{\rho_0^2(2h_0+(3-\gamma)\rho_0c_0^2)}{4(2h_0+c_0^2\rho_0)^2},\quad L=\frac{c_0^2\rho_0^2(h_0(4\gamma-2)+(\gamma+1)\rho_0c_0^2)}{4(2h_0+c_0^2\rho_0)^2}$$
$$N=\frac{h_0\rho_0(6h_0+(5-\gamma)c_0^2\rho_0}{2(2h_0+c_0^2\rho_0)^2} \tag{22}$$

These constants coincide with the well-known nonlinear corrections which make the Riemann wave isentropic [10]. In an unmagnetized ideal gas, $K=\frac{(3-\gamma)\rho_0}{4c_0^2}$, $L=\frac{\gamma+1}{4}\rho_0$, and $N=0$. An equation governing the velocity in the first magnetoacoustic planar wave which propagates in the positive direction of axis x, takes the form:

$$\frac{\partial v_1}{\partial t}+c_0\frac{\partial v_1}{\partial x}+\varepsilon_m v_1\frac{\partial v_1}{\partial x}+\frac{h_0}{\beta\rho_0}v_1(x,t)-$$
$$\frac{c_0h_0}{\beta^2\rho_0}\int\limits_{-\infty}^{x}\exp(-(x-x')c_0/\beta)v_1(x',t)dx'=0 \tag{23}$$

where

$$\varepsilon_m=\frac{6h_0+(\gamma+1)c_0^2\rho_0}{4h_0+2c_0^2\rho_0} \tag{24}$$

Equation (23) coincides with the Earnshaw equation when h_0 tends to zero and hence ε_m tends to $\varepsilon=\frac{\gamma+1}{2}$ [10]. It describes nonlinear distortions of a wave and also dispersion and attenuation associating with the finite electrical conductivity of plasma. Equation (23) may be simplified in the case of the low-frequency sound, $\beta k\ll c_0$:

$$\frac{\partial v_1}{\partial t}+c_{m,0}\frac{\partial v_1}{\partial x}+\varepsilon_m v_1\frac{\partial v_1}{\partial x}=0 \tag{25}$$

and in the case of the high-frequency sound, $\beta k\gg c_0$ [15]:

$$\frac{\partial v_1}{\partial t}+c_0\frac{\partial v_1}{\partial x}+\varepsilon_m v_1\frac{\partial v_1}{\partial x}+\frac{h_0}{\beta\rho_0}v_1=0 \tag{26}$$

The low-frequency sound does not experience attenuation in the leading order. Equation (25) may be solved by the method of characteristics. Equation (26) readily rearranges in the new variables

$$\bar v_1=\exp\left(\frac{h_0x}{\beta\rho_0c_0}\right)v_1,\quad X=\frac{\beta\rho_0c_0}{h_0}\left(1-\exp\left(-\frac{h_0x}{\beta\rho_0c_0}\right)\right),\quad \tau=t-x/c_0 \tag{27}$$

into the following leading-order equation

$$\frac{\partial\bar v_1}{\partial X}-\frac{\varepsilon_m}{c_0^2}v_1\frac{\partial\bar v_1}{\partial\tau}=0 \tag{28}$$

which in turn may be solved by the method of characteristics.

4. Magnetoacoustic heating

The first magnetosonic mode, which is an analogue of the Riemann wave in a gas with electrical conductivity, is represented in the leading order by the sum

$\psi_1+\psi_{nl,1}$. The projecting row P_3 points a way of linear combining of Equations (8) in order to eliminate foreign terms in the linear part of the equation which describes dynamics of ρ_3'. The nonlinear terms originating from the finite electrical conductivity in $\psi_{nl,1}$ and P_3, form the "magnetoacoustic force" of heating. The application of P_3 results in an equation which governs an excess density in the entropy mode:

$$\frac{\partial\rho_3'}{\partial t}=F_{m,h}= \frac{(\gamma-1)h_0}{c_0^3\beta^2}\left(c_0(\beta\partial v_1/\partial x-c_0v_1)\int\limits_{-\infty}^{x}\exp(-(x-x')c_0/\beta)v_1(x',t)dx'+\beta v_1(c_0v_1-2\beta\partial v_1/\partial x)\right) \tag{29}$$

Equation (29) may be simplified in the case of low-frequency magnetoacoustic perturbations, $\beta k\ll c_0$,

$$\frac{\partial\rho_3'}{\partial t}=\frac{(\gamma-1)h_0}{c_0^3}v_1(x,t)\frac{\partial v_1(x,t)}{\partial x} \tag{30}$$

and in the case of high-frequency magnetoacoustic perturbations, $\beta k\gg c_0$ [15]:

$$\frac{\partial\rho_3'}{\partial t}=2\frac{(\gamma-1)h_0}{c_0^3}v_1(x,t)\frac{\partial v_1(x,t)}{\partial x} \tag{31}$$

The general conclusion is that the periodic sound is not effective in producing heating in both the limits. In the leading order, the average over the sound period of the magnetoacoustic force of heating equals approximately zero. At least, the harmonic sound is not effective in producing heating at all. An example of a waveform

$$v_1(x,t)=V_0\exp\big(k(x-c_0t)\big) \tag{32}$$

represents a harmonic wave with wavelenght $2\pi/k$. In view of the complexity of establishing solution to Equation (23), we make use of the solution of the linear wave equations without dispersion exemplified by Equation (32). Equation (29) yields the magnetoacoustic force of heating

$$F_{m,h}=-V_0^2\frac{(\gamma-1)h_0\beta k^2}{c_0^3(c_0^2+\beta^2k^2)}\left(c_0\cos(2k(x-c_0t))+\beta k\sin(2k(x-c_0t))\right) \tag{33}$$

which is zero on average.

5. Acoustic streaming in a two-dimensional flow

In this section, we consider velocity in the plane (x,y), that is, $\vec{v}=(v_x(x,y,t),\, v_y(x,y,t),\, 0)$ perpendicular to the magnetic field

$\vec{H}=(0,0,H_z(x,y,t))$. The momentum equation consists of two compounds which take the leading-order forms:

$$\begin{aligned}\frac{\partial v_x}{\partial t}+\frac{1}{\rho_0}\frac{\partial(p'+h')}{\partial x}&=-(\vec{v}\cdot\vec{\nabla})v_x+\frac{\rho'}{\rho_0^2}\frac{\partial(p'+h')}{\partial x}\\ \frac{\partial v_y}{\partial t}+\frac{1}{\rho_0}\frac{\partial(p'+h')}{\partial y}&=-(\vec{v}\cdot\vec{\nabla})v_y+\frac{\rho'}{\rho_0^2}\frac{\partial(p'+h')}{\partial y}\end{aligned} \tag{34}$$

The most important case is a weakly diffracting magnetoacoustic beam which propagates, for definiteness, in the positive direction of axis x. A small parameter k_y/k_x measures the ratio of characteristic scales of perturbations in the longitudinal and transverse directions, so that we make use of the leading-order series

$$\sqrt{k_x^2+k_y^2}\approx k_x\left(1+\frac{k_y^2}{2k_x^2}\right) \tag{35}$$

The new kind of fluid motion appears in the two-dimensional flow, ordered as fifth. It is stationary in the linear flow. It reflects the existence of an incompressible rotational flow of a gas with a velocity the divergence of which is zero, $\vec{\nabla}\cdot\vec{v}_5=0$. The solenoidal velocity may be decomposed from the total velocity by applying the operator $P_{vor,\vec{v}}$ at the vector of the overall velocity:

$$P_{vor,\vec{v}}\vec{v}=\Delta^{-1}\begin{pmatrix}\frac{\partial^2}{\partial y^2} & -\frac{\partial^2}{\partial x\partial y}\\ -\frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial x^2}\end{pmatrix}\begin{pmatrix}\sum_{i=1}^5 v_{x,i}\\ \sum_{i=1}^5 v_{y,i}\end{pmatrix}=\begin{pmatrix}v_{x,5}\\ v_{y,5}\end{pmatrix} \tag{36}$$

Applying $P_{vort,\vec{v}}$ at the momentum equation Equation (34) and considering the dominant rightwards progressive mode, one arrives at the equation governing the magnetoacoustic streaming [33]:

$$\frac{\partial\vec{v}_{vort}}{\partial t}=-\frac{1}{\rho_0}P_{vort,\vec{v}}\left(\rho_1'\frac{\partial\vec{v}_1}{\partial t}\right) \tag{37}$$

An equivalent leading-order form of Equation (34) in terms of vorticity $\overrightarrow{\Omega}=\overrightarrow{\nabla}\times\vec{v}_{vort}$ sounds:

$$\begin{aligned}\frac{\partial\overrightarrow{\Omega}}{\partial t}&=\frac{1}{\rho_0}\overrightarrow{\nabla}\times\left(-\rho_1\frac{\partial}{\partial t}\vec{v}_1\right)=\\ &\frac{2c_0h_0}{\beta^2\rho_0^2}\overrightarrow{\nabla}\rho_1\times\left(\int\limits_{-\infty}^{x}\exp\left(-\frac{c_0(x-x')}{\beta}\right)\vec{v}_1dx'-\frac{\beta}{c_0}\vec{v}_1\right)\end{aligned} \tag{38}$$

The leading-order average form of Equation (38) in the case of the periodic magnetoacoustic wave, may be expressed in terms of magnetoacoustic pressure as

$$\overline{\frac{\partial v_{x,vort}}{\partial t}}=F_{m,s}=\frac{2h_0}{\beta^2\rho_0^3c_0^3}\overline{p_1'\left(\frac{\beta}{c_0}p_1'-\int\limits_{-\infty}^{x}\exp\left(-\frac{c_0(x-x')}{\beta}\right)p_1'dx'\right)} \tag{39}$$

The upper line designates the average over the sound period. The limits of large scale and small-scale magnetoacoustic perturbations are readily traced. In the case $\beta k \ll c_0$,

$$F_{m,s} = \frac{2h_0}{\rho_0^3 c_0^5} \overline{p_1' \left(\frac{\partial p_1'}{\partial x} \right)} \tag{40}$$

and in the case $\beta k \gg c_0$,

$$F_{m,s} = \frac{2h_0}{\beta \rho_0^3 c_0^4} \overline{p_1^2} \tag{41}$$

The low-frequency sound is not effective in the excitation of magnetoacoustic streaming. An example of a periodic waveform with a characteristic mediate wavelength of the order of β/c_0 represents the magnetoacoustic pressure

$$p_1'(x,t) = P_0 \sin((x - c_0 t)c_0/\beta) \tag{42}$$

It yields the magnetoacoustic force of streaming

$$F_{m,s} = \frac{h_0 P_0^2}{2\beta \rho_0^3 c_0^4} \tag{43}$$

which makes the longitudinal velocity of streaming (which coincides with the direction of propagation of sound) to increase with time.

6. Concluding Remarks

The equations which govern the magnetoacoustic wave and magnetoacoustic heating and streaming, are derived in this study. The main results are Equations (23), (29), (38), (39). The unperturbed magnetic pressure must satisfy Equation (15). Equation (23) which describe the dynamics of sound, accounts for nonlinearity, frequency-dependent dispersion and attenuation due to finite electrical conductivity. Equations (29), (38) determine instantaneous excitation of the secondary modes in the field of the intense sound, they are valid for the periodic and aperiodic sound. They describe the dynamics of magnetoacoustic heating and streaming independently of the spectrum of the magnetoacoustic wave.

The general conclusion is that finite electrical conductivity alone does not lead to noticeable acoustic heating for all frequencies of the magnetoacoustic wave, at least for the periodic magnetoacoustic perturbations. This concerns the mediate characteristic scale of perturbations of the order of β/c_0 which ensures the strongest attenuation at the wavelength, and also small-scale and large-scale limits. The acoustic streaming, in turn, is effective when induced by the high-frequency periodic sound, as well by the sound of mediate frequencies.

The influence of the Newtonian attenuation due to the shear and bulk viscosity of a gas as well as the thermal conduction of plasma are not taken into account in this study. They might be of importance. The influence of the Newtonian attenuation and the thermal conduction are well understood with regard to sound propagation and acoustic heating and streaming. Viscosity and thermal conduction of plasma should be considered as functions of temperature [34–36]. They may be considered independently of the effects which originate from the electrical conductivity of plasma. The acoustic forces of heating and streaming in a Newtonian fluid which are caused by the periodic sound, are proportional to the summary attenuation. Acoustic heating is governed by the non-uniform diffusivity equation in thermoconducting plasma. Acoustic streaming is also governed by the non-uniform diffusivity equation with a coefficient of diffusivity proportional to the shear viscosity.

The author applies the method of projecting an initial system of conservation equations into systems of coupling equations of interacting modes. It allows deriving the dynamic equations for the secondary modes which are excited in the field of the dominant wave mode independently of its spectrum and periodicity. Nonlinear effects of the aperiodic sound in a Newtonian fluid have been discussed in a number of papers by the author and co-authors [12, 33, 16]. Equations (29), (38) are not averaged over the sound period, they make use of an instantaneous magnetoacoustic source. This allows following the development of the secondary perturbations in detail. Magnetoacoustic perturbations may be periodic or not.

The Alfvén wave is represented by the fourth root of the dispersion equation in Equations (14). The projector P_4 which distinguishes the perturbation of the specific magnetic pressure h_4',

$$P_4(\rho' \quad v \quad p' \quad h')^T = h_4 \tag{44}$$

takes the form

$$P_4\left(0 \quad -\frac{2i\beta h_0 k}{c_0^2+\beta^2k^2} \quad -\frac{2h_0}{(c_0^2+\beta^2k^2)\rho_0} \quad 1-\frac{2(c_0^2-\beta^2k^2)h_0}{(c_0^2+\beta^2k^2)^2\rho_0}\right) \tag{45}$$

The dynamic equation for h_4' depends on the spectrum of sound. If $\beta k \ll c_0$, it takes the form

$$\frac{\partial h_4'}{\partial t} = \frac{\beta h_0}{c_0^2}\left((7-\gamma)\left(\frac{\partial v_1(x,t)}{\partial x}\right)^2 + (5-\gamma)v_1\frac{\partial^2 v_1(x,t)}{\partial x^2}\right) \tag{46}$$

This makes the harmonic sound

$$V = V_0 \sin(\omega(t - x/c_{m,0})) \tag{47}$$

effective in producing a positive perturbation in the magnetic pressure corresponding to the fourth mode:

$$\overline{\frac{\partial h_4'}{\partial t}} = \frac{\beta\omega^2 h_0 V_0^2}{c_0^4} \tag{48}$$

The case $\beta k \gg c_0$ gives

$$\frac{\partial h_4'}{\partial t} = \frac{\beta h_0(5-\gamma)}{c_0^2}\left(\left(\frac{\partial v_1(x,t)}{\partial x}\right)^2 + v_1\frac{\partial^2 v_1(x,t)}{\partial x^2}\right) \tag{49}$$

and, hence, the generation of magnetic perturbations by the periodic sound is insignificant. The same conclusion can be drawn for the periodic sound of medium wavelengths $k \approx c_0/\beta$.

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